

1. Convert the decimal number 2017 to binary.

- A.  $11111100001_2$     B.  $10000111111_2$     C.  $11011110001_2$     D.  $11110110001_2$     E. NOTA

2. Evaluate the sum:  $\sum_{n=0}^{\infty} \frac{2n}{2^n}$ .

- A. 6                      B. 4                      C. 3                      D. 2                      E. NOTA

3. Consider the function  $f(x) = \frac{x+2}{x-2}$ . Find a function  $g(x)$ , such that  $f(g(x)) = x$ .

- A.  $\frac{x-2}{x+2}$               B.  $\frac{2+2x}{x-1}$               C.  $\frac{1+2x}{x-1}$               D.  $\frac{1+2x}{1-x}$               E. NOTA

4. Consider  $y = x^2 + bx + c$ . Call the roots of the equation  $r$  and  $s$  and the  $y$ -intercept  $y$ . What unique relationship exists between  $r$ ,  $s$ , and  $y$  for all integer values of  $b$  and  $c$ ?

- A.  $rs = y$               B.  $r + s = y$               C.  $ry = s$               D.  $rsy = 1$               E. NOTA

5. Call a number *boring* if it is:

1) Not divisible by any perfect squares (besides 1).

2) A non-negative even number less than 100.

Compute the sum of all *boring* numbers.

- A. 920                      B. 930                      C. 940                      D. 968                      E. NOTA

6. How many five digit numbers exist such that each digit is strictly greater than the digit on its left? I.e. 12345 would count, but 12235 would not.

- A. 45000                  B. 22500                  C. 126                      D. 63                      E. NOTA

7. Consider the recursive function  $f(n) + 4f(n-1) - 8f(n-2) = 4n$ . Given that  $f(-2) = 4$  and  $f(-1) = 0$ , compute the value of  $f(2)$ .

- A. 960                      B. 888                      C. 732                      D. 760                      E. NOTA

8. What is the sum of all of the 2017<sup>th</sup> roots of unity?

- A. 2017                      B. 1                      C. -1                      D. 0                      E. NOTA

9. A regular dodecagon is inscribed in a circle of radius 4. Compute the ratio of the area enclosed by the dodecagon to the area enclosed by the circle.

- A.  $\frac{6}{\pi}$                       B.  $\frac{4}{\pi}$                       C. 1                      D.  $\frac{3}{\pi}$                       E. NOTA

10. Find the distance from the line  $y = 3x + 15$  to the circle  $x^2 - 4x = 36 - y^2$ .

- A.  $\frac{1}{\sqrt{10}}$       B.  $\frac{21}{\sqrt{10}}$       C.  $2\sqrt{10}$       D.  $\sqrt{10}$       E. NOTA

11. If  $\sin(x) + \cos(x) = \frac{17}{15}$ , compute  $\sin(2x)$ .

- A.  $\frac{6}{25}$       B.  $\frac{74}{225}$       C.  $\frac{64}{225}$       D.  $\frac{289}{225}$       E. NOTA

12. The length of the latus rectum of the equation  $4x^2 - 32x - 25y^2 + 100y = 72$  is:

- A.  $\frac{2}{5}$       B.  $\frac{12}{25}$       C.  $\frac{4}{5}$       D.  $\frac{24}{25}$       E. NOTA

13. The 2017 least positive perfect squares are summed together and the result is multiplied by 6. Let's call this number  $x$ . What are the last two digits of  $x$ ?

- A. 00      B. 10      C. 70      D. 80      E. NOTA

14. A cone has base circumference of  $20\pi$  meters and a height of 12 meters. The cone is placed with its vertex facing down and filled with water from above at a rate of 250 cubic meters per minute. What is the height of the water after 472 seconds?

- A.  $\frac{3\pi}{4}$       B.  $\frac{3\pi}{2}$       C.  $\frac{3}{2\pi}$       D. The water is overflowing      E. NOTA

15. What is  $\sum_{n=1}^5 \left| \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix}^n \right|$ ?

- A. 63      B. 64      C. 31      D. 32      E. NOTA

16. It is well known that  $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$ . Given this, compute  $\sum_{n=1}^{\infty} \frac{1}{(2n-1)^2}$ .

- A.  $\frac{\pi^2}{8}$       B.  $\frac{\pi^2}{12}$       C.  $\frac{\pi}{6}$       D.  $\frac{\pi}{3}$       E. NOTA

17. Find the distance between the circumcenter and the incenter of a right triangle with legs of length 24 and 70.

- A. 18.5      B. 20      C. 24      D. 25      E. NOTA

18. A ladder of length 50 is sliding down the side of a building located along the  $y$ -axis. Initially, the top of the ladder is at  $(0, 50)$  and the bottom at  $(0, 0)$ . After some time, the top is at  $(0, y_1)$  and the bottom at  $(x_1, 0)$ , where  $x_1$  and  $y_1$  are integers, with  $x_1 > 0$  and  $y_1 < 50$ . After some more time, the top is at  $(0, y_2)$  and the bottom at  $(x_2, 0)$ , where  $x_2$  and  $y_2$  are integers, with  $x_2 > x_1$  and  $y_2 < y_1$ . Finally, the ladder comes to rest with the top at  $(0, 0)$  and the bottom at  $(50, 0)$ . Compute the sum  $x_1 + x_2 + y_1 + y_2$  given that each of the four values  $x_1, x_2, y_1,$  and  $y_2$  is distinct.

