Answers:

0. 24
1. $\frac{10}{121}$
2. 5
3. 0.479 (must be a decimal)
4. 16
5. 10π
6. 14400
7. $42+10\sqrt{17}$
8. $(-\infty,-5]\cup(-2,1)\cup[2,\infty)$ (must be in interval notation)
9. 1709
10. 16144
11. -2
12. 149
13. 3032
14. $12-7i$

Solutions:

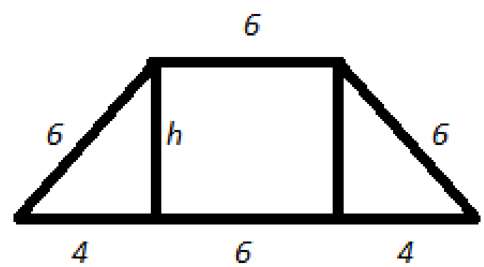
0. $\frac{30\text{ mi}}{\text{hr}}\cdot\frac{5280\text{ ft}}{\text{mi}}\cdot\frac{\text{hr}}{60\text{ min}}\cdot\frac{\text{min}}{60\text{ s}}=\frac{44\text{ ft}}{\text{s}}$, so $A=44$.

$$B=\frac{8!}{2!2!2!}=5040$$

$\frac{5040}{44}=114+\frac{24}{44}$, so the remainder is 24.

1. Using Heron's formula, $A=\sqrt{5(5-3)^2(5-4)}=2\sqrt{5}$.

Based on the picture to the right, $h=\sqrt{6^2-4^2}=2\sqrt{5}$, so the area enclosed by the trapezoid is $B=\frac{1}{2}(6+14)\left(2\sqrt{5}\right)=20\sqrt{5}$.



$$\frac{A\cdot B}{(A+B)^2}=\frac{2\sqrt{5}\cdot 20\sqrt{5}}{\left(2\sqrt{5}+20\sqrt{5}\right)^2}=\frac{200}{\left(22\sqrt{5}\right)^2}=\frac{10}{121}$$

2. Based on the given information, 9 lawyers can process 3 files in 7 hours, so 3 lawyers can process 1 file in 7 hours. Since 4 files need to be processed in 7 hours, this would require $A=12$ lawyers.

The digits of *B*, from left to right, can be written as *x*, 2*x*, 2*y*, *y*, 2*y*, 2*x*, *x*, based on the given information. This means that $6x+5y=26$, and the only solution with nonnegative integers is $x=1$, $y=4$. Therefore, $B=1284821$.

$\frac{1284821}{12}=107068+\frac{5}{12}$, so the remainder is 5.

3. Let *m* and *f* be the number of left-handed male and female students, respectively, in the sixth grade at Mu Alpha Theta Middle School. This means that $9m$ and $7f$ are the number of right-handed male and female students, respectively. Therefore, $m+f=24$ and $9m+7f=190$. Solving this system yields $m=11$ and $f=13$. The probability that a

right-handed student is female is $\frac{91}{190} = 0.4789...$, so this probability, when rounded to the nearest thousandth, is 0.479.

4. $A = \frac{\sum_{n=1}^{16} n^2}{\sum_{n=1}^{33} n^2} = \frac{\frac{16 \cdot 17 \cdot 33}{6}}{\frac{33 \cdot 34 \cdot 67}{6}} = \frac{8}{67}$ (which is easiest to find by canceling factors rather than multiply out the products)

$$B = \begin{vmatrix} 1 & 5 & 8 \\ -3 & 0 & 2 \\ 6 & -4 & -2 \end{vmatrix} = 0 + 60 + 96 - 0 - (-8) - 30 = 134$$

