For all questions, E “NOTA” means none of the above answers is correct.

1. Suppose the function $f(x)-f(2x)$ has derivative 5 at $x = 1$ and derivative 7 at $x = 2$. Find the derivative of $f(x)-f(4x)$ at $x=1$.

   a) 8 b) 12 c) 16 d) 19 e) NOTA

2. A totally spherical cow is being pulled out of the bottom of Niagara Falls. The bottom of the falls is 100 feet down and the cow and all the water on her weighs 200 pounds. She is being hauled up at a constant rate with a chain which weighs 2 pounds per foot. The water on the cow drips off the cow at the rate of ½ pounds per foot as she is being hauled up. How much work (in ft-lbs) is required to rescue the cow?

   a) 25500 b) 27500 c) 32500 d) 35500 e) NOTA

3. Evaluate: $\lim\limits_{x\to 0}\dfrac{\frac{d}{dx}\frac{\sin x}{x}}{x}$

   a) $\dfrac{-1}{3}$ b) $\dfrac{1}{3}$ c) $\dfrac{-1}{2}$ d) $\dfrac{1}{2}$ e) NOTA

4. A balloon’s circular cross-section has radius $y=\sqrt{2x-x^2}\cdot e^{\frac{-x}{2}}$ for this function’s entire domain. What is the volume of the balloon?

   a) $\dfrac{4}{e^2}$ b) $\dfrac{2\pi}{e^2}$ c) $\dfrac{4}{e}$ d) $\dfrac{2\pi}{e}$ e) NOTA

5. Water flows into a tank at 3 gallons per minute. The tank initially contains 100 gallons of water, with 50 pounds of salt. The tank is well-mixed, and drains at a rate of 2 gallons per minute. How many pounds of salt are left after one hour?

   a) $\dfrac{25}{16}$ b) $\dfrac{625}{16}$ c) $\dfrac{625}{32}$ d) $\dfrac{625}{64}$ e) NOTA

6. In Buffalo, there is a man named Calvin who has an infinite amount of time. This year, he is walking continuously at a speed of $\dfrac{1}{1+t^2}$, starting at time t = 0. If he continues to walk for an infinite amount of time, how far will he walk?

   a) $\dfrac{\pi}{4}$ b) $\dfrac{\pi}{2}$ c) π d) $\dfrac{3\pi}{2}$ e) NOTA

7. A rectangular pyramid tower is being built on a circular island of radius two. The height of the tower is equal to its width. What is the maximum volume of the tower?

   a) $\dfrac{64\sqrt{3}}{9}$ b) $\dfrac{128\sqrt{3}}{9}$ c) $\dfrac{64\sqrt{3}}{27}$ d) $\dfrac{128\sqrt{3}}{27}$ e) NOTA

8. Evaluate: $\int_0^{10} \left[(x-5)+(x-5)^2+(x-5)^3\right]dx$

a) $\frac{250}{3}$ b) $\frac{250}{9}$ c) $\frac{125}{3}$ d) $\frac{125}{9}$ e) NOTA

9. The set of all points (x, y) in the plane satisfying $x^{2/5}+|y|=1$ form a curve enclosing a region. Compute the area of this region.

a) $\frac{2}{7}$ b) $\frac{4}{7}$ c) $\frac{6}{7}$ d) $\frac{8}{7}$ e) NOTA

10. Silas does nothing but sleep, drink coffee, and prove theorems, and he never does more than one at a time. It takes 5 minutes to drink a cup of coffee. When doing math, Silas proves $(s+\ln c)$ theorems per hour, where c is the number of cups of coffee he drinks per day, and s is the number of hours he sleeps per day. How much coffee should Silas drink in a day to prove the most theorems?

a) 5 b) 6 c) 9 d) 12 e) NOTA

11. If $f'(x)$ and $g'(x)$ exist and $f'(x)>g'(x)$ for all real x, then the graph of $y=f(x)$ and the graph of $y=g(x)$

a) Intersect exactly once b) Intersect no more than once c) Do not intersect d) May intersect more than once e) NOTA

12. At $t=0$, a particle starts at rest and moves along a line in such a way that at time t its acceleration is $24t^2$ ft/sec². Through how many feet does the particle move during the first 2 seconds?

a) 32 b) 48 c) 64 d) 96 e) NOTA

13. If $f''(x)-f'(x)-2f(x)=0,\ f'(0)=-2$ and $f(0)=2,$ then $f(1)=$

a) 0 b) 1 c) e^2 d) $2e^{-1}$ e) NOTA

14. Find the complete interval of convergence for the series $\sum_{k=1}^{\infty}\frac{(x+1)^k}{k^2}$

a) $0<x<2$ b) $0\le x\le 2$ c) $-2<x\le 0$ d) $-2\le x\le 0$ e) NOTA

15. Which of the following is true about the graph of $y=\ln\left|x^2-1\right|$ in the interval (-1, 1)?

a) It is increasing b) It attains a relative minimum at (0, 0) c) It has a range of all reals d) It is concave down e) NOTA

16. Find the area of the region enclosed by the polar graph $r = 1 - \cos\theta$.

a) $\frac{3\pi}{4}$ b) π c) $\frac{3\pi}{2}$ d) 2π e) NOTA

17. Suppose $g'(x) < 0$ for all $x \geq 0$ and $F(x) = \int_0^x t g'(t)\,dt$ for all $x \geq 0$. Which of the following statements is FALSE?