- A. 122                      B. 132                      C. 140                      D. 200                      E. NOTA

19. Maximize the value of  $5x + 3y$  given that  $x \geq 0, y \geq 0, 2x + 3y \leq 12, 3x - 4y \leq 12$ .

- A. no maximum            B. 12                      C. 20                      D.  $\frac{456}{17}$                       E. NOTA

20. The odometer on Everett's car is malfunctioning. For every mile that Everett drives, the odometer increases by  $n$  instead of 1, where  $n$  is the value that the odometer currently reads. Unfortunately, the odometer is only able to show 2 digits. If Everett's odometer shows the number 01, what does his odometer read after Everett drives 20 miles?

- A. 76                      B. 24                      C. 04                      D. 44                      E. NOTA

21. For  $-\pi \leq \theta \leq \pi$ , what value of  $\theta$  maximizes  $|e^{i\theta}|$ ?

- A.  $-\pi$                       B.  $\pi$                       C.  $\frac{\pi}{2}$                       D. 0                      E. NOTA

22. Give the solution for  $\frac{x^2 - 7x + 1}{x^2 - 2} \leq x + 4$  in interval notation.

- A.  $(-\sqrt{2}, 1] \cup (\sqrt{2}, \infty)$             B.  $[-\sqrt{2}, 1] \cup [\sqrt{2}, \infty)$             C.  $(-\infty, -\sqrt{2}) \cup [1, \sqrt{2})$             D.  $(-\infty, \sqrt{2}] \cup [1, \sqrt{2}]$             E. NOTA

23. Compute the cosine of the smaller angle formed by the two lines described by:  $3x - 4y = \frac{15672}{223}$  and

$$y = \frac{5}{12}x - \frac{16277}{3}.$$

- A.  $\frac{56}{65}$                       B.  $\frac{63}{65}$                       C.  $\sqrt{\frac{63}{65}}$                       D.  $\sqrt{\frac{56}{65}}$                       E. NOTA

24. Compute  $\sum_{n=1}^{\infty} \frac{3n+4}{n^3+3n^2+2n}$ .

- A. 2                      B.  $\frac{5}{2}$                       C. 3                      D.  $\infty$                       E. NOTA

25. An isosceles triangle has sides of 39, 39, and 24. Its angles are  $x$ ,  $x$ , and  $y$ . Compute  $\cos(y)$ .

- A.  $\frac{-137}{338}$       B.  $\frac{-137}{169}$       C.  $\frac{137}{169}$       D.  $\frac{137}{338}$       E. NOTA

26. Three coins are placed in a bag. One of the coins has a  $\frac{1}{2}$  chance of turning up heads when flipped. Another one of the coins has a  $\frac{2}{3}$  chance of turning up heads when flipped. The last coin has a  $\frac{3}{4}$  chance of turning up heads when flipped. If one of the coins is drawn at random (each coin is equally likely to be drawn), what is the probability that tails is flipped?

- A.  $\frac{23}{36}$       B.  $\frac{7}{12}$       C.  $\frac{5}{12}$       D.  $\frac{13}{36}$       E. NOTA

27. Calculate  $\text{proj}_{\vec{v}} \vec{u}$  given that  $\vec{u} = \langle 2, 1, 4 \rangle$  and  $\vec{v} = \langle -3, -1, 2 \rangle$ .

- A.  $\langle \frac{-3}{14}, \frac{-1}{14}, \frac{1}{7} \rangle$       B.  $\langle \frac{-3}{\sqrt{14}}, \frac{-1}{\sqrt{14}}, \frac{2}{\sqrt{14}} \rangle$       C.  $\langle \frac{3}{14}, \frac{1}{14}, \frac{-1}{7} \rangle$       D.  $\langle \frac{3}{\sqrt{14}}, \frac{1}{\sqrt{14}}, \frac{-2}{\sqrt{14}} \rangle$       E. NOTA

28.  $f(x)$  is a cubic polynomial with  $f(0) = 120, f(1) = 129, f(2) = 70, f(3) = -51$ . What is  $f(4)$ ?

- A. -228      B. 0      C. 1      D. 228      E. NOTA

29. The entire graph of which of the following can be parametrized into the equations:

$$x(t) = 4 \cosh(t) + 5$$

$$y(t) = 6 \sinh(t) - 2$$

- A. Hyperbola      B. Parabola      C. Ellipse      D. Two lines      E. NOTA

30. Elliot lives in a house at the point  $(3, 2)$  on the Cartesian plane. There is a river flowing at  $y = 10$ . Elliot needs to go to the river and go to visit Anthony's house at  $(-5, 3)$ . What is the length of the shortest path Elliot can travel?

- A. 17      B. 15      C.  $\sqrt{65}$       D. 34      E. NOTA