

“For all questions, answer choice “E. NOTA” means none of the above answers is correct.”

1. Answer E. In the form $r = a \cos(n\theta)$ the value of a corresponds to the length of the petals and n to the number (the number of petals is n when n is odd, and $2n$ when n is even). Because of this, this equation cannot produce a rose curve with a number of petals that is twice any odd number.

2. Answer A. This could be solved by plugging in the points and creating a system of two equations and two variables, but a quicker way would be to notice that $h(8) = 125h(5) = 5^3h(5)$ therefore, the base of the exponential function must be 5 and you can find the other two quantities by dividing the given quantities by 5 in both cases, giving you 100 and $\frac{4}{5}$ respectively (to essentially take a step back in the geometric sequence produced by the y -coordinates).

3. Answer C. The ordered pair $(-6, -12)$ makes the second equation the same as the first, generating infinitely many solutions.

In problems 4-6 Solve for x .

4. Answer A. After applying the even/odd identities to make all the arguments $x - 9$ one should notice that the fraction takes the form $\frac{a-1}{1-a} = -1$.

5. Answer B. By creating a right triangle with an acute angle $\theta = \arctan\left(\frac{24}{x}\right)$ such that the side opposite is 12 and the hypotenuse is 13, you get that $\tan \theta = \frac{12}{5}$ and therefore $x = 10$.

6. Answer E. The left hand side of the equation must always be positive since the range of sine and cosine only includes real numbers and therefore can never equal $-\frac{1}{2}$ for any value of x .

7. Answer B. The exponent in the equation can only be zero or negative. Because of this, the largest the first term will be is 4 while it can approach as close to zero as possible without being zero. The -1 reduces this range by one.

8. Answer D. The system represents the Law of Cosines applied to a 30-60-90 right triangle whose short leg is of length 3. The expression we are asked to find represents twice the area of said triangle, which is $9\sqrt{3}$.

9. Answer C. Converting the right side to $2^{3x^2-15x-111}$ reduces the problem to the following quadratic: $3x^2 - 16x - 109 = 0$. From here we could simply solve the quadratic or note that the product of the solutions is equal to the constant term divided by the leading coefficient

10. Answer D. Multiplying by $\cot(\theta)$ on both sides and subtracting 1 leaves us with $3 \cos^2(\theta) = \sin^2(\theta)$. Finally, dividing both sides by $\cos^2(\theta)$ and square-rooting both sides gives us $\tan(\theta) = \pm\sqrt{3}$, the solutions to which on the given interval are: $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3},$ and $\frac{5\pi}{3}$.

11. Answer C. Adding all 4 equations together yields the desired expression.

12. Answer A. Since $x = 1$ is a zero of the numerator $x - 1$ is a factor, allowing us to simplify the left hand side using polynomial division to yield the equation: $x^2 + x - 12 = 18$. This is a simple quadratic which is factorable, yielding the solutions -6 and 5.

13. Answer A. The A and D are irrelevant in this problem and can be canceled out immediately. Using the fact that sine is an odd function we can reduce the problem to finding a value of C such that $\sin(2Bx + C) = -\sin(Bx)$. Using the sum of angles formula for sine on the left hand side should make it obvious that π is the desired value.

14. Answer B. We can employ the same method used in the problem above.

15. Answer C. The period has a length of 8 giving us a value of $\frac{\pi}{4}$ for B. The middle of the sine function is at $y = 1$ giving us the vertical displacement $D = 1$. Finally, the function deviates from this by a max of 4 units, which is the value of the amplitude A.

16. Answer A. The right hand side can be simplified to 1, provided none of our proposed values of x cause the base to be zero (in which we have 0^0 which is undefined). With this in mind, there are three ways the left hand side can be made to equal 1. First, we could cause the exponent to be zero. This occurs for $x = -7$ and 3. Second, we could cause the base to be equal to 1, which occurs for $x = -2$. Finally, we could cause the base to be -1 provided the exponent is also even for the same value of x, which is the case for $x = -3$. The sum of all these values is -9 .

17. Answer A. This polynomial is easily factorable by grouping, giving us $(2x^2 - 1)(x + 1) = 0$. Since the solutions generated by the first factor are additive inverses they can be ignored in the sum. Therefore the sum is simply the third solution which is -1 .

18. Answer E. The radicand happens to be a perfect square binomial which allows us to simplify the equation as follows: $|x^2 - 3| = 2x$. This is a simple factorable quadratic with solutions $x = \pm 1$ and ± 3 . However, the negative solutions are extraneous in the original equation (the result of a positive radical cannot be negative). Therefore, the sum is 4.

19. Answer A. Since $f(x)$ is 4th degree it has constant 4th order finite differences. If we call the missing y-intercept y, we can use finite differences to set up an equation to solve for y. The finite differences, starting with the first order differences (doing right minus left) would be:

$$\begin{array}{ccccccc}
 y - 144 & & -12 - y & & 6 & & 6 & & -6 \\
 & 132 - 2y & & y + 18 & & 0 & & -12 & \\
 & & 3y - 114 & & -y - 18 & & -12 & & \\
 & & & -4y + 96 & & y + 6 & & &
 \end{array}$$

This last pair of expressions must be equal because they are the fourth order finite differences. Solve for y in that simple linear equation yields $y = 18$.