a) F is not increasing b) F is continuous for all $x > 0$ c) $F(x) = xg(x) - \int_0^x g(t)\,dt$

d) $F'(x)$ exists for all $x > 0$ e) NOTA

18. Let f be the function satisfying $f'(x) = 4x - 2xf(x)$ for all real x, with $f(0) = 5$ and $\lim_{x\to\infty} f(x) = 2$. Find the value of $\int_0^\infty (4x - 2xf(x))\,dx$.

a) -7 b) -3 c) 3 d) 7 e) NOTA

19. If k is a positive constant, which of the following is a logistic differential equation?

a) $\frac{dy}{dt} = kt$ b) $\frac{dy}{dt} = ky$ c) $\frac{dy}{dt} = kt(1-t)$ d) $\frac{dy}{dt} = ky(1-t)$ e) NOTA

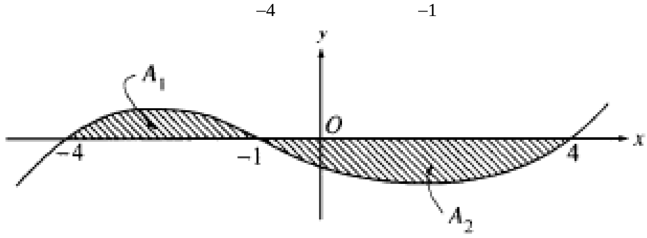
20. The area of a circular region is increasing at a rate of 96π square meters per second. When the area of the region is 64π square meters, how fast in meters per second, is the radius of the region increasing?

a) 6 b) 8 c) 16 d) $4\sqrt{3}$ e) NOTA

21. The general solution of the differential equation $y' = y + x^2$ is $y =$

a) Ce^x b) $Ce^x + x^2$ c) $-x^2 - 2x - 2 + C$ d) $Ce^x - x^2 - 2x - 2$ e) NOTA

22. The graph of $y = f(x)$ is shown below. If A_1 and A_2 are positive numbers that represent the area of the shaded regions, then in terms of A_1 and A_2, $\int_{-4}^{4} f(x)\,dx - 2\int_{-1}^{4} f(x)\,dx =$



a) A_1 b) $A_1 - A_2$ c) $2A_1 - A_2$ d) $A_1 + A_2$ e) NOTA

23. Evaluate $\lim\limits_{x\to 0}\dfrac{\sin^2(5x)\tan^3(4x)}{(\ln(2x+1))^5}$.

a) 0 b) 1 c) 10 d) 50 e) NOTA

24. Calvin, Inc can produce at most 24 perfectly spherical cow statues a week (yes, like the one at the bottom of Niagara Falls). He sets his price at $D = 110 - 2n$, where n is the number of statues produced that week. Producing n statues costs $600 + 10n + n^2$ dollars. How many statues should he make each week in order to maximize profit? He sells all his cows every week, and he can't divide them into smaller than whole number units.

a) 15 b) 16 c) 17 d) 18 e) NOTA

25. Evaluate $\displaystyle\int_{-2}^{2}\frac{1+x^2}{1+2^x}\,dx$.

a) $\dfrac{8}{3}$ b) $\dfrac{14}{3}$ c) $\dfrac{16}{3}$ d) $\dfrac{28}{3}$ e) NOTA

26. Let f be a function defined and continuous on the interval $[a,b]$. If f has a relative maximum at c and $a < c < b$, which of the following statements is true?

I. $f'(c)$ exists II. If $f'(c)$ exists, then $f'(c) = 0$. III. If $f''(c)$ exists, then $f''(c) \le 0$

a) II only b) III only c) I and II only d) I and III only e) NOTA

27. If $\displaystyle\sum_{n=0}^{\infty} a_n x^n$ is a Taylor series that converges to $f(x)$ for all real x, then $f'(1) =$

a) 0 b) a_1 c) $\displaystyle\sum_{n=0}^{\infty} a_n$ d) $\displaystyle\sum_{n=1}^{\infty} na_n$ e) NOTA

28. If $f(x) = g(x) + 7$ for $3 \le x \le 5$, then $\displaystyle\int_3^5 [f(x) + g(x)]\,dx =$

a) $2\displaystyle\int_3^5 g(x)dx + 7$ b) $2\displaystyle\int_3^5 g(x)dx + 14$ c) $2\displaystyle\int_3^5 g(x)dx + 28$ d) $\displaystyle\int_3^5 g(x)dx + 7$ e) NOTA

29. Let f be a twice differentiable function such that $f(1)=2$ and $f(3)=7$. Which of the following must be true for the function f on the interval $1 \le x \le 3$?

I. The average rate of change of f is $\frac{5}{2}$.

II. The average value of f is $\frac{9}{2}$.

III. The average value of f' is $\frac{5}{2}$.

a) None b) I only c) III only d) I and III only e) NOTA

30. If $s_n = \left(\frac{(5+n)^{100}}{5^{n+1}}\right)\left(\frac{5^n}{(4+n)^{100}}\right)$, to what number does the sequence $\{s_n\}$ converge?

a) $\frac{1}{5}$ b) 1 c) $\frac{5}{4}$ d) $\left(\frac{5}{4}\right)^{100}$ e) NOTA