Relay 1	Relay 2	Relay 3	Relay 4	Relay 5
1. 3	**1.** 15	**1.** 7	**1.** 45	**1.** 60
2. 4	**2.** 17	**2.** 140	**2.** 7	**2.** 24
3. 22	**3.** 2	**3.** 5	**3.** 25	**3.** 2
4. 320	**4.** 1792	**4.** 395	**4.** 14	**4.** 3
5. 78	**5.** 4088	**5.** 189	**5.** 9	**5.** 9
6. 5	**6.** 30	**6.** 7	**6.** 20	**6.** 200

Relay 1

1. Only 3 regular polygons can tessellate a plane—triangle, square, and hexagon.
2. If the three plane are parallel, then four regions are created.
3. $y=-5x+b \rightarrow 2=-5(4)+b \rightarrow b=22$.
4. $4x^2+24x-17+5y^2-10y-22=0 \rightarrow 4x^2+24x+5y^2-10y=39$

   $4\left(x^2+6x+9\right)+5\left(y^2-2y+1\right)=39+36+5=80 \rightarrow \frac{(x+3)^2}{20}+\frac{(y-1)^2}{16}=1$

   Area $=\sqrt{20}\sqrt{16}\pi=\sqrt{320}\pi$.
5. $\left\lfloor \frac{320}{125} \right\rfloor+\left\lfloor \frac{320}{25} \right\rfloor+\left\lfloor \frac{320}{5} \right\rfloor=2+12+64=78$.
6. $78-6=72 \rightarrow \frac{360}{72}=5$.

Relay 2

1. Area $=\frac{1}{2}r^2\theta=\frac{1}{2}(144)\theta=6\pi \rightarrow \theta=\frac{\pi}{12}=15°$.
2. $\frac{15}{8}=1 \text{ R}7 \rightarrow 17$.
3. $17^4=83521$ or use mods.
4. $\frac{8!}{3!5!}(1)^3(2)^5=1792$.
5. $1792=2^8 7^1$. Sum of positive divisors: $(1+2+4+8+16+32+64+128+256)(1+7)=4088$.
6. Let t represent Sam's time. Then $84t+56(t-2)=4088 \rightarrow t=30$.

Relay 3

1. $(x-y)^2=x^2-2xy+y^2=39 \rightarrow x^2+y^2=39+2(-16)=7$.
2. $\frac{L}{M}=\frac{17}{23}=\frac{119}{161}$. 1/7 of 161 is 23. $\frac{119+23}{161-23}=\frac{142}{138}=\frac{71}{69}$. Sum is 140.
3. An integer with three divisors must be the square of a prime number. There are five of these in this interval: 4, 9, 25, 49, and 121.
4. $C=20$. $\sqrt{(400-9)(400-1)+16}=\sqrt{(395-4)(395+4)+16}=395$.
5. $395(\log 3)\approx 395(0.4771)\approx 188.46 \rightarrow 189$.

6. The perimeter of the equilateral triangle is $\sqrt{189}=3\sqrt{21}$, so each side has length $\sqrt{21}$. The triangle area is $\frac{s^2\sqrt{3}}{4}=\frac{21\sqrt{3}}{4}=\frac{h^2\sqrt{3}}{3}\rightarrow 63\sqrt{3}=4h^2\sqrt{3}\rightarrow h^2=\frac{63}{4}\rightarrow h=\frac{3\sqrt{7}}{2}$. The circumscribed circle has radius $\frac{2}{3}h$, which will be $\sqrt{7}$. The area of the circle will be 7π.

Relay 4

1. Let $\dfrac{x}{44+\dfrac{x}{44+\dfrac{x}{44+\text{O}}}}=c$, so that $\frac{x}{44+c}=c$. Cross multiplying, we get $c^2+44c-x=0$.

