1. Since $\angle DEG$ is complementary to $\angle GEF$, $m \angle DEG + m \angle GEF = 90^\circ$. Since $\angle DEG \cong \angle GEF$, $m \angle DEG = m \angle GEF$. Substitute the given angle measures into these equations.

$(x+3y)+(2x+y)=90$ and $x+3y=2x+y$ and solve for $x$ and $y$. $x = 18$ and $y = 9$.

$x + y = 27$.

2. \[ \frac{n(n+1)}{2} + 1 = \frac{5(5+1)}{2} + 1 = 16 \]

16

3. $A = r^2 = (3\sqrt{2})^2 = 18$

$18\pi$

4. $APF \sim CQF \quad \frac{AP}{CQ} = \frac{2}{3}$ Since the ratio of the sides is $\frac{2}{3}$, the ratio of the areas is $\frac{2^2}{3^2} = \frac{4}{9}$.

$\frac{4}{9}$

5. \[ \frac{13}{5} = \frac{5}{x} \quad 13x = 25 \quad x = \frac{25}{13} \]

\[ \frac{13}{12} = \frac{25}{13} \quad \frac{25}{13} = \frac{144}{13} \]

\[ \frac{25}{13} = \frac{a}{144} \quad \frac{a^2}{13} = \frac{25 \times 44}{169} \quad a = \frac{60}{13} \]

\[ \frac{5+12}{13} = \frac{281}{13} \]

6. The radius of the larger of two concentric spheres is $r$. The radius of the smaller of two concentric spheres is $\frac{r}{2}$. The probability the randomly selected point is nearer to the center
The probability that the randomly selected point is nearer to the surface of the sphere than to the center of the sphere is \( \frac{7}{8} \).

7. There are two solutions.

\[ m \angle B_1 = 45^\circ, m \angle C_1 = 105^\circ, AB_1 = 6 + 6\sqrt{3} \]
\[ m \angle B_2 = 135^\circ, m \angle C_2 = 15^\circ, AB_2 = 6\sqrt{3} - 6 \]

8. \( x \) is the radius of the cone.

\[ 16 = \frac{2 \times 4 \times x}{2} \]
\[ x^2 = 4 \quad \Rightarrow \quad SA = x^2 + \sqrt{x} \cdot \sqrt{17} \]
\[ x = 2 \quad \Rightarrow \quad SA = 4 + 4\sqrt{17} \]

9. \[ n = 4 + 4\sqrt{17} \]
10. To minimize the distance, reflect point $F$ (or point $E$) over the river. Using the Pythagorean Theorem, $FD = 10$. Since $ACDF$ is a rectangle,

$$AC = 10, \quad AB' \sim CBE \quad \text{Using} \quad \frac{10}{14} = \frac{x}{x} \quad \text{Pythagorean Theorem}$$

$$FB = \frac{65}{6} \quad \text{and} \quad FB = \frac{65}{6} \quad \text{and} \quad EB = \frac{91}{6}$$

Cowboy Bob would walk 26 meters.

11.

$$\frac{9}{x} = \frac{12}{14} \quad \text{x}$$

6 and 8

12. The common external tangent measures 16.

16

13. The triangle formed is a 30°-60°-90° triangle. The angle across from 3 is a 30° angle. The angle in the sector is 60°.

$$\frac{3 \cdot \sqrt{3}}{2} + \frac{1}{6} \cdot \pi (6)^2 = \frac{9 \sqrt{3}}{2} + 6\pi$$

$$\frac{9 \sqrt{3} + 12\pi}{2}$$

14. Put both equations in the form $Ax + By + C = 0$. $4x + 3y \quad 15 = 0$ and $4x + 3y \quad 20 = 0$ The distance between two parallel lines can be found using the formula

$$\frac{|C_2 - C_1|}{\sqrt{A^2 + B^2}} \quad \frac{|15 - 20|}{\sqrt{4^2 + 3^2}} = 1$$
15. $x + \sqrt{3}x + 2x = 15 + 5\sqrt{3}$, so $x = 5$. Each edge of the regular octahedron measures 10. The formula for the volume of a regular octahedron is $V = \frac{s^3 \sqrt{2}}{12} = \frac{10^3 \sqrt{2}}{12} = \frac{500 \sqrt{2}}{3}$

16. Using Ceva’s Theorem $\frac{CF}{FB} = \frac{BE}{EA} = \frac{AD}{DC} = 1$

$\frac{AG}{AF} = \frac{9}{19}$

17. The diagonals of a rhombus are the perpendicular bisectors of each other. The sides of the rhombus measure $\frac{3\sqrt{3}}{2}$. The perimeter is $6\sqrt{3}$.

$6\sqrt{3}$

18. Tangents to a circle are perpendicular to the radii at the point of tangency. Radii in the same circle are congruent and tangents from the same point to the same circle are congruent. $OPAQ$ is a kite. $A$ is supplementary to $O$. $\widehat{mPQ} = m\angle O$

Area of the sector $= \frac{m \text{arc} \cdot \pi r^2}{360^\circ} = \frac{108^\circ \pi \cdot 10^2}{360^\circ} = 30\pi$

30

19. $s =$ semiperimeter of the triangle

$A = \sqrt{s(s-a)(s-b)(s-c)}$

$A = \sqrt{38(38-22)(38-25)(38-29)}$

$A = 12\sqrt{494}$

$12\sqrt{494}$
20. The length of the midsegment of a trapezoid equals the average of the bases.
\[ \frac{x^2}{2} + \frac{x^2}{3} = 2x^2 + 2x \]
Substituting \( \frac{1}{2} \) for \( x \) in \( RS \) gives a negative answer.
\[ x = \frac{1}{2}, 3 \]
\[ 2\left( \frac{3}{2} \right)^2 + 2\left( \frac{3}{3} \right) = 9 \]

21. The height of smaller pyramid \( \frac{5}{8} \) of the height of the larger pyramid.

\[ \text{Height of smaller pyramid} : \text{Height of larger pyramid} = \frac{5}{8} \]

- Ratio of areas: \( \frac{25}{64} \)
- Area of \( ABCD = 25 \)
- Volume of frustum = Volume of larger pyramid - Volume of smaller pyramid
\[ \frac{64 \times 8}{3} - \frac{25 \times 8}{3} = 129 \]

22.
\[ 6\hat{i} + 20\hat{j} - 2\hat{k} - 8\hat{k} + 10\hat{i} + 3\hat{j} = 4\hat{i} + 23\hat{j} - 10\hat{k} \]
\[ 4\hat{i} + 23\hat{j} - 10\hat{k} \]

23. The circumcenter is equidistant from the vertices. The triangle is a right triangle with the right angle at \((3, 3)\). The midpoint of the hypotenuse is equidistant from the vertices.
\[ \left( \frac{-3 + 6}{2}, \frac{5 + 12}{2} \right) = \left( \frac{3}{2}, \frac{17}{2} \right) \]
\[ \left( \frac{3}{2}, \frac{17}{2} \right) \text{ or } \left( \frac{11}{2}, \frac{1}{2} \right) \text{ or } (1.5, 8.5) \]

24. \( b^2 = a^2 + b^2 \)
\[ \cos B \]
\[ b^2 = 7^2 + 5^2 \]
\[ \cos 45^\circ \]
\[ b^2 = 74 - 35\sqrt{2} \]
25. Each interior angle is supplementary to each exterior angle. \[ 180 - 144 = 36 \]
\[ \frac{360}{n} = 36 \]
\[ n = 10 \]

decagon