#0 Mu Bowl
MAΘ National Convention 2018

$$A=\lim_{x\to 1}\left(\frac{2x^2-3x+3}{4x^2-2x-1}\right)$$

$$B=\lim_{x\to 2}\left(\frac{x^2+x-6}{2x^2-3x-2}\right)$$

Evaluate: $A+B$

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Evaluate: $A+B$

#1 Mu Bowl
MAΘ National Convention 2018

$A = \lim_{x \to 0} \left(\frac{\sin x}{\tan x} \right)$

$B = \lim_{x \to 1^-} \left(\frac{\arcsin x}{\arctan x} \right)$

Evaluate: $A + B$

#1 Mu Bowl
MAΘ National Convention 2018

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Evaluate: $A + B$

#2 Mu Bowl
MAΘ National Convention 2018

The tangent to $y = e^x$ at the point (a, e^a) has slope $a+1$. The *y*-intercept of this tangent is $(0, b)$, the *x*-intercept of this tangent is $(c, 0)$, and the slope of a line perpendicular to this tangent has slope *d*. Find the value of $|a|+|b|+|c|+|d|$.

#2 Mu Bowl
MAΘ National Convention 2018

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#3 Mu Bowl
MAΘ National Convention 2018

Consider the function $f(x) = x^3 + 6x^2 - 36x + 40$. Find all values of *x*, written in interval notation, such that *f* is simultaneously positive, decreasing, and concave downward.

#3 Mu Bowl
MAΘ National Convention 2018

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#4 Mu Bowl
MAΘ National Convention 2018

A rectangle is to be drawn so that two of its vertices lie on the *x*-axis while the other two vertices are above the *x*-axis on the parabola $y = 36 - 2x^2$. If the side length of the rectangle on the *x*-axis must have length from 2 to 8, inclusive, find the value of $A - B$, where *A* is the maximum area enclosed by such a rectangle and *B* is the minimum area enclosed by such a rectangle.

#4 Mu Bowl
MAΘ National Convention 2018

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#5 Mu Bowl
MAΘ National Convention 2018

Two people start walking from the same point at the same time. One walks east at 3 miles per hour while the other walks northeast at 2 miles per hour (thus the angle between their paths is 45°). If the rate of change of the distance between the people, in miles per hour, at the moment when they have been walking for 15 minutes is written in the form $\sqrt{A-B\sqrt{C}}$, where *A*, *B*, and *C* are positive integers, and *C* is not divisible by the square of any integer greater than 1, find the value of $A \cdot B + C$. (HINT: You will need to use the Law of Cosines.)

#5 Mu Bowl
MAΘ National Convention 2018

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#6 Mu Bowl
MAΘ National Convention 2018

$$A=\lim_{n\to\infty}\sum_{i=1}^{n}\left(\left(1+\frac{i}{n}\right)^2\cdot\frac{1}{n}\right)$$

$$B=\lim_{n\to\infty}\sum_{i=1}^{n}\left(\sin\left(\frac{\pi}{4}+\frac{\pi i}{4n}\right)\cdot\frac{\pi}{4n}\right)$$

$$C=\lim_{n\to\infty}\sum_{i=1}^{n}\left(\frac{2}{n}\cdot\ln\left(1+\frac{2i}{n}\right)\right)$$

If $\lfloor x \rfloor$ represents the greatest integer less than or equal to *x*, and if $\lceil x \rceil$ represents the least integer greater than or equal to *x*, find the value of $\lfloor A \rfloor+\lceil B \rceil+\lceil C \rceil$.

#6 Mu Bowl
MAΘ National Convention 2018

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#7 Mu Bowl
MAΘ National Convention 2018

$A=\int_0^1 \frac{1}{x^2+1}dx$

$B=\int_0^1 \frac{x}{x^2+1}dx$

$C=\int_0^1 \frac{x}{x^4+1}dx$

$D=\int_0^1 \frac{x^3}{x^4+1}dx$

List the letters *A*, *B*, *C*, and *D* in increasing numerical order.

#7 Mu Bowl
MAΘ National Convention 2018

$A=\int_0^1 \frac{1}{x^2+1}dx$

$B=\int_0^1 \frac{x}{x^2+1}dx$

$C=\int_0^1 \frac{x}{x^4+1}dx$

$D=\int_0^1 \frac{x^3}{x^4+1}dx$

List the letters *A*, *B*, *C*, and *D* in increasing numerical order.

#8 Mu Bowl
MAΘ National Convention 2018

$$A = \int_0^{4\sqrt{6}} \sin\left(\arctan\frac{x}{2}\right)dx$$

$$B\sqrt{C} + \ln\left(D + \sqrt{E}\right) = \int_0^4 \sqrt{1 + \frac{1}{2x}}\ dx$$

Given that *B*, *C*, *D*, and *E* are positive integers such that *C* is not divisible by the square of any integer greater than 1, find the value of $\frac{B \cdot C + D \cdot E}{A}$.