   $x=45$ is the smallest integer that produces integer values.
2. Let $M^2=B^2-45$. $B^2-M^2=45\rightarrow(B+M)(B-M)=45$. We now have three options for (B, M):

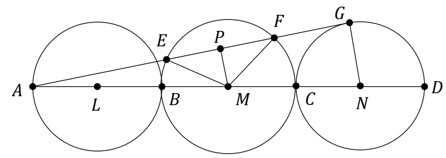
   $\begin{cases}B+M=45\\B-M=1\end{cases}\rightarrow(23, 22)$ $\begin{cases}B+M=15\\B-M=3\end{cases}\rightarrow(9, 6)$ $\begin{cases}B+M=9\\B-M=5\end{cases}\rightarrow(7, 2)$ The smallest possible B is 7.
3. $(x-7)^2-2(y-3)^2=8\rightarrow\frac{(x-7)^2}{8}-\frac{(y-3)^2}{4}=1$. The hyperbola opens horizontally so the directrices are vertical. Using $x=h\pm\frac{a^2}{c}$, we have $x=7\pm\frac{8}{2\sqrt{3}}\rightarrow x=\frac{21\pm4\sqrt{3}}{3}$. $21+4=25$.
4. Using a system of equations, we have one equation for monetary values and one equation for number of coins: $P+10D+25Q=200$ and $P+D+Q=50$. Subtracting, we get $9D+24Q=150$, or $3D+8Q=50$. The only feasible possibilities for (D, Q) are (6, 4) and (14, 1), so the maximum possible number of dimes is 14.
5. $f(n+1)=f(n)+\frac{1}{2}$, generating the arithmetic sequence 2, 2.5, 3, 3.5, ….

   $f(15)=f(1)+14(.5)=2+7=9$.
6. Using properties of logarithms we can simplify the expression to $40\log_9\frac{6x-5}{2x+1}$. This can be rewritten again as $40\log_9\left(\frac{-8}{2x+1}+3\right)$. As x approaches infinity, the expression approaches the value $40\log_9 3\rightarrow 40\left(\frac{1}{2}\right)=20$.

Relay 5

1. Let the fourth root be d. Since there is no x^3 term, the sum of the roots is 0, so $d=-6$. The value of a is the sum of the product of the roots taken two at a time, so $a=(1)(2)+(1)(3)+(1)(-6)+(2)(3)+(2)(-6)+(3)(-6)=-25$. The c value is the product of the roots, which is –36. $|a+c|-1\rightarrow 61-1=60$.

2. Construct radii from M to E and F and one perpendicular $\overline{EF}$ at P. By a property of circles, P will be the midpoint of $\overline{EF}$. Also construct a radius from N to G. We now have $\triangle APM \sim \triangle AGN$, so that $\frac{PM}{AM}=\frac{GN}{AN}\rightarrow\frac{PM}{45}=\frac{15}{75}=\frac{1}{5}\rightarrow PM=9$. $(PM)^2+(PF)^2=(FM)^2\rightarrow 81+(PF)^2=225\rightarrow PF=12$, so $EF=24$.



3. $B=24$, so $a=2, b=4$. $p*2=\frac{4^p}{2^2}=4\rightarrow 4^p=16\rightarrow p=2$.

4. The one-, two-, and three-digit integers beginning with three (and the total number of digits for each group) are 3 (1), 30-39 (20), and 300-399 (300), for a total of 321 digits. 2018–321=1697. The remaining integers will be four-digit integers. 1697 has a remainder of 1 when divided by four, so the 2018th digit will be a 3.

5. $\frac{3}{\log_{a^3}(ab)}+\frac{3}{\log_{b^3}(ab)}\rightarrow\frac{9}{\log_a(ab)}+\frac{9}{\log_b(ab)}\rightarrow\frac{9\log a}{\log(ab)}+\frac{9\log b}{\log(ab)}\rightarrow\frac{9\log(ab)}{\log(ab)}=9$.

6. The height of the shaded trapezoid is $\frac{1}{10}$ the height of the triangle, h. If the base of the triangle is b, then the trapezoid area, 38, is found by $\frac{1}{2}\left(\frac{1}{10}h\right)(b+0.9b)$. Simplifying we get the area of the triangle, $\frac{1}{2}bh=200$.