1. D

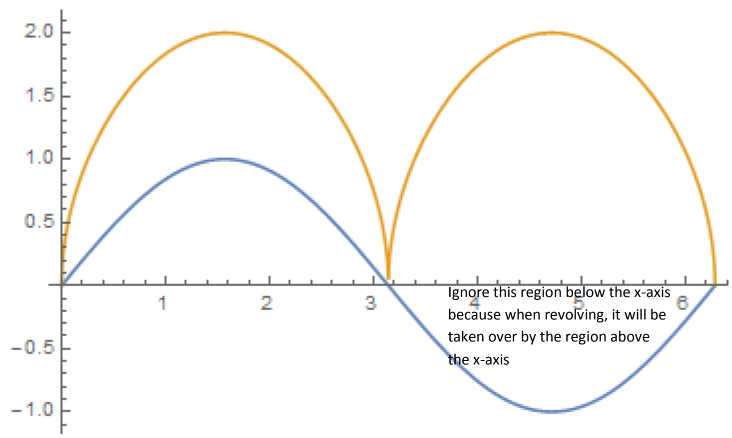
$$\int_1^{\sqrt[2018]{2017}} x^{2017}\,dx = \frac{x^{2018}}{2018}\bigg]_1^{\sqrt[2018]{2017}} = \frac{2017}{2018} - \frac{1}{2018} = \frac{1008}{1009}$$

2. E

$$\int_1^{2017^{\frac{1}{2018}}} y^{\frac{1}{2017}}\,dy = \frac{2017}{2018} y^{\frac{2018}{2017}}\bigg]_1^{2017^{\frac{1}{2018}}} = \frac{2017(\sqrt[2017]{2017} - 1)}{2018}$$

3. D

The first period from $x = 0$ to 2π of this region is plotted below with $2\sqrt{|\sin(x)|}$ being yellow:



We can find the volume revolved about the $x-$axis for this period and multiply the result by 2016/2 to find the volume from $x = 0$ to $x = 2016\pi$ by solving: $1008\pi\left(\int_0^{\pi}(4\sin[x] - \sin[x]^2)\,dx + \int_0^{\pi}(4\sin[x])\,dx\right) = 16128\pi - 504\pi^2$. Finally, we add the volume from $x = 2016\pi$ to 2017π by adding $\pi\left(\int_0^{\pi}(4\sin[x] - \sin[x]^2)\,dx\right) = 8\pi - \frac{\pi^2}{2}$. Thus, $16128\pi - 504\pi^2 + 8\pi - \frac{\pi^2}{2} = 16136\pi - \frac{1009\pi^2}{2}$

4. B

We can split the octagon into 8 isosceles triangles with vertex angle 45° with opposite side "s=2017". We start by using law of cosines to find the length of the other legs, "x", which will mean that the area of the triangle will be $\frac{1}{2}x^2\sin(45°)$.

From law of cosines, $x^2 + x^2 - 2x^2\sin(45°) = s^2$ so $x^2 = \frac{s^2}{2-\sqrt{2}} = \frac{s^2(2+\sqrt{2})}{2}$. Thus, the area of the triangle is $\frac{s^2(2+2\sqrt{2})}{8}$. Thus, the total area of the octagon is $s^2(2+2\sqrt{2})$. Plugging in 2017 gives $2017^2(2+2\sqrt{2})$.

5. C

Using the octagon area formula $s^2(2+2\sqrt{2})$, we have $\int_0^{2017} s^2(2+2\sqrt{2})\,dy =$

$$\int_0^{2017} (2017^y)^2(2+2\sqrt{2})\,dy = (2+2\sqrt{2})\int_0^{2017} (2017^2)^y\,dy$$

$$= (2+2\sqrt{2})\left[\frac{(2017^2)^y}{\ln(2017^2)}\right]_0^{2017} = \frac{(2+2\sqrt{2})(2017^{4034}-1)}{2\ln(2017)}$$

6. B

$\frac{x}{\sqrt{4+x^2}} = \frac{x}{\sqrt{9-x^2}}$ when $x = \pm\sqrt{\frac{5}{2}}, x = 0$. Area is $2\int_0^{\sqrt{\frac{5}{2}}} \frac{x}{\sqrt{4+x^2}} - \frac{x}{\sqrt{9-x^2}}\,dx =$ $2\int_0^{\sqrt{\frac{5}{2}}} \frac{x}{\sqrt{4+x^2}}\,dx - 2\int_0^{\sqrt{\frac{5}{2}}} \frac{x}{\sqrt{9-x^2}}\,dx = 2\sqrt{26} - 10$ using two trivial u substitutions of $4 + x^2$ and $9 - x^2$.

