#0 Alpha Bowl
MAΘ National Convention 2017

$$A = (x-a)(x-b)(x-c)\cdots(x-z)$$
$$B = \cos 1^\circ \cos 2^\circ \cos 3^\circ \cdots \cos 179^\circ$$
$$C = \tan 1^\circ \tan 2^\circ \tan 3^\circ \cdots \tan 89^\circ$$

Compute $A+B+C$.

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$$C = \tan 1^\circ \tan 2^\circ \tan 3^\circ \cdots \tan 89^\circ$$

Compute $A+B+C$.

#1 Alpha Bowl
MAΘ National Convention 2017

Let $\vec{u} =< 2, 4, 3 >, \vec{v} =< 4, 4, 2 >, \vec{w} =< 5, 0, 1 >$.

$$A = \vec{u} \cdot (\vec{v} \times \vec{w})$$
$$B = \vec{w} \cdot (\vec{v} \times \vec{u})$$
$$C = \vec{u} \times (\vec{v} \times \vec{w})$$
$$D = \vec{w} \times (\vec{v} \times \vec{u})$$

Define $f(X)$ as a function on quantity X such that $f(X)$ equals X if X is a scalar, and $f(X)$ equals the sum of the components of X if X is a vector. Compute $f(A) - f(B) + f(C) - f(D)$.

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Let $\vec{u} =< 2, 4, 3 >, \vec{v} =< 4, 4, 2 >, \vec{w} =< 5, 0, 1 >$.

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$$B = \vec{w} \cdot (\vec{v} \times \vec{u})$$
$$C = \vec{u} \times (\vec{v} \times \vec{w})$$
$$D = \vec{w} \times (\vec{v} \times \vec{u})$$

Define $f(X)$ as a function on quantity X such that $f(X)$ equals X if X is a scalar, and $f(X)$ equals the sum of the components of X if X is a vector. Compute $f(A) - f(B) + f(C) - f(D)$.

#2 Alpha Bowl
MAΘ National Convention 2017

A = the period of $y = \sin^2(\pi x)$
B = the amplitude of $y = \sin^2(\pi x)$
C = the period of $y = \sin\left(\frac{\pi x}{2}\right) + \cos\left(\frac{\pi x}{3}\right)$
D = the amplitude of $y = 5\sin(\pi x) + 4\cos(\pi x)$
Compute $ABCD$.

#2 Alpha Bowl
MAΘ National Convention 2017

A = the period of $y = \sin^2(\pi x)$
B = the amplitude of $y = \sin^2(\pi x)$
C = the period of $y = \sin\left(\frac{\pi x}{2}\right) + \cos\left(\frac{\pi x}{3}\right)$
D = the amplitude of $y = 5\sin(\pi x) + 4\cos(\pi x)$
Compute $ABCD$.

#3 Alpha Bowl
MAΘ National Convention 2017

Wash and Book are running around a circular track, each at a constant rate, and starting at a common starting point at the same time. If they run in the same direction, it takes Wash 7 minutes to overtake Book. If they run in the opposite direction, it takes them 3 minutes to pass each other.

A = The time, in seconds, it takes for Wash to run one lap around the track.

B = The time, in seconds, it takes for Book to run one lap around the track.

C = The time, in minutes, it takes for Wash and Book to be at the starting point at the same time again the first time after they start, if they run in the same direction.

D = The time, in minutes, it takes for Wash and Book to be at the starting point at the same time again the first time after they start, if they run in opposite directions.

E = The number of times Wash and Book are at the same point on the track during the time interval $[0, C]$ minutes, if they run in the same direction.

F = The number of times Wash and Book are at the same point on the track during the time interval $[0, D]$ minutes, if they run in opposite directions.

Compute $A + B + C + D + E + F$.

#3 Alpha Bowl
MAΘ National Convention 2017

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F = The number of times Wash and Book are at the same point on the track during the time interval $[0, D]$ minutes, if they run in opposite directions.

Compute $A + B + C + D + E + F$.

#4 Alpha Bowl
MAΘ National Convention 2017

In ΔABC, points *D*, *E*, and *F* are on sides *BC*, *CA*, and *AB*, respectively. $AF:FB = 2:1, BD:DC = 3:2, CE:EA = 3:1$, and the area of ΔABC is 120.

