0. 1
1. -38
2. $6\sqrt{41}$
3. 936
4. $\frac{1}{3}$
5. 111
6. $\frac{35\pi}{6}$
7. $6\sqrt{2}$
8. $\frac{81}{64}$
9. 8
10. 1
11. CABD
12. 175
13. 26
14. 15

0. **Answer: 1**

$A = 0$ since one of the factors is $(x - x) = 0$.
$B = 0$ since one of the factors is $\cos 90° = 0$.
$C = 1$ since $\tan x° = 1/\tan(90 - x)°$ and $\tan 45° = 1$.
So $A + B + C = 1$

1. **Answer: -38**

Since $\vec{v} \times \vec{w}$ and $\vec{v} \times \vec{u}$ are each in two separate parts, they should be computed first:

$$\vec{v} \times \vec{w} = \begin{vmatrix} i & j & k \\ 4 & 4 & 2 \\ 5 & 0 & 1 \end{vmatrix} = 4i + 6j - 20k$$

$$\vec{v} \times \vec{u} = \begin{vmatrix} i & j & k \\ 4 & 4 & 2 \\ 2 & 4 & 3 \end{vmatrix} = 4i - 8j + 8k$$

Then

$$A = 8 + 24 - 60 = -28$$

$$B = 20 + 0 + 8 = 28$$

$$C = \begin{vmatrix} i & j & k \\ 2 & 4 & 3 \\ 4 & 6 & -20 \end{vmatrix} = -98i + 52j - 4k$$

$$D = \begin{vmatrix} i & j & k \\ 5 & 0 & 1 \\ 4 & -8 & 8 \end{vmatrix} = 8i - 36j - 40k$$

So $f(A) - f(B) + f(C) - f(D) = -28 - 28 - 50 + 68 = -38$.

2. **Answer: $6\sqrt{41}$**

$$\sin^2 \pi x = \frac{1 - \cos 2\pi x}{2}$$

So $A = 1, B = \frac{1}{2}$.

C is the least common multiple of the period of $\sin\left(\frac{\pi x}{2}\right)$, which is 4, and $\cos\left(\frac{\pi x}{3}\right)$, which is 6. So $C = 12$.

$D = \sqrt{5^2 + 4^2} = \sqrt{41}$

So $ABCD = 1 \cdot \frac{1}{2} \cdot 12 \cdot \sqrt{41} = 6\sqrt{41}$.

3. **Answer: 936**

Let the speed for Wash and Book be w and b respectively, measured in laps/minute. Then

$$w - b = \frac{1}{7}$$
$$w + b = \frac{1}{3}$$

Solving to get $w = \frac{5}{21}, b = \frac{2}{21}$.
Therefore

$$A = \frac{1}{w} = \frac{21}{5} min = 252 sec$$
$$B = \frac{1}{b} = \frac{21}{2} min = 630 sec$$

The direction they run in is irrelevant for *C* and *D*, which are both the least common multiple of *A* and *B*, which is 21 minutes.
If they run in the same direction, Wash catches up to Book every 7 minutes. So over 21 minutes, he catches up 3 times. Add in the initial starting point, $E = 4$.
If they run in the same direction, they meet every 3 minutes. So over 21 minutes, they meet 7 times. Add in the initial starting point, $F = 8$.
Finally, $A + B + C + D + E + F = 252 + 630 + 21 + 21 + 4 + 8 = 936$.

4. **Answer: $\frac{1}{3}$**

$$W = \frac{1}{2} AF \cdot AE \cdot \sin A$$

By the ratios given, $AF = \frac{2}{3}AB$, $AE = \frac{1}{4}AC$. Therefore,

$$W = \frac{1}{2}\left(\frac{2}{3}AB\right)\left(\frac{1}{4}AC\right)\sin A = \frac{1}{6}\left(\frac{1}{2}AB \cdot AC \cdot \sin A\right)$$

Notice that $\frac{1}{2}AB \cdot AC \cdot \sin A$ is the area of ΔABC, so $W = 20$.
Similarly, $X = \frac{3}{5} \cdot \frac{1}{3} \cdot 120 = 24$, $Y = \frac{2}{5} \cdot \frac{3}{4} \cdot 120 = 36$.
ΔDEF is the remaining region in the middle, so $Z = 120 - 20 - 24 - 36 = 40$.
So $\frac{WX}{YZ} = \frac{(20)(24)}{(36)(40)} = \frac{1}{3}$.