20. Answer E. You can use the table above to generate a system of four equations and four variables (a, b, c , and d). If you correctly solved the preceding problem this cuts it down to only three equations and three variables (since you know $d = 18$). If you use the first two table values to generate two equations, you will notice that these two equations cancel the a and c terms allowing you to directly solve for b as -12 . After this it is simply a system of two equations and two variables. Solving the whole system yields the following polynomial: $f(x) = x^4 - 12x^3 + 47x^2 - 66x + 18$. Alternatively, if you solved the preceding problem correctly, you can discern from the table that $1 + a + b + c + d = 144$. If you subtract 1 from both sides and $2d = 36$ from both sides, you get the desired expression is equal to 107.

21. Answer C. $f(x) = (x - 2)^5$ therefore, the inverse is easy to find, simply fifth root both sides and add 2, giving us: $f^{-1}(x) = \sqrt[5]{x} + 2$

22. Answer C. Simplifying for extra revolutions we get $\sec\left(\frac{\pi}{6} - \theta\right) = 2$ which means we want $\frac{\pi}{6} - \theta = \frac{\pi}{3}$ or $-\frac{\pi}{3} + 2\pi n$. This means $\theta = -\frac{\pi}{6}$ or $\frac{\pi}{2} + 2\pi n$. Now since both arguments in the expression are multiples of 3 this means the angle in each trig function (sine and cosine) will be some multiple of $\frac{\pi}{2}$, meaning that the value of these will be either 1 or 0. In the case of the sine term, we are always guaranteed to have an odd multiple of $\frac{\pi}{2}$ meaning that this term will always have a value of 1 no matter which angle is used as a . Since 18 has only one factor of two and also has two 3s as factors, we are guaranteed an odd multiple of π for the expression $18b$, which means the cosine will always be either 1 or -1 , but since it's squared, there is no difference and this term too will also always have a value of 1.

23. Answer A. The right hand side simplifies to $\sin^2 \theta$ which if you subtract it to the other side and combine it with the $\cos^2 \theta$ that is already there you get the Double Angle Identity for cosine producing, after factoring, the equation: $(2 \sin(2\theta) + 1)(\cos(2\theta)) = 0$. From here you can solve for each factor independently. The first yields the solutions $\theta = \frac{7\pi}{12}$ and $\frac{11\pi}{12} + \pi n$. The second, $\theta = \frac{\pi}{4}$ and $\frac{3\pi}{4} + \pi n$. However, since we are asked only for solutions on the interval $[0, \pi]$ the $+\pi n$ is superfluous.

24. Answer A. Squaring the first equation produces $a^2 + b^2 + c^2 + 2ab + 2bc + 2ac = 36$. The first two terms are equal to 2 from the third equation and the last three terms are equal to 26 (using twice the second equation). This means $c^2 = 8$ and therefore $c = 2\sqrt{2}$.

25. Answer B. Multiplying by $\cos(\theta)$ on both sides reveals an easy conversion on both sides of the equation. The left would be $r\cos(\theta)$ which is equal to x . The right would be $\tan(\theta)$ which is equal to $\frac{y}{x}$. Solving for y gives us $y = x^2$.

26. Answer E. Given the information above we know that the equation for the circle is: $(x - 2)^2 + (y - 3)^2 = 25$. The hyperbola has an a value of 3 (because the vertex mentioned must be on $(2,0)$ which is three units away) and a c value of 5 (because the foci are on the circle which has a radius of 5 and shares a center with the hyperbola). Therefore the equation for the hyperbola is: $\frac{(y-3)^2}{9} - \frac{(x-2)^2}{16} = 1$. Solving for $(y - 3)^2$ in the second equation and substituting we get: $(x - 2)^2 + 9 + \frac{9(x-2)^2}{16} = 25$ which simplifies to: $\frac{25(x-2)^2}{16} = 16$. Once we square-root both sides, multiply by $\frac{4}{5}$, and add 2 we get $x = 2 \pm \frac{16}{5}$. Because of the position of these conics on the coordinate plane, we know the negative solution ($x = -\frac{6}{5}$) will be the desired one, which we can then plug into the circle equation to get $\frac{256}{25} + (y - 3)^2 = 25$ which simplifies to $(y - 3)^2 = \frac{369}{25}$. Solving for y and applying the same logic, we get the y -coordinate to be $3 - \frac{3\sqrt{41}}{5}$.

27. Answer D. Adding the last two equations together yields $a=1.5$. This makes $b+c=4.5$, and we are looking for $(a+b+c)^2=6^2=36$.

28. Answer B. This equation is quadratic in form which can be revealed using the substitution $y = 3^x$. This yields: $y^2 - \frac{28}{3}y + 3 = 0$, which is factorable to: $(y - 9)\left(y - \frac{1}{3}\right) = 0$. This gives us $y = 9$ and $\frac{1}{3}$ which means $x = 2$ and -1 .

29. Answer C. Simplifying and moving the second term on the left over to the right, gives us: $\sqrt{x + 2} = 2 - \sqrt{x}$. Squaring both sides gives us: $x + 2 = 4 - 4\sqrt{x} + x$ which simplifies to: $\sqrt{x} = \frac{1}{2}$ giving us $x = \frac{1}{4}$

30. Answer B. We can use the change of base formula to rewrite the equation as follows: $9 \ln(e^2) \log_3 \sqrt{x + 1} = 1$. Using simplifications and rules of logs we get: $9 \log_3(x + 1) = 1$. Solving for x gives us $x = \sqrt[9]{3} - 1$