W = area of ΔAFE

X = area of ΔBDF

Y = area of ΔCED

Z = area of ΔDEF

Compute $\frac{WX}{YZ}$.

#4 Alpha Bowl
MAΘ National Convention 2017

In ΔABC, points *D*, *E*, and *F* are on sides *BC*, *CA*, and *AB*, respectively. $AF:FB = 2:1, BD:DC = 3:2, CE:EA = 3:1$, and the area of ΔABC is 120.

W = area of ΔAFE

X = area of ΔBDF

Y = area of ΔCED

Z = area of ΔDEF

Compute $\frac{WX}{YZ}$.

#5 Alpha Bowl
MAΘ National Convention 2017

Positive integers a, b, c, d, m, n, p, q satisfy the relations

$$a + \cfrac{1}{a + \cfrac{1}{a + \cfrac{1}{a + \cdots}}} = m + \sqrt{17}$$

$$b + \cfrac{1}{b + \cfrac{1}{b + \cfrac{1}{b + \cdots}}} = n + \sqrt{170}$$

$$c + \cfrac{1}{c + \cfrac{1}{c + \cfrac{1}{c + \cdots}}} = p + \sqrt{26}$$

$$d + \cfrac{1}{d + \cfrac{1}{d + \cfrac{1}{d + \cdots}}} = q + \sqrt{226}$$

Compute $a + b + c + d + m + n + p + q$.

#5 Alpha Bowl
MAΘ National Convention 2017

Positive integers a, b, c, d, m, n, p, q satisfy the relations

$$a + \cfrac{1}{a + \cfrac{1}{a + \cfrac{1}{a + \cdots}}} = m + \sqrt{17}$$

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$$d + \cfrac{1}{d + \cfrac{1}{d + \cfrac{1}{d + \cdots}}} = q + \sqrt{226}$$

Compute $a + b + c + d + m + n + p + q$.

#6 Alpha Bowl
MAΘ National Convention 2017

For each part, find the sum of the solutions to the equation on the interval $[0, 2\pi)$. If an equation has no solutions, that part is $-\pi$. Compute $A + B + C + D$.

A. $\sin\left(2x + \frac{\pi}{3}\right) = \frac{1}{2}$

B. $\frac{\tan^2 x - 1}{1 - \tan x} = -2$

C. $4\cos x = 3\sin^2 x + 1$

D. $\sin x + \cos x = \frac{7}{5}$

#6 Alpha Bowl
MAΘ National Convention 2017

For each part, find the sum of the solutions to the equation on the interval $[0, 2\pi)$. If an equation has no solutions, that part is $-\pi$. Compute $A + B + C + D$.

A. $\sin\left(2x + \frac{\pi}{3}\right) = \frac{1}{2}$

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C. $4\cos x = 3\sin^2 x + 1$

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#7 Alpha Bowl
MAΘ National Convention 2017

Express

$$\frac{(4+3i)^{53}\left(i+\sqrt{3}\right)^{28}(2+i)^{21}\left(\operatorname{cis}\frac{7\pi}{8}\right)^{14}}{(2-2i)^{11}(1+3i)^{21}(7+24i)^{26}\left(\operatorname{cis}\frac{7\pi}{30}\right)^{20}}$$

as $a+bi$, then compute $a+b$.

#7 Alpha Bowl
MAΘ National Convention 2017

Express

$$\frac{(4+3i)^{53}\left(i+\sqrt{3}\right)^{28}(2+i)^{21}\left(\operatorname{cis}\frac{7\pi}{8}\right)^{14}}{(2-2i)^{11}(1+3i)^{21}(7+24i)^{26}\left(\operatorname{cis}\frac{7\pi}{30}\right)^{20}}$$

as $a+bi$, then compute $a+b$.

#8 Alpha Bowl
MAΘ National Convention 2017

6 fair coins are flipped.
A = the probability that exactly 3 coins land heads.
B = the probability that more than half of the coins land heads.
C = the probability that an even number of coins land heads.
D = the probability that no more than 1 coin land heads.
Compute $A + B + C + D$.

#8 Alpha Bowl
MAΘ National Convention 2017

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A = the probability that exactly 3 coins land heads.
B = the probability that more than half of the coins land heads.
C = the probability that an even number of coins land heads.
D = the probability that no more than 1 coin land heads.
Compute $A + B + C + D$.

