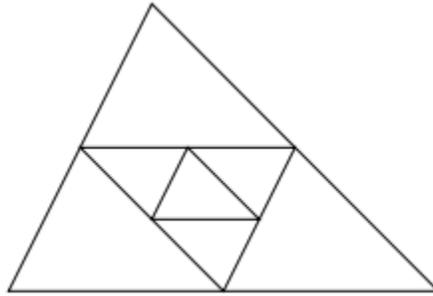


For all questions, E "NOTA" means none of the above answers is correct.

- As we know, a mathematician is a device for turning coffee into theorems. Zach, a mathematician, can prove a theorem in six hours (given enough coffee, of which he has an infinite supply). Zach's invention, the Lemm-o-Matic 2017, can prove a theorem in five hours. Working together, how long will it take them to prove 100 total theorems? Express your answer to the nearest hour.  
a) 152                      b) 205                      c) 273                      d) 311                      e) NOTA
- Compute  $1 + 2 + 3 + \dots + n + \dots + 1000000$ .  
a) 10001000              b) 50005000              c) 1000010000              d) 5000050000              e) NOTA
- A mouse factory makes 3- and 5-button mice. The factory normally uses 207 buttons a day, but one day accidentally switches the orders and makes 5-button mice instead of 3-button mice, and vice versa, and ends up making 281 buttons. How many 3-button mice does the factory normally make?  
a) 49                      b) 50                      c) 51                      d) 52                      e) NOTA
- Given a random string of 33 bits (each bit is a 0 or 1), how many (they can overlap) occurrences of two consecutive 0's would you expect? (i.e. "100101" has 1 occurrence, "0001" has 2 occurrences)  
a) 6                      b) 8                      c) 10                      d) 12                      e) NOTA
- Harry is grazing his cows at the origin. There is a river that runs east to west 50 feet north of the origin. The barn is 100 feet to the south and 80 feet to the east of the origin. Harry leads his cows to the river to take a swim, then the cows leave the river from the same place they entered and Harry leads them to the barn. He does this using the shortest path possible, and the total distance he travels is  $d$  feet. Find the value of  $d$ .  
a)  $2\sqrt{70}$                       b)  $40\sqrt{29}$                       c)  $16\sqrt{35}$                       d)  $20\sqrt{60}$                       e) NOTA
- Sonny wants to buy a new TV and the one he wants just went on sale for 25% off. But, Sonny forgot about the 8% sales tax, so when the TV is rung up, he finds he is \$35 short. The store manager comes by and says, "Give it to him for 30% off." With that Sonny can buy the TV and he walks out with the TV and \$5.50 in his pocket. How much is the sales tax on his purchase?  
a) \$37.50                      b) \$40.50                      c) \$41.25                      d) \$42.00                      e) NOTA
- We draw all diagonals of a convex 12-gon. Each intersection of diagonals is contained in exactly two diagonals. How many intersections of diagonals are there inside the 12-gon?  
a) 216                      b) 480                      c) 495                      d) 500                      e) NOTA

8. There are three triangles of different sizes: small, medium and large. The small one is inscribed in the medium one such that its vertices are at the midpoints of the three edges of the medium one. The medium triangle is inscribed in the large triangle in the same way (see the figure below). If the small triangle has area 1, what is the sum of the areas of the three triangles?



- a) 14                      b) 16                      c) 19                      d) 21                      e) NOTA
9. A horse stands at the corner of a typical chessboard on a white square. With each jump, the horse can move either two squares horizontally and one vertically or two vertically and one horizontally (like a knight moves). The horse earns two carrots every time it lands on a black square, but it must pay one carrot in rent to a rabbit who owns the chessboard for every move it makes. When the horse reaches the square on which it began, it can leave. What is the maximum number of carrots the horse can earn without touching any square more than twice?
- a) 0                      b) 1                      c) 2                      d) 3                      e) NOTA
10. An archer misses the target on his first shot and hits the target on the next three shots. What is the least number of consecutive hits he must achieve following the first four shots in order to hit the target on more than nine tenths of his shots?
- a) 6                      b) 7                      c) 9                      d) 10                      e) NOTA
11. A treasure is located at a point along a straight road with towns A, B, C, and D in that order. A map gives the following instructions for locating the treasure: (a) Start at town A and go  $\frac{1}{2}$  of the way to C. (b) Then go  $\frac{1}{3}$  of the way towards D. (c) Then go  $\frac{1}{4}$  of the way towards B, and dig for the treasure. If  $AB = 6$  miles,  $BC = 8$  miles, and the treasure is buried midway between A and D, find the distance from C to D.
- a) 4                      b) 6                      c) 8                      d) 10                      e) NOTA
12. A cylindrical container 10 cm tall and 2 cm radius is partially filled with oil. The cylinder is tilted so that the oil level reaches 9 cm up the side of the cylinder at the highest but only 3 cm up the side at its lowest. Find the volume (in cubic cm) of oil in the cylinder.
- a)  $12\pi$                       b)  $24\pi$                       c)  $36\pi$                       d)  $48\pi$                       e) NOTA
13. Compute:  $201620152014^2 - 2(201620152011)^2 + 201620152008^2$
- a) 10                      b) 16                      c) 18                      d) 24                      e) NOTA

