

1. 3
2. -5
3. 59
4. 16
5.  $\frac{26}{3}$
6. 2.6
7. 452
8. 35
9. 288
10. 502
11.  $\frac{45}{4}x^3$
12. 2
13.  $\frac{20}{3}$
14. 3267
15. 12
16. 27
17. 6
18.  $y = -\frac{6}{5}x + \frac{53}{5}$
19. 432
20.  $\frac{48}{3125}$
21. 2
22. 4
23.  $x^2 - 9x + 16$
24.  $e^{2x} + e^{-2x}$
25. -3 and 1

- The fraction factors into  $y = \frac{(x+5)(x+2)(x-1)(x-3)}{(x+6)(x-3)(x-4)}$ , so there are vertical asymptotes at  $x = -6$  and  $x = 4$ . The slant asymptote can be found by long division to be  $y = x + 4$ .  $-6 + 4 + 1 + 4 = 3$ .
- The expression becomes  $\log_7 \left( \frac{1}{2} \right) \left( \frac{2}{3} \right) \left( \frac{3}{4} \right) \dots \left( \frac{16806}{16807} \right) \rightarrow \log_7 \left( \frac{1}{16807} \right) = -5$ .
- $AB = C \rightarrow \begin{bmatrix} a+4+5 & 5a+2b+0 & 3a+14+25 \\ 0+2+4 & 0+b+0 & 0+7+20 \\ 2+6+6 & 10+3b+0 & 6+21+30 \end{bmatrix} = \begin{bmatrix} 10 & 7 & 42 \\ 6 & 1 & 27 \\ 14 & 13 & c \end{bmatrix}$ . Using this, we get  $a = 1, b = 1, c = 57$ .  
Their sum is 59.
- $\sqrt{(\log_2 32768)(1+i)^8} + \sqrt{(\log_2 32768)(1+i)^8} + \dots \rightarrow \sqrt{(15)(16)} + \sqrt{(15)(16)} + \dots \rightarrow x = \sqrt{(15)(16)} + x \rightarrow$   
 $x^2 - x - (15)(16) = 0 \rightarrow x = \frac{1 + \sqrt{1 - 4(15)(-16)}}{2} = 16$ . Also, a nested square root whose radicand is the product of two consecutive integers yields the larger integer when adding inside the square root.
- $a = \frac{3}{x} \rightarrow \frac{3}{x} f(x) = 2 \rightarrow f(x) = \frac{2}{3}x \rightarrow f(13) = \frac{26}{3}$ .
- $|z_1| = \sqrt{5^2 + (-12)^2} = 13, |z_2| = \sqrt{3^2 + 4^2} = 5 \rightarrow \frac{13}{5} = 2.6$ .
- Using common summation formulas we have  $2 \left[ \frac{8(9)(17)}{6} \right] + \frac{(8)(9)}{2} + 8 \rightarrow 408 + 36 + 8 = 452$ .
- $\begin{cases} a^3 + b^3 = 468 \\ (a+b)^3 = 1728 \rightarrow a+b = 12 \end{cases}$ .  $(a+b)^3 = a^3 + b^3 + 3ab(a+b) \rightarrow 1728 = 468 + 3ab(12) \rightarrow ab = 35$ .
- $f(x) = (x-3)(x+3)(x+4) = (x^2 - 9)(x+4) = x^3 + 4x^2 - 9x - 36$ . Here,  $f(0) = -36$ , so multiply all terms by 2.  
 $P(x) = 2x^3 + 8x^2 - 18x - 72 \rightarrow P(5) = 2[125 + 4(25) - 9(5) - 36] = 2(144) = 288$ .
- $\left\lfloor \frac{2017}{625} \right\rfloor + \left\lfloor \frac{2017}{125} \right\rfloor + \left\lfloor \frac{2017}{25} \right\rfloor + \left\lfloor \frac{2017}{5} \right\rfloor \rightarrow 3 + 16 + 80 + 403 = 502$ .
- $\frac{5!}{2!3!} \left( \frac{x}{2} \right)^3 (3)^2 = \frac{45}{4} x^3$ .
- The sum of the reciprocals of the divisors is the quotient of the sum of the divisors and the number itself.  
 $496 = 2^4 \cdot 31^1$ . To find the sum of the positive divisor  $(2^0 + 2^1 + 2^2 + 2^3 + 2^4)(31^0 + 31^1) \rightarrow$   
 $(1+2+4+8+16)(1+31) \rightarrow (31)(32) = 992 \Rightarrow \frac{992}{496} = 2$ .
- The absolute value and the +12 only affects the range, so we only have to concentrate on the  $f(3x-6)$ .  
If the domain of  $f(x)$  is  $-48 \leq x \leq 56$ , then  $-48 \leq 3x-6 \leq 56 \rightarrow -42 \leq 3x \leq 62 \rightarrow -14 \leq x \leq \frac{62}{3}$ .  $a+b = \frac{20}{3}$ .
- The expanded form will be  $0+1+2+2+3+4+4+5+6+6+7+\dots+65+66$ . The sum of 1-65 is  $\frac{(65)(66)}{2} = 2145$

and the sum of the evens from 2-66 is  $\frac{33[2(2)+32(2)]}{2} = 1122 \Rightarrow 2145 + 1122 = 3267$ .

