This test consists of five relays of six questions each. "$TAFTPQITR$" stands for "The Answer From The Previous Question In The Relay", so if question 3 in a relay references $TAFTPQITR$, that is the answer from question 2 in that relay.

The answers to all parts of all Relays are integers.

Relay 1

(1) Evaluate: $\frac{d}{dx}\left[14\pi\sin\left(\sqrt{x}\right)\right]_{x=\pi^2}$

(2) The line tangent to $f(x) = e^{ax^2}$ at $x = 2$ has the equation $y = mx + (TAFTPQITR)e^{4a}$.

Find a.

(3) If $f(x) = \frac{x}{TAFTPQITR + \frac{x}{TAFTPQITR + \frac{x}{TAFTPQITR + \frac{x}{\cdots}}}}$ then find $25f'(6)$.

(4) Let $f(x) = |3x^4 - (TAFTPQITR)x^3 - 15x^2 + 25x|$. Let M be the number of relative maxima of $f(x)$, N be the number of relative minima of $f(x)$, and P be the number of points of inflection of $f(x)$. Find $M \cdot N \cdot P$.

(5) Find the number of zeroes at the end of $\frac{d^{TAFTPQITR}}{dx^{TAFTPQITR}}[\cos(x^2)]_{x=0}$.

(6) Let y satisfy the differential equation $y' = xy + x - y - 1$, and also let $y = TAFTPQITR$ when $x = 0$. When $x = 4$, $y = Ae^B + C$, with A, B, and C integers. What is $A \cdot B \cdot C$?

Relay 2

(1) Find the maximum area enclosed by a rectangle with a perimeter of 12 units.

(2) A tank in the shape of an inverted right circular cone starts out full of water. Water begins leaking out of the bottom of the tank at a rate of $TAFTPQITR\pi$ units cubed per second. If the tank is 100 units tall and 12 units wide at its widest point, what is the rate of change of the height of the water (in units per second) when the height is 50 units?

(3) Let D be the shortest distance between the curve $y = x^{\frac{3}{2}} - 1$ and the point $\left(\frac{1}{2}, TAFTPQITR\right)$. Find $\frac{7}{D^2}$.

(4) An ellipse centered at the origin has a semi-major axis of length $\sqrt{TAFTPQITR}$ concurrent with the x-axis and a semi-minor axis with length 6 concurrent with the y-axis. There are two lines tangent to this ellipse that go through the point $(12,0)$. Find the number of degrees in the angle between these two lines.

(5) A rectangle has one vertex at the origin, one on the x-axis, one on the y-axis, and one on the curve $y = 30 * TAFTPQITR - x^2$. Find the maximum possible area enclosed by this rectangle.

(6) A full 1,000 gallon tank initially contains $TAFTPQITR$ mg of salt. Water begins flowing out of the tank at 10 gallons per minute, and at the same time fresh water is flowing into the tank at the same rate. The volume of liquid in the tank remains constant, and the tank is perfectly mixed at all times. How much salt, in mg, is in the tank after $200\ln(3)$ minutes?

Relay 3

(1) Evaluate: $\int_1^2 (12x^3 - 6x^2 + 8x - 3)dx$

(2) If $\int_0^1 \frac{x^2+x+1}{2x^3+3x^2+6x+TAFTPQITR+4}dx = \frac{1}{A}\ln\left(\frac{B}{C}\right)$, where A, B, and C are positive integers and B and C are coprime, then find $A + B - C$.

(3) Evaluate: $3\int_0^{\frac{\pi}{2}} \sin^{TAFTPQITR-4}(x)\,dx$

(4) Evaluate: $\int_0^{TAFTPQITR} \frac{x^{2018}}{x^{2018}+(TAFTPQITR-x)^{2018}}dx$

(5) $\int_0^{TAFTPQITR} \frac{x^3}{\sqrt{x^4+1}}dx$ is a root of $y = 4x^2 + 4x + K$. Find K.

(6) Let $I(b) = \int_1^{\infty} e^{bx}dx$. Find $180e * I(TAFTPQITR)$.

Relay 4

(1) Find the area of the finite region bounded by the curves $f(x) = 20x^3$ and $g(x) = 5x^4$.

(2) The volume obtained when the finite region bounded by $f(x) = \sqrt{x}$ and $g(x) = \frac{1}{2}\sqrt{TAFTPQITR}x^2$ is revolved around the x-axis is $\frac{A}{B}\pi$, where A and B are coprime positive integers. Find the <u>sum of the digits</u> of $A + B$.

(3) The volume when the finite region bounded by the x-axis, the line $x = 1$, and the curve $y = x^{TAFTPQITR}$ is revolved around the line $x = 1$ is $\frac{2\pi}{B}$. Find B.

(4) The finite region between the x-axis and $f(x) = (TAFTPQITR)x - x^2$ is divided into two regions of equal area by the line $y = mx$. Find $\frac{m}{\left(1-\frac{1}{\sqrt[3]{2}}\right)}$.

(5) The area between $f(x) = 4x^3 + 3x^2 + 2x + TAFTPQITR$ and the x-axis between $x = 1$ and $x = 2$ is approximated using Simpson's Rule with $TAFTPQITR$ intervals of equal width. Find the resulting value.

(6) The region defined by $x^2 - 4x + y^2 + 2y \leq 11$ is revolved around the line $3x + 4y + TAFTPQITR = 0$. If the resulting volume is $\frac{32\pi^2}{5}K$, find K.

Relay 5

(1) Evaluate: $\sum_{n=1}^{\infty} \frac{4n}{3^n}$

(2) Evaluate: $\lim_{x\to\infty}\left((x^{TAFTPQITR} + 8x^{TAFTPQITR-1})^{\frac{1}{TAFTPQITR}} - (x^{TAFTPQITR} + 2x^{TAFTPQITR-1})^{\frac{1}{TAFTPQITR}}\right)$

(3) Find $\frac{d}{dx}\int_{x^2}^{x^4} \frac{\sin\left(\frac{\pi}{8}t\right)}{\sqrt{t}}\,dt$ evaluated at $x = TAFTPQITR$.

(4) Evaluate: $\lim_{n\to\infty} \frac{12}{\pi}\sum_{k=1}^{n} \frac{1}{\sqrt{(TAFTPQITR)^2 n^2 - k^2}}$

(5) Evaluate: $\frac{81}{e^3}\int_0^e x^{TAFTPQITR}\ln(x)\,dx$

(6) Let $P(x)$ be a polynomial of order $TAFTPQITR$. If $P(x) = q(x)P''(x)$ for some quadratic function $q(x)$, and if $P(x)$ has a root of multiplicity greater than one, then how many distinct roots does $P(x)$ have?