#9 Alpha Bowl
MAΘ National Convention 2017

For each part, find the number of non-overlapping enclosed areas for the graph of the polar equation. If a graph has an infinite number of enclosed areas, then let that part be -1. For example, if the graph is a circle, that part is 1. If the graph is a limacon with a loop, that part is 2. Find $A+B+C+D+E+F+G+H$.

A	$r=2$	E	$r=\cos 2\theta$
B	$r=2\theta$	F	$r^2=\cos 2\theta$
C	$r=2\cos\theta$	G	$r=2+\cos\theta$
D	$r=2\sec\theta$	H	$r=\dfrac{2}{1+2\cos\theta}$

#9 Alpha Bowl
MAΘ National Convention 2017

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D	$r=2\sec\theta$	H	$r=\dfrac{2}{1+2\cos\theta}$

#10 Alpha Bowl
MAΘ National Convention 2017

$$A = \lim_{x\to\infty} \frac{e^x + 3e^{-x}}{6e^x - e^{-x}}$$
$$B = \lim_{x\to-\infty} \frac{e^x + 3e^{-x}}{6e^x - e^{-x}}$$
$$C = \lim_{x\to 2} \frac{x^3 - 8}{x^2 - 4}$$
$$D = \lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{i}{n^2}\right)$$
$$E = \lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{i^2}{n^3}\right)$$

Compute $A + B + C + D + E$.

#10 Alpha Bowl
MAΘ National Convention 2017

$$A = \lim_{x\to\infty} \frac{e^x + 3e^{-x}}{6e^x - e^{-x}}$$
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$$E = \lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{i^2}{n^3}\right)$$

Compute $A + B + C + D + E$.

#11 Alpha Bowl
MAΘ National Convention 2017

$A = 2^a$, where a is the smallest odd number with at least 7 positive integral factors.

$B = 3^b$, where b is largest integer that leaves the same remainder when it is divided into 453, 1013, and 1503.

$C = 5^c$, where c is the number of positive integral factors of 3600.

$D = 7^d$, where d is the greatest possible value of x if $\frac{1}{x} + \frac{1}{y} = \frac{1}{6}$ for positive integers x and y.

Arrange A, B, C, D from smallest to largest. For example, if $A < B < C < D$, turn in $ABCD$.

#11 Alpha Bowl
MAΘ National Convention 2017

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#12 Alpha Bowl
MAΘ National Convention 2017

Merlin has 3 identical bottles containing 100 mL of potion with concentration of 2%. That is, 2% of the potion is herbal extract, and the remaining 98% is water. Both water and herbal extract can be freely added to the potion, but only water evaporates and herbal extract does not.
A = the amount of herbal extract, in mL, he needs to add to the first bottle double its concentration.
B = the amount of water, in mL, he needs to add to the second bottle to half its concentration.
C = the amount of water, in mL, that needs to be evaporated from the third bottle to double its concentration.
Compute $12A + B + C$.

#12 Alpha Bowl
MAΘ National Convention 2017

Merlin has 3 identical bottles containing 100 mL of potion with concentration of 2%. That is, 2% of the potion is herbal extract, and the remaining 98% is water. Both water and herbal extract can be freely added to the potion, but only water evaporates and herbal extract does not.
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C = the amount of water, in mL, that needs to be evaporated from the third bottle to double its concentration.
Compute $12A + B + C$.

#13 Alpha Bowl
MAΘ National Convention 2017

Let r, s, t be the three roots of $f(x) = x^3 - 4x - 1$. Compute

$$\sum_{n=-2}^{3} (r^n + s^n + t^n)$$

#13 Alpha Bowl
MAΘ National Convention 2017

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$$\sum_{n=-2}^{3} (r^n + s^n + t^n)$$

#14 Alpha Bowl
MAΘ National Convention 2017

Each of *A*, *B*, *C*, and *D* is the point corresponding to the solution of its system. Find the area of quadrilateral $ABCD$.

A $\begin{cases} 3x + y = 6 \\ x - 2y = -5 \end{cases}$

C $\begin{cases} x - 3y = 1 \\ 2x + y = 9 \end{cases}$

B $\begin{cases} 2x - 3y = -2 \\ -x + 2y = 3 \end{cases}$

D $\begin{cases} 3x - 5y = 7 \\ 5x - 3y = 1 \end{cases}$

#14 Alpha Bowl
MAΘ National Convention 2017

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