This test consists of five relays of six questions each. “TAFTPQITR” stands for “the answer from the previous question in this relay,” so if question 3 in a relay references TAFTPQITR, that is the answer from question 2 in that relay.

All answers on this test are integers.

RELAY 1

1. How many types of regular polygons can tile a plane?

2. Let $A = \text{TAFTPQITR}$. Find the minimum number of regions that $A$ distinct planes can separate three-dimensional space.

3. Let $B = \text{TAFTPQITR}$. Find the $y$-intercept ($y$-coordinate only) of the line parallel to $y = 9 - 5x$ that passes through $(B, 2)$.

4. Let $C = \text{TAFTPQITR}$. The area enclosed by $4x^2 + 24x - 17 + 5y^2 - 10y - C = 0$ is $\sqrt{D} \pi$. Find $D$.

5. Let $D = \text{TAFTPQITR}$. Find the number of consecutive terminating zeros in $D!$.

6. Let $E = \text{TAFTPQITR}$. A regular convex polygon has an exterior angle of measure $(E - 6)$ degrees. How many sides does the polygon have?

RELAY 2

1. The area of a sector of a circle of radius 12 is $6 \pi$. Find the degree measure of the arc of this sector.

2. Let $A = \text{TAFTPQITR}$. Find the base 8 representation of $A$. (Do not write the subscript base.)

4. Let $C = \text{TAFTPQITR}$. Find the coefficient of the sixth term in the expansion of $(x + C)^9$ when written in descending powers of $x$.

5. Let $D = \text{TAFTPQITR}$. Find the sum of the positive integral divisors of $D$.

6. Let $E = \text{TAFTPQITR}$. Sam and Ella are train conductors. Sam’s train, the *Ecoli Express*, travels at 84 mph and Ella’s train, the *Listeria Liner*, travels at 56 mph. If both trains leave from the same station heading in opposite directions on a straight track, how long, in hours, will Sam have to travel before the two trains are $E$ miles apart, if Ella’s train leaves 2 hours after Sam’s train?

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**RELAY 3**

1. If $(x - y)^2 = 39$ and $xy = -16$, what is the value of $x^2 + y^2$?

2. Let $A = \text{TAFTPQITR}$. The ratio of logarithm problems to matrix problems on a test is 17:23. If $1/A$ of the matrix problems are replaced with logarithm problems, the new ratio (when reduced) is $x:y$, where $x, y > 0$ are integers. Find the value of $x + y$.

3. Let $B = \text{TAFTPQITR}$. How many positive integers between 1 and $B$, inclusive, have exactly three divisors?

4. Let $C = 4(\text{TAFTPQITR})$. Evaluate $\sqrt{(C-3)(C-1)(C+1)(C+3)+(C-4)}$.

5. Let $D = \text{TAFTPQITR}$. Find the number of digits in the expansion of $3^D$.

6. Let $E = \text{TAFTPQITR}$. The perimeter of an equilateral triangle is $\sqrt{E}$. If the area enclosed by its circumscribed circle is $F\pi$, what is the value of $F$?
RELAY 4

1. Find the smallest positive integer value for $x$ such that \[ \frac{x}{44 + \frac{x}{44 + \frac{x}{44 + \ldots}}} \] is an integer.

2. Let $A = TAFTPQITR$. If $B$ is a positive integer and $B^2 - A$ is also a perfect square (integer), find the smallest possible value of $B$.

3. Let $B = TAFTPQITR$. The hyperbola $(x - B)^2 - 2(y - 3)^2 = 8$ has directrices, when simplified, \[ x = \frac{B \pm P \sqrt{Q}}{T}. \] Find the value of $B + P$.

4. Let $C = TAFTPQITR$. In Jack’s piggy bank, he has only pennies, dimes, and quarters. If he has $2C$ coins in the bank totaling $2.00, what is the greatest number of dimes that he can have?

5. Let $D = TAFTPQITR$. If \[ f(n+1) = \frac{2f(n)+1}{2} \] for $n \geq 1$ and $f(1) = 2$, find $f(D+1)$.

6. Let $E = TAFTPQITR$. If $x > 0$ and grows beyond all bounds, what value does the expression \[ 40 \log_b(6x - 5) - 40 \log_b(2x + 1) \] approach?

RELAY 5
1. If three of the roots of \( x^4 + ax^2 + bx + c = 0 \) are 1, 2, and 3, what is the value of \(|a+c| - 1\)?

2. Let \( Z = TAFTPQITR \). Points B and C lie on AD; and \( AB, BC, \) and \( CD \) are diameters of length \( Z/2 \) of circles L, M, and N, respectively. If \( AG \) is tangent to circle N at G and intersects circle M at E and F (per the diagram), what is the length of \( EF \)?

3. Let \( B = TAFTPQITR \). The tens digit of B is \( a \) and the units digit of B is \( b \). If \( x \cdot y = \frac{b^x}{a^y} \), find the value of \( p \) such that \( p \cdot a = b \).

4. Let \( C = TAFTPQITR + 1 \). All of the positive integers with an initial digit of \( C \) are written down in succession in increasing order. What is the 2018th digit thus written?

5. Let \( D = TAFTPQITR \). Find the value of \( \frac{D}{\log_b(ab)} + \frac{D}{\log_b(ab)} \).

6. Let \( E = TAFTPQITR \). \( E \) lines parallel to the base of a triangle divide the other sides into \( E + 1 \) congruent segments and the area into \( E + 1 \) distinct parts. If the area of the largest of these parts is 38, what is the area of the original triangle?