15. Multiply 0.5 by 5 to get 2.5. The first digit will be 2. Take the “remainder,” 0.5, and multiply it by 5. We are at 2.5 again. We will repeat this process five times, so we get six 2’s. The sum is 12.
16. Let  $N$  be number obtained by subtracting 100,000 from the integer in question. Using the information in the problem, we now have  $3(100,000 + N) = 10N + 1 \rightarrow 7N = 299,999 \rightarrow N = 42857$ . Since the integer is  $N + 100,000$ , our number is 142857, whose digit-sum is 27.
17. For the determinant we get  $(-6x + 4x + 15) - (15 + 24 - x^2) = 0 \rightarrow x^2 - 2x - 24 = 0 \rightarrow (x - 6)(x + 4) = 0 \rightarrow x = 6, -4$ . The only positive value is 6.

18. The slope of the segment connecting the center with the point of tangency is  $\frac{2-7}{-3-3} = \frac{5}{6}$ , so the slope of

the tangent line is  $-\frac{6}{5}$ . The tangent line through the point of tangency (in point-slope form) is

$$y - 7 = -\frac{6}{5}(x - 3) \rightarrow y = -\frac{6}{5}x + \frac{53}{5}.$$

19. The major axis length,  $2a$ , is 240 and the minor axis length,  $2b$ , is 144. The focal radii have total length  $2a$  as well. The distance between the foci is  $2c$ . To find  $c$ , we use  $a^2 - b^2 = c^2$ , so we get  $c = 96$ . The total distance that has been rowed is  $240 + 192 = 432$ .
20. There is  $1/5$  chance of choosing the correct answer and a  $4/5$  chance of choosing an incorrect answer.

$$\frac{6!}{4!2!} \left(\frac{1}{5}\right)^4 \left(\frac{4}{5}\right)^2 \rightarrow 15 \left(\frac{1}{625}\right) \left(\frac{16}{25}\right) = \frac{48}{3125}.$$

21. The equation simplifies to  $x^5 + 2x^3 + 8x^2 + 16 = 0 \rightarrow x^3(x^2 + 2) + 8(x^2 + 2) = 0 \rightarrow (x^2 + 2)(x + 2)(x^2 - 2x + 4) = 0$ . The first and third terms in the factorization yield imaginary numbers. Using Vieta’s formulas, the first roots from the first term sum to zero and the roots from the third term sum to 2.

22. Create a function  $g(x)$  which has roots  $-1 + \sqrt{2}$  and  $-1 - \sqrt{2}$ . Using sum and product properties,  $g(x) = x^2 + 2x - 1$ . Now use long division to divide  $g(x)$  into  $f(x)$ :

$$\begin{array}{r} x^8 - x^6 + 3 \\ x^2 + 2x - 1 \overline{) x^{10} + 2x^9 - 2x^8 - 2x^7 + x^6 + 0x^5 + 0x^4 + 0x^3 + 3x^2 + 6x + 1} \\ \underline{x^{10} + 2x^9 - x^8} \phantom{+ 0x^7 + 0x^6 + 0x^5 + 0x^4 + 0x^3 + 3x^2 + 6x + 1} \\ -x^8 - 2x^7 + x^6 + 0x^5 + 0x^4 + 0x^3 + 3x^2 + 6x + 1 \\ \underline{-x^8 - 2x^7 + x^6} \phantom{+ 0x^5 + 0x^4 + 0x^3 + 3x^2 + 6x + 1} \\ \phantom{-x^8 - 2x^7 + x^6} 3x^2 + 6x - 3 \\ \phantom{-x^8 - 2x^7 + x^6} \phantom{3x^2 + 6x - 3} 4 \end{array}$$

Since  $\sqrt{2} - 1$  is a root of  $g(x)$ , we get  $f(\sqrt{2} - 1) = [0][\text{something}] + 4 \Rightarrow 4$ .

23. Let  $x + 1 = c \rightarrow x = c - 1$ . Substitute this into the given function:  
 $(c - 1)^2 - 7(c - 1) + 8 \rightarrow c^2 - 9c + 16 \Rightarrow x^2 - 9x + 16$ .

24.  $\sqrt{(e^{2x} - e^{-2x})^2 + 4} \rightarrow \sqrt{e^{4x} - 2 + e^{-4x} + 4} \rightarrow \sqrt{e^{4x} + 2 + e^{-4x}} \rightarrow \sqrt{(e^{2x} + e^{-2x})^2} \rightarrow |e^{2x} + e^{-2x}|$ . Since all powers of  $e$  are positive, the result simplifies to  $e^{2x} + e^{-2x}$ .

$$25. 4x^2 + 8x - 2\sqrt{4x^2 + 8x - 3} = 6 \rightarrow 4x^2 + 8x - 3 - 2\sqrt{4x^2 + 8x - 3} = 3 \rightarrow a^2 - 2a - 3 = 0 \rightarrow (a - 3)(a + 1) = 0 \rightarrow a = 3, -1.$$

$$\sqrt{4x^2 + 8x - 3} = 3 \quad \sqrt{4x^2 + 8x - 3} = -1$$

$$4x^2 + 8x - 3 = 9 \quad \text{no real solution}$$

$$4x^2 + 8x - 12 = 0$$

$$x^2 + 2x - 3 = 0$$

$$(x + 3)(x - 1) = 0 \Rightarrow x = -3, 1$$