$$A \cdot B = \frac{8}{67} \cdot 134 = 16$$

5. Let a and b be the length of the legs and c be the length of the hypotenuse of the right triangle. The length of the median to the hypotenuse is $\frac{c}{2}$. The length of the angle bisector to the hypotenuse is $\sqrt{ab\left(\frac{(a+b)^2 - c^2}{(a+b)^2}\right)} = \sqrt{ab\left(\frac{a^2 + b^2 - c^2 + 2ab}{(a+b)^2}\right)} = \frac{ab\sqrt{2}}{a+b}$. The length of the altitude to the hypotenuse is $\frac{ab}{c}$. The product of the lengths of these three segments is $\frac{c}{2} \cdot \frac{ab\sqrt{2}}{a+b} \cdot \frac{ab}{c} = \frac{a^2b^2\sqrt{2}}{2(a+b)}$. Since either a or b equals 2 (let's say $b = 2$), then $\frac{4a^2\sqrt{2}}{2(a+2)} = 9\sqrt{2} \Rightarrow 2a^2 = 9a + 18 \Rightarrow 0 = 2a^2 - 9a - 18 = (2a+3)(a-6) \Rightarrow a = 6$ (since the value must be positive, being the length of the leg). This makes the hypotenuse length $\sqrt{2^2 + 6^2} = \sqrt{40} = 2\sqrt{10}$. Since this is also the diameter of the circumscribed circle, the radius of the circle is $\sqrt{10}$, making the enclosed area 10π.

6. You might think that you should arrange the officers in all possible ways $(5! = 120)$, arrange the other members in all possible ways $(5! = 120)$, then, treating the officers as one "person", account for every way the 6 "people" may be arranged around the table $(5! = 120)$, and multiply these three numbers together. However, arranging the non-officers in every possible way already accounts for every arrangement of the 6 "people".

Therefore, the number of distinct ways in which the people may sit around the table is $(5!)^2 = 14400$.

7. $A = \frac{x+y}{2}$ and $H = \frac{2xy}{x+y}$, so because the geometric mean of these two quantities is 8, we have $8 = \sqrt{\left(\frac{x+y}{2}\right)\left(\frac{2xy}{x+y}\right)} = \sqrt{xy}$, so by Vieta's formulas, $\sqrt{x}$ and $\sqrt{y}$ are the two solutions to $z^2 - 10z + 8 = 0 \Rightarrow z = 5 \pm \sqrt{17}$. Since *x* is the greater of *x* and *y*, $\sqrt{x} = 5 + \sqrt{17} \Rightarrow x = 42 + 10\sqrt{17}$.

8. $0 \geq \frac{x}{x+2} + \frac{1}{x-1} - \frac{3}{2} = \frac{2x(x-1) + 2(x+2) - 3(x+2)(x-1)}{2(x+2)(x-1)} = \frac{-x^2 - 3x + 10}{2(x+2)(x-1)}$
$= \frac{-(x+5)(x-2)}{2(x+2)(x-1)}$, and performing sign analysis on this expression shows that it is positive on the intervals $(-5,-2) \cup (1,2)$ (positive numerator and denominator). The expression is negative on the intervals $(-\infty,-5) \cup (2,\infty)$ (negative numerator, positive denominator) and $(-2,1)$ (positive numerator, negative denominator). Further, -5 and 2 should be included in the answer since the expression equals 0, while -2 and 1 should not be included since the expression is undefined. Therefore, the solution to the inequality is $(-\infty,-5] \cup (-2,1) \cup [2,\infty)$.

9. Using the standard terminology of conic sections, we know that $c = 8$, $a^2 - b^2 = 64$, and $ab = 255$. Solving this system yields $a = 17$, $b = 15$. Because the foci are on the same horizontal line as the center, the equation for the ellipse is $\frac{(x-2)^2}{289} + \frac{(y+3)^2}{225} = 1$, which can be rewritten as $225x^2 + 289y^2 - 900x + 1734y - 61524 = 0$, so $E = 61524$. Since the prime factorization of 61524 is $2^2 \cdot 3^2 \cdot 1709$, the greatest positive prime integral divisor is 1709.