7. A

First solve $\sin(4x) + \cos(2x) = 0$. $2\sin(2x)\cos(2x) + \cos(2x) = 0 \rightarrow$ $\cos(2x) = 0, \sin(2x) = -\frac{1}{2}$ which occurs at $x = \frac{\pi}{4}, \frac{7\pi}{12}$ for the range. Then calculate:

$$\left|\int_0^{\frac{\pi}{4}} \sin(4x) + \cos(2x)\,dx\right| + \left|\int_{\frac{\pi}{4}}^{\frac{7\pi}{12}} \sin(4x) + \cos(2x)\,dx\right|$$

$$+ \left|\int_{\frac{7\pi}{12}}^{\frac{2\pi}{3}} \sin(4x) + \cos(2x)\,dx\right|$$

$$= \left[-\frac{\cos(4x)}{4} + \frac{\sin(2x)}{2}\right]_0^{\frac{\pi}{4}} + \left[-\frac{\cos(4x)}{4} + \frac{\sin(2x)}{2}\right]_{\frac{\pi}{4}}^{\frac{7\pi}{12}} + \left[-\frac{\cos(4x)}{4} + \frac{\sin(2x)}{2}\right]_{\frac{7\pi}{12}}^{\frac{2\pi}{3}}$$

$$= \frac{1}{4} + \frac{1}{2} + \frac{1}{4} + \frac{1}{4} + \frac{1}{2} + \frac{1}{8} + \frac{1}{4} + \frac{1}{8} - \frac{\sqrt{3}}{4} + \frac{1}{8} + \frac{1}{4} = \frac{21 - 2\sqrt{3}}{8}$$

8. A

Solving for y when $x = 3$ gives $y = \pm\frac{\pi}{3}$. Thus the volume of revolution is equal to: $\pi\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}}(3-1)^2-(1+\sec(y)-1)^2dy = \frac{8\pi^2}{3}-2\pi\sqrt{3}$

9. C

Trivial: using cylindrical shells we have $\int_0^{\pi} 2\pi(distance-y)f(y)dy =$ $\int_0^{\pi} 2\pi(2018-y)\sqrt{2017\sin(y)}dy$

10. A

$$\frac{\frac{9-1}{2}}{3}\left(2+4\ \cdot 2^3+42\cdot 2^5+4\ \cdot 2^7+2^9\right)=748$$

11. C

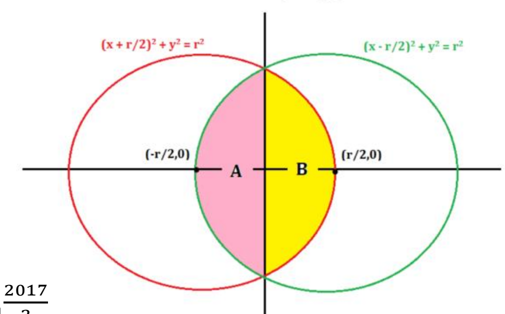
$x=\frac{1}{3}(y^2+2)^{\frac{3}{2}}\to(3x)^{\frac{2}{3}}-2=y^2, \frac{3^{\frac{3}{2}}}{3}\le x\le\frac{11^{\frac{3}{2}}}{3}. 2yy'=\frac{2}{3}\cdot 3(3x)^{-\frac{1}{3}}\to y'=\frac{(3x)^{-\frac{1}{3}}}{y}$