#8 Mu Bowl
MAΘ National Convention 2018

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Given that *B*, *C*, *D*, and *E* are positive integers such that *C* is not divisible by the square of any integer greater than 1, find the value of $\frac{B \cdot C + D \cdot E}{A}$.

#9 Mu Bowl
MAΘ National Convention 2018

Let $f(x)=\left(1+\frac{1}{x}\right)^x$ and $g(x)=(\log 5)10^{\log_5 x}$.

$A=\lim_{x\to\infty} f(x)$

$B=\lim_{x\to\infty} f'(x)$

$g'(x)=C^{\log_5 x}$

Find the value of $C\cdot(A+B)$.

#9 Mu Bowl
MAΘ National Convention 2018

Let $f(x)=\left(1+\frac{1}{x}\right)^x$ and $g(x)=(\log 5)10^{\log_5 x}$.

$A=\lim_{x\to\infty} f(x)$

$B=\lim_{x\to\infty} f'(x)$

$g'(x)=C^{\log_5 x}$

Find the value of $C\cdot(A+B)$.

#10 Mu Bowl
MAΘ National Convention 2018

A sequence $\left\{a_n\right\}_{n=1}^{\infty}$ is defined recursively in the following way: $a_1 = 5$, and for integers $n \geq 2$, $a_n = 3a_{n-1} - 2$. If *A* is added to each term in this sequence, the sequence becomes geometric with common ratio *B.*

A sequence $\left\{b_n\right\}_{n=1}^{\infty}$ is defined recursively in the following way: $b_1 = 5,\ b_2 = 13$, and for integers $n \geq 3$, $b_n = 3b_{n-1} - 2b_{n-2}$. If *C* is added to each term in this sequence, the sequence becomes geometric with common ratio *D*.

Find the value of $\left(A+B\right)^{C+D}$.

#10 Mu Bowl
MAΘ National Convention 2018

A sequence $\left\{a_n\right\}_{n=1}^{\infty}$ is defined recursively in the following way: $a_1 = 5$, and for integers $n \geq 2$, $a_n = 3a_{n-1} - 2$. If *A* is added to each term in this sequence, the sequence becomes geometric with common ratio *B.*

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Find the value of $\left(A+B\right)^{C+D}$.

#11 Mu Bowl
MAΘ National Convention 2018

The function $f(x)=x^4+4x^3-48x^2+Ax+B$ has two inflection points: one at $x=C$ and one at $x=D$. If these two inflection points have the same *y*-value *E*, find the value of $A+B-C\cdot D-E$.

#11 Mu Bowl
MAΘ National Convention 2018

The function $f(x)=x^4+4x^3-48x^2+Ax+B$ has two inflection points: one at $x=C$ and one at $x=D$. If these two inflection points have the same *y*-value *E*, find the value of $A+B-C\cdot D-E$.

#12 Mu Bowl
MAΘ National Convention 2018

Evaluate: $\sum_{n=1}^{\infty}\left(\arctan\left(\frac{1}{n^2+n+1}\right)\right)$

#12 Mu Bowl
MAΘ National Convention 2018

Evaluate: $\sum_{n=1}^{\infty}\left(\arctan\left(\frac{1}{n^2+n+1}\right)\right)$

#13 Mu Bowl
MAΘ National Convention 2018

Find the area enclosed by the graph of the polar equation $r + \frac{163}{r} = 16\cos\theta + 20\sin\theta$.

#13 Mu Bowl
MAΘ National Convention 2018

Find the area enclosed by the graph of the polar equation $r + \frac{163}{r} = 16\cos\theta + 20\sin\theta$.

#14 Mu Bowl
MAΘ National Convention 2018

$A=\int_{\frac{\pi}{6}}^{\frac{\pi}{2}}\left(\csc x - x\csc x\cot x\right)dx$

Using the second-degree Maclaurin polynomial for $\cos\left(\sqrt{x}\right)$, let B be the approximation of $\int_0^2 \cos\left(\sqrt{x}\right)dx$.

Find the value of $\frac{A}{B}$.

#14 Mu Bowl
MAΘ National Convention 2018

$A=\int_{\frac{\pi}{6}}^{\frac{\pi}{2}}\left(\csc x - x\csc x\cot x\right)dx$

Using the second-degree Maclaurin polynomial for $\cos\left(\sqrt{x}\right)$, let B be the approximation of $\int_0^2 \cos\left(\sqrt{x}\right)dx$.

Find the value of $\frac{A}{B}$.