1. Simplify the Boolean expression

$$x \cdot y' + y$$

A. $x \cdot y' + y$ B. x C. $x + y$ D. y E. NOTA

2. Consider

$$x^2 = x + x + x \ldots + x + x \ (1)$$

where there are x x's on the right-hand side.

If we take a derivative on both sides, we get

$$2x\ dx = (1 + 1 + 1 \ldots + 1 + 1)\ dx \ (2)$$

where there are x 1's on the right-hand side.

Simplifying, we get

$$2x\ dx = x\ dx \ (3)$$

which means

$$2 = 1 \ (4)$$

Which equation is the first to be incorrect?

A. (1) B. (2) C. (3) D. (4) E. NOTA

3. What theorem would you use to prove that there is at least one 0 in the function f between 0 and 1 given that $f(x) = x^3 + x^2 + x - 1$?

A. Extreme-Value Theorem	B. Mean Value Theorem	C. Rolle's Theorem	D. Intermediate Value Theorem	E. NOTA

4. What theorem would you use to show that there exists a value of x between 0 and 1 such that $f'(x) = 3$ given that $f(x) = x^3 + x^2 + x - 1$?

A. Extreme-Value Theorem	B. Mean Value Theorem	C. Rolle's Theorem	D. Intermediate Value Theorem	E. NOTA

5. Which Boolean function is represented by the following Karnaugh Map?

ab \ cd	00	01	11	10
00	0	1	0	1
01	1	0	1	0
11	0	1	0	1
10	1	0	1	0

A. $a + b + c + d$ B. $a + b + d$ C. $a \oplus b \oplus d$ D. $a \oplus b \oplus c \oplus d$ E. NOTA

6. Consider the following incorrect proof using $\int \frac{1}{x} dx$:

By using integration by parts with $u = \frac{1}{x}$ and $v = x$, $\int u dv = uv - \int v du$

$$\int \frac{1}{x} dx = \frac{1}{x} \cdot x - \int x \, d\frac{1}{x} \ (1)$$

$$\int \frac{1}{x} dx = \frac{1}{x} \cdot x + \int x \cdot \frac{1}{x^2} dx \ (2)$$

$$\int \frac{1}{x} dx = 1 + \int \frac{1}{x} dx \ (3)$$

$$0 = 1 \ (4)$$

Which step is the first to have an incorrect statement?

A. (1) B. (2) C. (3) D. (4) E. NOTA

7. How many elements are in the largest set of the integers from 1 to 100, inclusive that has no two numbers x and y such that x = 3y?

A. 33 B. 34 C. 66 D. 67 E. NOTA

8. I have two children. The younger one is a boy who was born last Thursday. What is the probability that I have two boys?

A. $\frac{1}{4}$ B. $\frac{1}{3}$ C. $\frac{1}{2}$ D. $\frac{2}{3}$ E. NOTA

9. Consider the following proof:

$$e^x = e^{i2\pi \frac{x}{i2\pi}} \ (1)$$

$$e^{i2\pi \frac{x}{i2\pi}} = \left(e^{i2\pi}\right)^{\frac{x}{i2\pi}} \ (2)$$

$$\left(e^{i2\pi}\right)^{\frac{x}{i2\pi}} = 1^{\frac{x}{i2\pi}} \ (3)$$

$$e^x = 1 \ (4)$$

Which step is incorrect?

A. (1) B. (2) C. (3) D. (4) E. NOTA

10. I asked 120 people to guess which permutation of 12345 I had thought of. Everyone guessed a valid permutation. Of them, 10 people guessed a permutation which was different from mine in all 5 places (for example, if my permutation was 54321 and they guessed 12453, then they would fall into this category). 45 people guessed a permutation which was different from mine in 4 places. 45 people guessed a permutation which was different from mine in 3 places. 15 people guessed a permutation which was different from mine in 2 places. Suppose M and N represent the number of people who guessed my number and who were wrong in exactly one place. What is M – N?

A. 5 B. 3 C. 1 D. Not Enough Information E. NOTA

11. From the last problem, how many different ways were there to guess a permutation which was different in all 5 places?

A. 119 B. 60 C. 24 D. 10 E. NOTA

12. Suppose 100 dogs and 100 cats are all racing. Each of them races independently of each other and the result is completely random. If you were to bet on what place the highest-ranking dog would place based on the highest probability, which place should you bet?

A. 1st B. 25th C. 50th D. 100th E. NOTA

13. Now suppose that we only race 5 dogs and 5 cats. You are placing wagers of \$10, \$8, \$6, and \$4 on the rank of the highest-ranking dog. What ranks should you place the bets on, based on the highest probabilities? (Answers are ordered with the first rank receiving \$10, second \$8, third \$6, and fourth \$4).

A. 1,2,3,4 B. 3,2,4,1 C. 2,3,1,4 D. 1,4,2,3 E. NOTA

14. Consider the following probability distribution function:

$f(x) = Cx^{-a}$ for $x \geq 1$ and 0 elsewhere.

Given that $a > 1$, what is the value of C?

A. a B. $\frac{1}{a-1}$ C. $a - 1$ D. Not enough Information E. NOTA

15. Jen wants to make a binary adder using simple gates (AND, OR, NOT, XOR). She takes three bits, which we will call X, Y, and Z and wants to add them. She realizes that the output might be 2 bits. Which of the following Boolean expressions below represents how she should do this? (S is the lower bit, C is the upper bit).

A.	B.	C.	D.	E. NOTA
$S = X \oplus Y \oplus Z$ $C = XYZ$	$S = X \oplus Y \oplus Z$ $C = XY + XZ + YZ$	$S = XY + XZ + YZ$ $C = X \oplus Y \oplus Z$	$S = XYZ$ $C = X \oplus Y \oplus Z$	