1. What is the remainder when 8,460,934,872 is divided by 9?

A. 0 B. 6 C. 7 D. 8 E. NOTA

2. Compute x given that x and y are real and $x + yi = \left(\sqrt{5} + 1 + i\left(\sqrt{10 - 2\sqrt{5}}\right)\right)^6$.

A. 4096 B. -4096 C. $1024\sqrt{5} + 1024$ D. $-1024 - 1024\sqrt{5}$ E. NOTA

3. Given that the shortest leg of a right triangle has length 103, which of the following could be values for the two remaining sides?

A. 5,152 & 5,154 B. 5,154 & 5,155 C. 5,302 & 5,304 D. 5,304 & 5,305 E. NOTA

4. Compute the sum of the coefficients of

$$(6x^3 + 4x^2y - 5xy^2 - 2y^3)^8$$

A. 65536 B. 6561 C. 256 D. 1 E. NOTA

5. Compute 1035^2.

A. 1,050,625 B. 1,060,925 C. 1,071,225 D. 1,081,625 E. NOTA

6. It is known that 405,224 is a perfect cube. What is its cube root?

A. 74 B. 78 C. 84 D. 88 E. NOTA

7. Convert 10111010110011_2 to hexadecimal.

A. CBD3 B. BAC3 C. 2EB3 D. 2FC3 E. NOTA

8. Let $A = \begin{bmatrix} 2 & 5 \\ 3 & 7 \end{bmatrix}, B = \begin{bmatrix} 4 & 7 \\ 3 & 5 \end{bmatrix}, C = \begin{bmatrix} 4 & 9 \\ 3 & 7 \end{bmatrix}$, and $D = \begin{bmatrix} 25 & 19 \\ 4 & 3 \end{bmatrix}$. Compute the determinant of $ADCB$.

A. -1 B. 0 C. 1 D. 2 E. NOTA

9. Compute $\sum_{n=1}^{5}(4n^3 + 6n^2)$

A. 343 B. 729 C. 900 D. 1230 E. NOTA

10. Elliot rolls a tetrahedral die, a hexahedral die, an octahedral die, a dodecahedral die, and an icosahedral die. If each die is fair and has numbers from 1 to the number of sides on the die on it, what is the expected value of the sum of all numbers showing on the top of each die?

A. 27.5 B. 25 C. 22.5 D. 20 E. NOTA

11. Given that $f(0) = -28$, $f(1) = -60, f(2) = -8, and\ f(3) = 134$, compute $f(4)$ if $f(x)$ is a monic cubic function.

A. 392 B. 382 C. 372 D. 362 E. NOTA

12. Given $x + \frac{1}{x} = 4$, compute $x^4 + \frac{1}{x^4}$.

A. 256 B. 250 C. 200 D. 194 E. NOTA

13. Write $\frac{2}{7*13*37}$ as a decimal.

A. $0.\overline{000594}$ B. $0.\overline{0005940}$ C. $0.0\overline{00594}$ D. $0.\overline{00059400}$ E. NOTA

For Problems 14-15: Imagine that you are inputting a 4-digit code into a lock.

14. How many codes have the property that each digit is strictly greater than the digit to its right?

A. 10 B. 210 C. 220 D. 5000 E. NOTA

15. How many codes have the property that the digits are neither in strictly increasing order nor in strictly decreasing order?

A. 10 B. 210 C. 220 D. 5000 E. NOTA

16. A triangle has sides of length 8, 10, and 12. Compute its inradius.

A. $\frac{\sqrt{7}}{2}$ B. $\sqrt{7}$ C. $2\sqrt{7}$ D. $4\sqrt{7}$ E. NOTA

17. If $\frac{x^2y-3x^2-7xy+21x+xy^2-3xy-7y^2+21y}{x+y} = 72$ and $0 < x < y$, what is the minimum possible value of $x + y$ for integers x, y?

A. 27 B. 28 C. 29 D. 30 E. NOTA

18. Vector $\boldsymbol{a}$ has a magnitude of 5. Vector $\boldsymbol{b}$ has a magnitude of 12. How many possible integer values are there for the magnitude of the cross-product, $\boldsymbol{a} \times \boldsymbol{b}$?

A. 13 B. 27 C. 60 D. 61 E. NOTA

19. Alan is graphing interesting functions that he found online. Instead of graphing function one at a time, he decides to pair them up and parametrize x and y.

If Alan uses

$$x(t) = \frac{e^t - e^{-t}}{2},$$

$$y(t) = \frac{e^t + e^{-t}}{2}$$

What is the equation of his graph in terms of x, y and constants?