10. Using the square as an example, the number of symmetries of a regular 2018-gon is 4036: 2018 rotations (each vertex maps to a point) and 2018 reflections (1009 diagonals, 1009 lines through opposite side midpoints). So $A = 4036$ (and, actually, it can be easily shown that for a regular polygon with *n* sides, there are 2*n* total symmetries). For the rectangle, there are 4 symmetries: 2 rotations (leave alone or rotate 180°) and 2 reflections (lines through opposite side midpoints). So $B = 4$.

Further, it is easy to see that a scalene triangle has only 1 symmetry: leaving it alone. So $C=1$. Therefore, $A \cdot B \cdot C = 4036 \cdot 4 \cdot 1 = 16144$.

11. Using a property of logarithms, we have that $\log_{100}\left(7x^2-2x+17\right)=\log\left(1-3x\right)$ $\log_{100}\left(1-3x\right)^2$, so because we now have the same base on the logarithms, we may now set the arguments of the logarithms equal to each other, solving for values of *x* that make both original logarithm arguments positive:

$7x^2-2x+17=1-6x+9x^2 \Rightarrow 0=2x^2-4x-16=2\left(x-4\right)\left(x+2\right) \Rightarrow x=4$ or $x=-2$, but only $x=-2$ makes the argument $\left(1-3x\right)$ positive, so that is the only solution (and, therefore, the sum).

12. Let *x* and *y* be the dimensions of any of the puzzles.

For the puzzles with the same number of interior as border pieces: We know that there are *xy* total pieces, and there are $\left(x-2\right)\left(y-2\right)$ interior pieces, so $xy=2\left(x-2\right)\left(y-2\right)$ $=2xy-4x-4y+8 \Rightarrow 8=xy-4x-4y+16=\left(x-4\right)\left(y-4\right)$, and the only ways to factor 8 where both factors are greater than -4 are $8 \cdot 1$ and $4 \cdot 2$. Therefore, setting $x-4$ and $y-4$ equal to these factors in both cases, we get that the puzzles could be 12×5 or 8×6.

For the puzzle with twice the number of border pieces as interior pieces: In a similar fashion, $xy=3\left(x-2\right)\left(y-2\right)=3xy-6x-6y+12 \Rightarrow 3=xy-3x-3y+9=\left(x-3\right)\left(y-3\right)$, and the only way to factor 3 where both factors are greater than -3 is $3 \cdot 1$. Therefore setting $x-3$ and $y-3$ equal to these factors, we get that the puzzle is 6×4.

For the puzzles with twice the number of interior pieces as border pieces: Again, in a similar fashion, $2xy=3\left(x-2\right)\left(y-2\right)=3xy-6x-6y+12 \Rightarrow 24=xy-6x-6y+36$ $=\left(x-6\right)\left(y-6\right)$, and the only ways to factor 24 where both factors are greater than -6 are $24 \cdot 1$, $12 \cdot 2$, $8 \cdot 3$, and $6 \cdot 4$. Therefore, setting $x-6$ and $y-6$ equal to these factors in all four cases, we get that the puzzles could be 30×7, 18×8, 14×9, or 12×10.

$12+5+8+6+6+4+30+7+18+8+14+9+12+10=149$

13. The results of this problem can best be found through synthetic division, but here are the results by brute force:

$g(1) = 3 - 4 + 7 - 12 - 8 = -14$ $g(2) = 96 - 64 + 56 - 24 - 8 = 56$

$g(3) = 729 - 324 + 189 - 36 - 8 = 550$ $g(4) = 3072 - 1024 + 448 - 48 - 8 = 2440$

$g(1) + g(2) + g(3) + g(4) = -14 + 56 + 550 + 2440 = 3032$

14. $\frac{535 + 283i}{23 + 37i} \cdot \frac{23 - 37i}{23 - 37i} = \frac{12305 - 19795i + 6509i + 10471}{23^2 + 37^2} = \frac{22776 - 13286i}{1898} = 12 - 7i$