5. **Answer: 111**

All 4 equations are in the same form. If we let x be equal to the right side and let k be equal to $a, b, c,$ or d, we can write a common form as $k + \frac{1}{x} = x$, or $x^2 - kx = 1$. This equation can be solved by completing the square. Since none of the right hand side contains a fraction, k must be even. For simplicity, let $k = 2k'$. Then we have

$$(x - k')^2 = 1 + (k')^2$$
$$x = k' + \sqrt{1 + (k')^2}$$

For the first equation, $1 + (k')^2 = 17$, so $m = k' = 4$ and $a = 2k' = 8$.
Similarly, $n = 13, b = 26, p = 5, c = 10, q = 15, d = 30$.
Finally, $a + b + c + d + m + n + p + q = 8 + 26 + 10 + 30 + 4 + 13 + 5 + 15 = 111$.

6. **Answer: $\frac{35\pi}{6}$**

Part A: To solve for x on $[0, 2\pi)$, it is equivalent to solving for $2x + \frac{\pi}{3}$ on $\left[\frac{\pi}{3}, \frac{13\pi}{3}\right)$. So

$$2x + \frac{\pi}{3} = \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}, \frac{25\pi}{6}$$
$$2x = \frac{3\pi}{6}, \frac{11\pi}{6}, \frac{15\pi}{6}, \frac{23\pi}{6}$$

So $A = \frac{1}{2}\left(\frac{3\pi}{6} + \frac{11\pi}{6} + \frac{15\pi}{6} + \frac{23\pi}{6}\right) = \frac{13\pi}{3}$.
Part B: The numerator is a difference of squares, so

$$\frac{(\tan x + 1)(\tan x - 1)}{1 - \tan x} = -2$$
$$-(\tan x + 1) = -2$$
$$\tan x = 1$$

However, when $\tan x = 1$, the denominator is 0, so $B = -\pi$.
Part C: Replace $\sin^2 x$ with $1 - \cos^2 x$ to get a quadratic in terms of $\cos x$:

$$4\cos x = 3 - 3\cos^2 x + 1$$
$$3\cos^2 x + 4\cos x - 4 = 0$$
$$(3\cos x - 2)(\cos x + 2) = 0$$

Since $\cos x \neq -2$, we only have $\cos x = \frac{2}{3}$. There are two solutions, one in the first quadrant, and one in the fourth quadrant with the same reference angle. So $C = 2\pi$.
Part D: $\sin x + \cos x = \sqrt{2}\sin\left(x + \frac{\pi}{4}\right)$, so $\sin\left(x + \frac{\pi}{4}\right) = \frac{1.4}{\sqrt{2}}$
$\sqrt{2} \approx 1.414$, so $x + \frac{\pi}{4} = \theta, \pi - \theta$, where θ is very close to $\frac{\pi}{2}$.
Therefore, $D = \left(\theta - \frac{\pi}{4}\right) + \left(\pi - \theta - \frac{\pi}{4}\right) = \frac{\pi}{2}$.
Finally, $A + B + C + D = \frac{13\pi}{3} - \pi + 2\pi + \frac{\pi}{2} = \frac{35\pi}{6}$

7. **Answer: $6\sqrt{2}$**

$$\left(i+\sqrt{3}\right)^{28} = \left(2\operatorname{cis}\frac{\pi}{6}\right)^{28} = 2^{28}\operatorname{cis}\frac{14\pi}{3} = 2^{28}\operatorname{cis}\frac{2\pi}{3}$$

$$\left(\operatorname{cis}\frac{7\pi}{8}\right)^{14} = \operatorname{cis}\frac{49\pi}{4} = \operatorname{cis}\frac{\pi}{4}$$

$$(2-2i)^{11} = \left(2\sqrt{2}\operatorname{cis}\left(-\frac{\pi}{4}\right)\right)^{11} = 2^{\frac{33}{2}}\operatorname{cis}\left(-\frac{11\pi}{4}\right) = 2^{\frac{33}{2}}\operatorname{cis}\frac{5\pi}{4}$$

$$\left(\operatorname{cis}\frac{7\pi}{30}\right)^{20} = \operatorname{cis}\frac{14\pi}{3} = \operatorname{cis}\frac{2\pi}{3}$$

The remaining 4 quantities cannot be directly converted to polar form. However, note that $(2+i)$ and $(1+3i)$ have identical exponents, and the exponent on $(4+3i)$ is approximately twice that of $(7+24i)$, so it makes sense to manipulate those 4 quantities in 2 pairs. Note that

$$\frac{2+i}{1+3i} = \frac{(2+i)(1-3i)}{(1+3i)(1-3i)} = \frac{5-5i}{10} = \frac{1}{2} - \frac{1}{2}i$$