Using the surface area formula $2\pi\int_{\frac{3^{\frac{3}{2}}}{3}}^{\frac{11^{\frac{3}{2}}}{3}} y\sqrt{1+(y')^2}dx = 2\pi\int_{\frac{3^{\frac{3}{2}}}{3}}^{\frac{11^{\frac{3}{2}}}{3}}\sqrt{y^2+(3x)^{-\frac{2}{3}}}dx$

$$= 2\pi\int_{\frac{3^{\frac{3}{2}}}{3}}^{\frac{11^{\frac{3}{2}}}{3}}\sqrt{(3x)^{\frac{2}{3}}-2+(3x)^{-\frac{2}{3}}}dx = 2\pi\int_{\frac{3^{\frac{3}{2}}}{3}}^{\frac{11^{\frac{3}{2}}}{3}}\frac{\sqrt{(3x)^{\frac{4}{3}}-2(3x)^{\frac{2}{3}}+1}}{(3x)^{\frac{1}{3}}}dx = 2\pi\int_{\frac{3^{\frac{3}{2}}}{3}}^{\frac{11^{\frac{3}{2}}}{3}}\frac{(3x)^{\frac{2}{3}}-1}{(3x)^{\frac{1}{3}}}dx.$$

$$u=(3x)^{\frac{2}{3}}-1, \frac{du}{2}=\frac{dx}{(3x)^{\frac{1}{3}}}\to 2\pi\int_{\frac{3^{\frac{3}{2}}}{3}}^{\frac{11^{\frac{3}{2}}}{3}}\frac{(3x)^{\frac{2}{3}}-1}{(3x)^{\frac{1}{3}}}dx = \pi\int_2^{10}u\,du = 48\pi$$

12. B



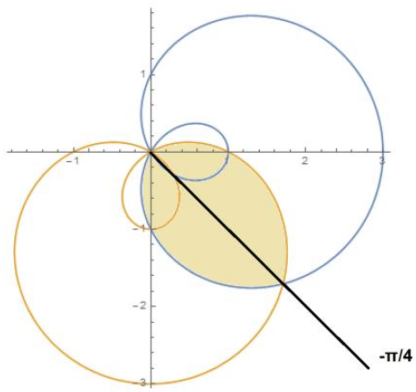
Place the cross sections of the great circles as such: half the volume is calculated by revolving the upper half of B about the x-axis which is calculated with

$$\pi\int_0^{\frac{2017}{2}} 2017^2-\left(x+\frac{2017}{2}\right)^2dx = 2017^2x-\frac{\left(x+\frac{2017}{2}\right)^3}{3}\Bigg]_0^{\frac{2017}{2}}\pi = \frac{2017^3}{2}-\frac{2017^3}{3}+\frac{2017^3}{24} =$$

$2017^3\cdot\frac{5}{24}$. So full volume is $2017^3\cdot\frac{5}{12}$

13. B

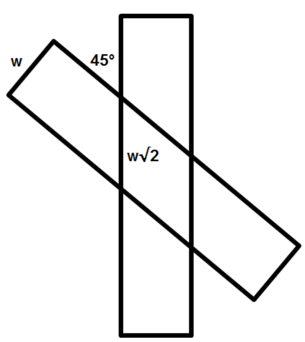
A rough sketch should give:

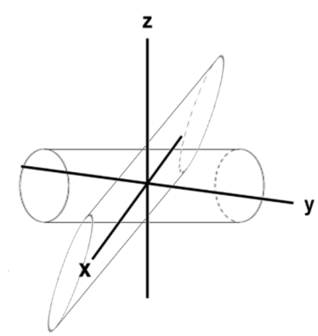


Thus, in order to find the area we can simply calculate twice the area of $1 - 2\sin(\theta)$ from $-\frac{\pi}{4}$ to $\frac{\pi}{6} = \int_{-\frac{\pi}{4}}^{\frac{\pi}{6}}(1 - 2\sin(\theta))^2 d\theta = -1 - 2\sqrt{2} + \frac{3\sqrt{3}}{2} + \frac{5\pi}{4}$.