14. Consider the statement  $\begin{bmatrix} -1 & \sin(2x) \\ \cos(x) & -\sin(x) \end{bmatrix} \begin{bmatrix} \cos(x) \\ \sin(x) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$  on  $0 \leq x < 2\pi$ . What is the sum of the values that make this statement true?  
 a)  $\pi$                       b)  $2\pi$                       c)  $3\pi$                       d)  $4\pi$                       e) NOTA
15. On the first day of her precalculus class last year, Sam was surprised that her text book had almost 1000 pages. She was more surprised when the teacher said to read any set of consecutive pages with the page numbers totaling exactly 1456 when added together. What is the smallest number of pages that Sam can read in order to complete the assignment?  
 a) 4                      b) 7                      c) 11                      d) 13                      e) NOTA
16. Two Hawaiian boys paddled a canoe 6 miles down a river to the point where the river flowed into an extinct volcano with a lake; then they paddled 4 miles across the lake to an old sacred village where the Fukuhara Ohana started. Later they returned over the same route to their starting point. If the total trip to the sacred village required 2 hours, the return trip required 4 hours, and there was no current in the lake, find the rate of the current (in mph) in the river?  
 a)  $5/4$                       b) 2                      c)  $5/2$                       d) 4                      e) NOTA
17. Four boxes each contain 5 items. The first box contains 1 piece of celery and 4 buffalo wings, the second box contains 2 pieces of celery and 3 buffalo wings, the third box contains 3 pieces of celery and 2 buffalo wings, and the fourth box contains 4 pieces of celery and 1 buffalo wing. One of the boxes is chosen at random and 2 items are drawn randomly from it without replacement. What is the probability that both items chosen are buffalo wings? Each box is equally likely to be selected.  
 a)  $1/5$                       b)  $1/4$                       c)  $1/2$                       d)  $1/3$                       e) NOTA
18. A certain device makes a “beep” each 60 seconds. A second device makes a “beep” each 62 seconds. They both “beep” at 10 AM exactly. The time they will make the next “beep” together is ?  
 a) 10:26 AM                      b) 10:30 AM                      c) 10:31 AM                      d) 10:36 AM                      e) NOTA
19. How many odd integers between 450 and 700 can be formed using only the digits 3, 4, 5, 6, 7, and 8?  
 a) 32                      b) 37                      c) 45                      d) 48                      e) NOTA
20. Evaluate:  $\lim_{x \rightarrow 0} \frac{\sin(0.5x)}{x}$   
 a)  $1/2$                       b) 1                      c) 2                      d) 3                      e) NOTA
21. If the following fraction  $\frac{8x-26}{x^2-6x+8}$  were decomposed into 2 separate fractions with linear denominators of  $(x-2)$  and  $(x-4)$ , what would be the numerator of the fraction with  $(x-4)$  in it?  
 a) 3                      b) 5                      c) 7                      d) 8                      e) NOTA
22. If a spherical section of dough with radius 4 can make 8 cinnamon buns, how many cinnamon buns can a spherical section of dough with radius 6 make?  
 a) 12                      b) 24                      c) 27                      d) 48                      e) NOTA

23. The arithmetic mean between 2 positive numbers is 17 and their positive geometric mean is 15. What is the smaller of these 2 numbers?  
 a) 3                      b) 6                      c) 9                      d) 12                      e) NOTA
24. A network of ranger towers in the forest use flags on a pole to communicate their observations to one another. If a tower has nine flags, three each of three different colors, and a message consists of all nine flags one above another on the pole, how many possible messages can be conveyed? Assume different sequences of flags convey different messages.  
 a) 1240                      b) 1680                      c) 1820                      d) 2020                      e) NOTA
25. The second, sixth, twenty-second and last term of an increasing arithmetic progression, taken in this order, form a geometric progression. Find the number of terms in the arithmetic progression.  
 a) 64                      b) 86                      c) 94                      d) 102                      e) NOTA
26. Evaluate:  $\left( \frac{1 \cdot 2 \cdot 4 + 2 \cdot 4 \cdot 8 + 3 \cdot 6 \cdot 12 + \dots + n \cdot 2n \cdot 4n + \dots}{1 \cdot 3 \cdot 9 + 2 \cdot 6 \cdot 18 + 3 \cdot 9 \cdot 27 + \dots + n \cdot 3n \cdot 9n + \dots} \right)^{1/3}$   
 a)  $\frac{1}{4}$                       b)  $\frac{1}{3}$                       c)  $\frac{1}{2}$                       d)  $\frac{2}{3}$                       e) NOTA
27. There is only one way for a cheater not to be expelled after getting caught cheating on a math test. He is given 18 white balls and 6 black balls. He must divide all balls among 3 boxes with at least one ball in each box. Then, blindfolded, he must choose a box at random and then select a ball from within this chosen box. He does not get expelled only if the chosen ball is white. Assuming the student distributed the balls in the most favorable manner, what is the probability that he does not get expelled?  
 a)  $7/12$                       b)  $2/3$                       c)  $4/5$                       d)  $10/11$                       e) NOTA
28. What is the smallest possible surface area of an object constructed by joining the faces of five cubes of edge length one?  
 a) 10                      b) 15                      c) 20                      d) 25                      e) NOTA
29. An ellipse has semi-major axis 2 and semi-minor axis 1. Find the distance between its foci.  
 a)  $\sqrt{3}$                       b)  $2\sqrt{3}$                       c)  $\sqrt{5}$                       d)  $2\sqrt{5}$                       e) NOTA
30. The equation  $\begin{pmatrix} 1 & 2 & 2 \\ 1 & 3 & 4 \\ 3 & 4 & k \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$  has a solution for  $(x, y, z)$  besides  $(0, 0, 0)$ . Find the value of k.  
 a) 1                      b) 2                      c) 3                      d) 4                      e) NOTA