So

$$\left(\frac{2+i}{1+3i}\right)^{21}\left(\frac{1}{\sqrt{2}}\operatorname{cis}\left(-\frac{\pi}{4}\right)\right)^{21} = \frac{1}{2^{\frac{21}{2}}}\operatorname{cis}\left(-\frac{21\pi}{4}\right) = \frac{1}{2^{\frac{21}{2}}}\operatorname{cis}\frac{3\pi}{4} = \frac{1}{2^{\frac{21}{2}}\operatorname{cis}\frac{5\pi}{4}}$$

Also note that

$$(4+3i)^2 = 16+24i+9i^2 = 7+24i$$

So

$$\frac{(4+3i)^{53}}{(7+24i)^{26}} = \frac{(4+3i)^{52}(4+3i)}{(7+24i)^{26}} = 4+3i$$

Putting everything together, the expression is

$$\frac{\left(2^{28}\operatorname{cis}\frac{2\pi}{3}\right)\left(\operatorname{cis}\frac{\pi}{4}\right)(4+3i)}{\left(2^{\frac{21}{2}}\operatorname{cis}\frac{5\pi}{4}\right)\left(2^{\frac{33}{2}}\operatorname{cis}\frac{5\pi}{4}\right)\left(\operatorname{cis}\frac{2\pi}{3}\right)} = \left(2\operatorname{cis}\left(-\frac{9\pi}{4}\right)\right)(4+3i)$$

$$= \left(\sqrt{2}-\sqrt{2}i\right)(4+3i) = 7\sqrt{2}-\sqrt{2}i$$

So $a+b = 6\sqrt{2}$

8. **Answer: $\frac{81}{64}$**

$$A = \binom{6}{3}\left(\frac{1}{2}\right)^6 = \frac{20}{64}$$
$$B = \left(\binom{6}{4} + \binom{6}{5} + \binom{6}{6}\right)\left(\frac{1}{2}\right)^6 = \frac{22}{64}$$
$$C = \left(\binom{6}{0} + \binom{6}{2} + \binom{6}{4} + \binom{6}{6}\right)\left(\frac{1}{2}\right)^6 = \frac{32}{64}$$
$$D = \left(\binom{6}{0} + \binom{6}{1}\right)\left(\frac{1}{2}\right)^6 = \frac{7}{64}$$
$$A + B + C + D = \frac{81}{64}$$

Alternatively, A, B, C, D combine to cover every possible outcome once except 0, 4, and 6 heads. So $A + B + C + D = 1 + \frac{1+15+1}{64} = \frac{81}{64}$.

9. **Answer: 8**

$A = 1$, graph is a circle.
$B = -1$, graph is spirals of Archimedes, which has an infinite number of enclosed regions.
$C = 1$, graph is a circle.
$D = 0$, graph is a vertical line.
$E = 4$, graph is a 4-petal rose.
$F = 2$, graph is a lemniscate.
$G = 1$, graph is a convex limacon.
$H = 0$, graph is a hyperbola.
So $A + B + C + D + E + F = 9$.

10. **Answer: 1**

$$A = \frac{1}{6}, B = -3$$
$$C = \lim_{x\to 2}\frac{(x-2)(x^2+2x+4)}{(x-2)(x+2)} = \frac{12}{4} = 3$$
$$D = \lim_{n\to\infty}\frac{1}{n^2}\sum_{i=1}^{n} i = \lim_{n\to\infty}\frac{1}{n^2}\left(\frac{n(n+1)}{2}\right) = \frac{1}{2}$$
$$E = \lim_{n\to\infty}\frac{1}{n^3}\sum_{i=1}^{n} i^2 = \lim_{n\to\infty}\frac{1}{n^3}\left(\frac{n(n+1)(2n+1)}{6}\right) = \frac{1}{3}$$
$$A + B + C + D + E = \frac{1}{6} - 3 + 3 + \frac{1}{2} + \frac{1}{3} = 1$$

11. **Answer: CABD**

Part A – We should consider 7, 8, and 9 positive integral factors. For 7 factors, a needs to be in the form p^6, the smallest value is $3^6 = 729$. For 8 factors, a needs to be in the form $p_1 p_2 p_3$, the smallest value is $3 \cdot 5 \cdot 7 = 105$. For 9 factors, a needs to be in the form $p_1^2 p_2^2$. The smallest value is $3^2 \cdot 5^2 = 225$. Any amount beyond 9 will either require more primes or require higher power on the primes. Therefore, $A = 2^{105}$.

Part B – We'll call the same remainder r. Then

$$453 = bq_1 + r$$
$$1013 = bq_2 + r$$
$$1503 = bq_3 + r$$

If we subtract the first equation from second, and second from the third, we have

$$560 = b(q_2 - q_1)$$
$$490 = b(q_3 - q_2)$$

We are then looking for the greatest common divisor of 560 and 490. So $B = 3^{70}$.