14. E

The intersection will have perpendicular cross sections of rhombuses with a circular base. If we look at the cross section of the cylinder as rectangles we can see:





The widths of the rectangle is a function of the diameter in relationship to the height of the cylinder , z. The circular cross section of one of the cylinders can be written as $z^2 + x^2 = r^2 \rightarrow z = \sqrt{r^2 - x^2}$. So the width of the rectangle is expressed as twice that: $2\sqrt{r^2 - x^2}$. Thus the side of the rhombuses is: $2\sqrt{2} \cdot \sqrt{r^2 - x^2}$. The area of the rhombuses is calculated (through two isosceles triangle) using $\left(2\sqrt{2} \cdot \sqrt{r^2 - x^2}\right)^2 \sin\left(\frac{\pi}{4}\right) = 4\sqrt{2}(r^2 - x^2)$. The volume is then calculated using $\int A(x)dx = \int_{-r}^{r} 4\sqrt{2}(r^2 - x^2)dx = \frac{16\sqrt{2}}{3}r^3$.

15. E

Rewrite the expression as: $r = \left(\frac{-e^{-\frac{3i\theta}{2}}+e^{\frac{3i\theta}{2}}}{2i}\right)\left(\frac{1}{2}e^{-\frac{3i\theta}{2}}+\frac{1}{2}e^{\frac{3i\theta}{2}}\right) = \sin\left(\frac{3\theta}{2}\right)\cos\left(\frac{3\theta}{2}\right) = \frac{1}{2}\sin(3\theta)$. Area is $\frac{1}{2}\int_0^{\pi}(\frac{1}{2}\mathrm{Sin}[3\theta])^2\,d\theta = \frac{\pi}{16}$, or simply $\frac{a^2}{4}\pi = \frac{\left(\frac{1}{2}\right)^2}{4}\pi$

16. D

The standard form of a rotated conic is given by $A(x-h)^2 + B(x-h)(y-k) + C(y-k)^2 - f = 0$ where (h,k) is the center of the conic. When expanded, the standard form is $-f + ah^2 + bhk + ck^2 - 2ahx - bkx + ax^2 - bhy - 2cky + bxy + cy^2 = 0$. From this, we know that $a = 2, b = 1, c = 3, -2ah - bk = 2, -bh - 2ck = -11, -f + ah^2 + bhk + ck^2 = 7$. Now, substituting a, b, c to solve for h, k, f yields the system of equations $-4h - k = 2, -h - 6k = -11, -f + 2h^2 + hk + 3k^2 = 7,$ which gives $h = -1, k = 2, f = 5$.

17. A

The area of an ellipse given by $A(x-h)^2 + B(x-h)(y-k) + C(y-k)^2 - f = 0$ is given by the formula $\frac{2\pi i}{\frac{1}{f}b^2 - 4ac}$. Plugging in our values found in question 16, we get $\frac{2\pi i}{\frac{1}{5}\sqrt{-23}} = \frac{10\pi}{\sqrt{23}}$.

18. C

Using Pappus's Centroid Theorem, $V = 2\pi Ar, A = \frac{10\pi}{\sqrt{23}}, r =$ distance from $(-1,2)$to $x = y \to \frac{|-1\cdot 1 - 2\cdot(-1)|}{\sqrt{1^2+1^2}} = \frac{3}{\sqrt{2}}$. $V = 2\pi\cdot\frac{10\pi}{\sqrt{23}}\cdot\frac{3}{\sqrt{2}} = \frac{60\pi^2}{\sqrt{46}}$.

19. A

Rewrite as: $x^4 + 2x^2y^2 + y^4 = -2xy \to (x^2+y^2)^2 = -2xy \to$ $(r^2)^2 = -2r\sin(\theta)\, r\cos(\theta) = -r^2\sin(2\theta) \to r^2 = -\sin(2\theta)$. $-\sin(2\theta)$ is positive from $-\frac{\pi}{2}$ to 0 and $\frac{3\pi}{2}$ to 2π. Area= $2\cdot\frac{1}{2}\int_{-\frac{\pi}{2}}^{0}\sin[2\theta]\,d\theta = 1$

20. C

The line intersects the parabola at $(9,-1)$ and $(1,1)$. Thus the volume is calculated using washers by $\int_{-1}^{1}(5-4y+1)^2 - ((y-2)^2+1)^2 dy =$ $\pi\int_{-1}^{1}(36-48y+16y^2) - (25-40y+26y^2-8y^3+y^4)dy$. We can ignore the odd powered terms and proceed to simplify the integral to:

$2\pi \int_0^1 (36+16y^2)-(25+26y^2+y^4)dy = 2\pi \int_0^1 11-10y^2-y^4 dy = \frac{224\pi}{15}$.