A. $x^2 + y^2 = 1$ B.$4x^2 + 4y^2 = 1$ C.$4x^2 - 4y^2 = 1$ D. $x^2 - y^2 = 1$ E. NOTA

For questions 20 and 21, moving directly right means moving from the point (x,y) to the point (x+1,y), moving directly left means moving from the point (x,y) to the point (x-1,y), moving directly up means moving from the point (x,y) to the point (x,y+1), and moving directly down means moving from the point (x,y) to the point (x,y-1).

20. Justin is travelling back home from the point (0,0) and can only move directly right one space or directly up one space. His greatest rival is at his favorite fro-yo place, Razzy Fresh, at (2,3) so that Justin cannot move through the space. How many different paths could Justin take to get to his house at (4,6)?

A. 100 B. 110 C. 200 D. 210 E. NOTA

21. Justin has gotten back home, but learns that Alex left Razzy Fresh. Justin really wants bubble tea, so now he decides that he is also able to take shortcuts to get there. If Justin now can move one space directly left, one space directly down, or diagonally down and left one space, how many ways can Justin get to Razzy Fresh?

A. 10 B. 12 C. 25 D. 27 E. NOTA

22. Elliot and his dice are back. And now he's playing a game with his friend Jennifer. He rolls his dodecahedral die while Jennifer rolls his octahedral die. Elliot decides that in this game, if he rolls a higher number, then he wins. What is the probability that Elliot wins?

A. $\frac{1}{2}$ B. $\frac{13}{24}$ C.$\frac{7}{12}$ D.$\frac{5}{8}$ E. NOTA

23. The quartic $x^4 - 115x^3 + 180x^2 - 11x + 396 = 0$ has 4 roots, call them $a, b, c,$ and d. What is the sum of their reciprocals, or $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} + \frac{1}{d}$?

A. 36 B. -36 C. $\frac{1}{36}$ D.$-\frac{1}{36}$ E. NOTA

24. Rickie loves climbing staircases, but he can never make up his mind if he likes to go 2 steps at a time better or 1 step at a time better. So, he decides to mix the two up, randomly choosing if he will go one step or two. If Rickie walks up 10 stairs (not necessarily step on 10 stairs), in how many unique ways can he walk up the stairs?

A. 55 B. 89 C. 90 D. 144 E. NOTA

25. Irene draws a circle with two chords inside that intersect and are perpendicular to one another. If one chord is a perpendicular bisector of another and the chords have lengths 12 and 16, what is the area enclosed by the circle?

A. 36π B. 64π C. 100π D. Not Enough Information E. NOTA

26. What shape is defined by the polar form equation: $r = \frac{4}{2-5\cos(\theta)}$?

A. Circle B. Parabola C. Ellipse D. Hyperbola E. NOTA

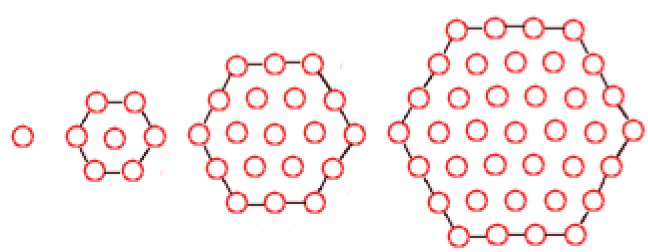
27. How many digits are in the product: $2^{12}3^{13}5^{12}$? Note: log(2) = .301, log(3) = .477.

A. 18 B. 19 C. 20 D. 21 E. NOTA

28. Alec, Bryan, Cole, Dexter, and Fletcher never make mistakes when doing math. One day, they decide to form a train of sorts and calculate some interesting numbers. Alec starts by calling out the number 1 to Bryan. Bryan starts with 0 and every time Alec calls out a number, he adds Alec's number to his own and then calls it out to Cole. Cole also starts with 0, and every time Bryan calls out a number, he adds that number to his own and calls it out to Dexter. Dexter starts with 0, and every time Cole calls out a number, Dexter adds it to his own and shouts it out to Fletcher. Fletcher starts with 0 and every time Dexter calls out a number, he add Dexter's number to his own and then calls it out to Alec, who promptly ignores him and continues to call out 1 to Bryan. At time 0, Alec calls out his first 1. After 45 seconds, Alec calls out his last 1. What is the last number that Fletcher calls out?

A. 145 B. 220 C. 495 D. 715 E. NOTA

29. Hex numbers are defined as such: we start with a single point, then we draw a hexagon around it, then another hexagon around that. Such that, the first four iterations are as below.



What is the sum of the first 7 hex numbers?

A. 49 B. 127 C. 255 D. 343 E. NOTA

30. Let $\frac{4x}{(1-x^2)(1-\frac{4x^2}{(1-x^2)^2})} = \sqrt{3}$. Let y denote the positive root with the largest magnitude. Let z denote the negative root with the largest magnitude. Compute $y + z$.

A. 2 B. -2 C. 0 D. $\sqrt{3}$ E. NOTA