Part C – $3600 = 2^4 \cdot 3^2 \cdot 5^2$, so the number of positive integral factors of 3600 is $(4+1)(2+1)(2+1) = 45$, and $C = 5^{45}$.

Part D – Multiply both sides of the equation by $6xy$, then rearrange to get

$$6y + 6x = xy$$
$$xy - 6x - 6y = 0$$
$$xy - 6x - 6y + 36 = 36$$
$$(x-6)(y-6) = 36$$

Since x and y are both integers, so are $(x-6)$ and $(y-6)$, which are then both factors of 36. The largest x can be is when 36 is broken down as $36 \cdot 1$, which gives us $D = 7^{42}$.

Now we must compare $2^{105}, 3^{70}, 5^{45}, 7^{42}$. The greatest common divisor for the four exponents is 1, so the problem cannot be reduced in a manner to compare all 4 numbers at once. The next best thing is to make pair-wise comparison. It makes sense to compare the quantities where the exponents have a relatively large gcd to minimize the powers we have to raise the bases to.

2^{105} with 3^{70}: $2^{105} = (2^3)^{35}, 3^{70} = (3^2)^{35}$. Since $2^3 < 3^2$, $2^{105} < 3^{70}$.

3^{70} with 7^{42}: $3^{70} = (3^5)^{14}, 7^{42} = (7^3)^{14}$. Since $3^5 < 7^3$, $3^{70} < 7^{42}$.

2^{105} with 5^{45}: $2^{105} = (2^7)^{15}, 5^{45} = (5^3)^{15}$. Since $5^3 < 2^7$, $5^{45} < 2^{105}$.

From smallest to largest, we have $C < A < B < D$.

12. **Answer: 175**

In the potion, there is 98mL of water, and 2mL of herbal extract. For each of the parts, we will consider the quantity that does not change. For each of the parts, let *x* denote the final amount of potion in milliliters.
Part A – the 98mL of water need to become 96% of the potion So $\frac{98}{x} = \frac{96}{100}$, or $x = \frac{9800}{96}$.
The amount of extract that need to be added is $\frac{9800}{96} - 100 = \frac{25}{12}$.
Part B – the 2mL of extract need to become 1% of the potion. So $\frac{2}{x} = \frac{1}{100}$, or $x = 200$. The amount of water that need to be added is $200 - 100 = 100$.
Part C – the 2mL of extract need to become 4% of the potion. So $\frac{2}{x} = \frac{4}{100}$, or $x = 50$. The amount of water that need to be evaporated is $100 - 50 = 50$.
Finally, $12A + B + C = 175$

13. **Answer: 26**

With the given information, we have $f(x) = (x - r)(x - s)(x - t)$. Reverse and negate the coefficients of $f(x)$ to produce $g(x) = x^3 + 4x^2 - 1$. Then by symmetry, $g(x) = (rx - 1)(sx - 1)(tx - 1)$, so the three roots of $g(x)$ are $\frac{1}{r}, \frac{1}{s}, \frac{1}{t}$.
We will handle the summation by taking it apart based on values of *n*. Before we do so, by applying Vieta's on $f(x)$, $r + s + t = 0$, $rs + st + tr = -4$, $rst = 1$. By applying Vieta's on $g(x)$, $\frac{1}{r} + \frac{1}{s} + \frac{1}{t} = -4$, $\frac{1}{rs} + \frac{1}{st} + \frac{1}{tr} = 0$.
$n = -2$: $\frac{1}{r^2} + \frac{1}{s^2} + \frac{1}{t^2} = \left(\frac{1}{r} + \frac{1}{s} + \frac{1}{t}\right)^2 - 2\left(\frac{1}{rs} + \frac{1}{st} + \frac{1}{tr}\right) = 16$.
$n = -1$: $\frac{1}{r} + \frac{1}{s} + \frac{1}{t} = -4$.
$n = 0$: $r^0 + s^0 + t^0 = 3$.
$n = 1$: $r + s + t = 0$.
$n = 2$: $r^2 + s^2 + t^2 = (r + s + t)^2 - 2(rs + st + tr) = 8$.
$n = 3$: Substitute r into $f(x)$, and since it is a root, we have $r^3 - 4r - 1 = 0$, or $r^3 = 4r + 1$. Similarly, $s^3 = 4s + 1, t^3 = 4t + 1$.
Therefore $r^3 + s^3 + t^3 = 4(r + s + t) + 3 = 3$.
The final summation is $16 - 4 + 3 + 0 + 8 + 3 = 26$

14. **Answer: 15**

Solving the systems of equations to get $A\ (1, 3), B\ (5, 4), C\ (4, 1), D\ (-1, -2)$.
Apply shoelace theorem to get the area to be 15.