21. B

We can convert this into $y = 1-x^2$ for $x \geq 0$. The area is simply calculated with $\int_0^1 (1-x^2)dx = \frac{2}{3}$

22. B

Start with $\int_0^1 \frac{\ln(x^2+2x+1)}{x^2+1}dx = \int_0^1 \frac{\ln((x+1)^2)}{x^2+1}dx = 2\int_0^1 \frac{\ln(x+1)}{x^2+1}dx$. $u = \arctan(x) \rightarrow du = \frac{1}{1+x^2}dx \rightarrow 2\int_0^1 \frac{\ln(x+1)}{x^2+1}dx = 2\int_0^{\frac{\pi}{4}} \ln(\tan(u)+1)\,du$.

Let $I = \int_0^{\frac{\pi}{4}} \ln(\tan(u)+1)\,du$. Make a substitution $u = \frac{\pi}{4} - v \rightarrow du = -dv \rightarrow$ $\int_0^{\frac{\pi}{4}} \ln(\tan(u)+1)\,du = \int_{\frac{\pi}{4}}^0 -\ln\left(\tan\left(\frac{\pi}{4}-v\right)+1\right)dv = \int_0^{\frac{\pi}{4}} \ln\left(\tan\left(\frac{\pi}{4}-v\right)+1\right)dv$ which we can rewrite as $\int_0^{\frac{\pi}{4}} \ln\left(\tan\left(\frac{\pi}{4}-u\right)+1\right)du = \int_0^{\frac{\pi}{4}} \ln\left(\frac{1-\tan(u)}{1+\tan(u)}+1\right)du =$ $\int_0^{\frac{\pi}{4}} \ln\left(\frac{2}{1+\tan(u)}\right)du = I$. $I + I = 2I = \int_0^{\frac{\pi}{4}} \ln(\tan(u)+1)\,du + \int_0^{\frac{\pi}{4}} \ln\left(\frac{2}{1+\tan(u)}\right)du =$ $\int_0^{\frac{\pi}{4}} \ln(\tan(u)+1) + \ln\left(\frac{2}{1+\tan(u)}\right)du = \int_0^{\frac{\pi}{4}} \ln(2)\,du = \frac{\pi}{4}\ln(2)$. We wanted:

$2\int_0^{\frac{\pi}{4}} \ln(\tan(u)+1)\,du = 2I = \frac{\pi}{4}\ln(2)$.

23. C

Set two squares to have sides $\sin(x)$ and $\cos(x)$ and assume $\sin(x) \geq \cos(x) \geq 0$. Then, the rectangle we have will have one side equal to $\sin(x)$ and the other side equal to $\sin(x)+\cos(x)$. The area, $N = \sin(x)(\sin(x)+\cos(x)) = \frac{1-\cos(2x)+\sin(2x)}{2}$.

Taking the derivative and setting it equal to 0 yields $\sin(2x)+\cos(2x) = 0 \rightarrow$ $\tan(2x) = -1 \rightarrow x = \frac{3\pi}{8}, \frac{7\pi}{8}, \frac{11\pi}{8}, \frac{15\pi}{8}$. Because we want $\sin(x) \geq \cos(x) \geq 0$, only $\frac{3\pi}{8}$ works. Thus, $N = \frac{1-\cos(2x)+\sin(2x)}{2} = \frac{1+\sqrt{2}}{2}$

24. C

Convert everything to polar to get: $\frac{1}{9}x^2 + y^2 = 1 \rightarrow \frac{r^2\cos^2(\theta)}{9} + r^2\sin^2(\theta) = 1 \rightarrow$ $r^2 = \frac{9}{\cos^2(\theta)+9\sin^2(\theta)}$, $y = \frac{1}{2}x \rightarrow \theta = \arctan\left(\frac{1}{2}\right)$. The area is $\frac{1}{2}\int_0^{\arctan\left(\frac{1}{2}\right)} r^2 d\theta =$

$\frac{1}{2}\int_0^{\arctan\left(\frac{1}{2}\right)} \frac{9}{\cos^2(\theta)+9\sin^2(\theta)}\,d\theta = \frac{1}{2}\int_0^{\arctan\left(\frac{1}{2}\right)} \frac{9}{\cos^2(\theta)+9\sin^2(\theta)} \cdot \frac{\sec^2(\theta)}{\sec^2(\theta)}\,d\theta =$

$\frac{1}{2}\int_0^{\arctan\left(\frac{1}{2}\right)} \frac{9\sec^2(\theta)}{1+9\tan^2(\theta)}\,d\theta$. Make the substitution $u = \tan(\theta)$ to get

$\frac{1}{2}\int_0^{\frac{1}{2}} \frac{9}{1+9u^2}\,du = \frac{1}{2}[3\arctan(3u)]_0^{\frac{1}{2}} = \frac{3}{2}\arctan(\frac{3}{2})$

25. A

We solve for m in $\frac{1}{2}\int_{\arctan(m)}^{\frac{\pi}{2}} \frac{9\sec^2(\theta)}{1+9\tan^2(\theta)}\,d\theta = \frac{3}{2}\arctan\left(\frac{3}{2}\right)$. $\frac{1}{2}\int_{\mathrm{Arctan}(m)}^{\frac{\pi}{2}} \frac{9\sec^2(\theta)}{1+9\tan^2(\theta)}\,d\theta =$

$\frac{1}{2}\int_m^{\infty} \frac{9}{1+9u}\,du = \frac{1}{2}[3\arctan(3u)]_m^{\infty} = \frac{1}{2}\left(\frac{3\pi}{2} - 3\arctan(3m)\right) = \frac{3}{2}\arctan\left(\frac{3}{2}\right) \rightarrow$

$\frac{\pi}{2} - \arctan(3m) = \arctan\left(\frac{3}{2}\right) \rightarrow \tan\left(\frac{\pi}{2} - arctan(3m)\right) = \frac{3}{2} \rightarrow$

$\cot(arctan(3m)) = \frac{3}{2} \rightarrow \frac{1}{3m} = \frac{3}{2} \rightarrow m = \frac{2}{9}$

26. C

From symmetry, all we have to do is find twice the area above the x-axis and the first curve which is found by $\int_{-1}^{\infty}\left[\left(\frac{x^{1513}}{1+x^{2018}}\right)^2\right]dx$. Let's make the substitution that $x^{1009} = \tan(u)$. $1009x^{1008}dx = \sec^2(u)du$. Then, our integral becomes

$\int_{-\frac{\pi}{4}}^{\frac{\pi}{2}}\left[\frac{x^{3026}}{(1+\tan^2(u))^2}\right]dx = \int_{-\frac{\pi}{4}}^{\frac{\pi}{2}}\left[\left(\frac{x^{1008}x^{2018}dx}{\sec^4(u)}\right)\right] = \int_{-\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{\tan^2(u)}{1009\sec^2(u)}\,du$

$= \frac{1}{1009}\int_{-\frac{\pi}{4}}^{\frac{\pi}{2}} \sin^2(u)\,du = \frac{1}{1009}\int_{-\frac{\pi}{4}}^{\frac{\pi}{2}}\left(\frac{1-\cos(2u)}{2}\right)du = \frac{1}{1009}\left[\frac{2u+\sin(2u)}{4}\right]_{-\frac{\pi}{4}}^{\frac{\pi}{2}} = \frac{-2+3\pi}{8072}$. Twice that is $\frac{-2+3\pi}{4036}$.

27. E

Arc length of $g(x) = I_1 = \int_a^b \sqrt{1 + \left(\sqrt{\ln(e\cot(x^2)) - 1}\right)^2}\,dx = \int_a^b \ln(e\cot(x^2))\,dx =$ p. Area under $f(x) = I_2 = \int_a^b \ln(e\,tan(x^2))\,dx = p$. $I_1 + I_2 = 2p =$

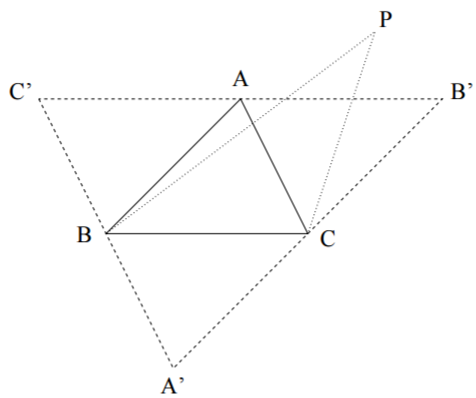
$\int_a^b \ln(e\,tan(x^2))\,dx + \int_a^b \ln(e\cot(x^2))\,dx = \int_a^b \ln(e\,tan(x^2)) + \ln(e\cot(x^2))\,dx$

$= \int_a^b \ln(e\,tan(x^2)\cdot e\cot(x^2))\,dx = \int_a^b \ln(e^2)\,dx = \int_a^b 2dx = 2b - 2a = 2p$.

$p = b - a$

28. D

Pick three points A, B, C in S such that the triangle ΔABC has maximum area. Draw lines A′B′, B′C ′, C ′A′ parallel to AB, BC, CA respectively. For any point P above line B′C , the triangle ΔPBC will have larger area than ΔABC because it has the same base but more height than ΔABC, hence all points of S must below line B′C ′. A similar argument leads to the conclusion that all points of S must be on the triangle ΔA′B′C ′. Also note that triangles ΔABC and ΔCB′A are each half of the parallelogram ABCB′, so they are equal. For the same reason ΔBAC′ and ΔA′CB are also equal to ΔABC. Since the area of ΔABC is at most 1, then the area of ΔA′B′C ′ is at most 4.



29. B

We can solve this using Theorem of Pappus. We begin by finding the area of the triangle by finding two vectors formed by the three points and computing the cross product. Choosing (1,1,1) as the end point (tail) of the vectors gives $\langle 0,1,2\rangle$ and $\langle -3,-1,1\rangle$. $\langle 0,1,2\rangle \times \langle -3,-1,1\rangle = \langle 3,-6,3\rangle$. The area of the triangle is half of the cross product's magnitude giving $\frac{1}{2}\sqrt{9+36+9} = \frac{3\sqrt{6}}{2}$. The centroid is just the average of the three points: (0,1,2). To find the distance from the point to the line we begin by finding the magnitude of the vector from $(4,1,-2)$ to $(0,1,2) \rightarrow \langle 4,0,-4\rangle \rightarrow \left|\left|\langle 4,0,-4\rangle\right|\right| = 4\sqrt{2}$. This vector makes an angle, θ, with the line vector $\langle 3,1,-1\rangle$ such that $\cos(\theta) = \frac{\langle 3,1,-1\rangle\cdot\langle 4,0,-4\rangle}{|\langle 4,0,-4\rangle|\,|\langle 3,1,-1\rangle|} = 2\sqrt{\frac{2}{11}} \rightarrow \sin(\theta) = \frac{\sqrt{3}}{\sqrt{11}}$. The distance to the line is then calculated as $4\sqrt{2}\sin(\theta) = 4\sqrt{\frac{6}{11}}$. The volume is then calculated as

$$V = 2\pi Ar = 2\pi \cdot \frac{3\sqrt{6}}{2} \cdot 4\sqrt{\frac{6}{11}} = 72\sqrt{\frac{1}{11}}\pi.$$

30. D

Essentially as n becomes large in $y = \sqrt[n]{x}$ and $y = x^n$, the graph looks close to a 1x1 square, thus the centroid should be as close to $(0.5,0.5)$ as possible thus the answer is $\left(\frac{1008}{2017}, \frac{1008}{2017}\right)$.