1. D—The derivative of \( f(x) - f(2x) \) is \( f'(x) - 2f'(2x) \). So, \( f'(1) - 2f'(2) = 5 \), \( f'(2) - 2f'(4) = 7 \). Thus, \( f'(1) - 4f'(4) = (f'(1) - 2f'(2)) + 2(f'(2) - 2f'(4)) = 5 + 2(7) = 19 \).

2. B—Let \( x \) indicate the distance the cow has yet to travel. Then the work for a distance \( 1 \) is \( (2x + 200 - \frac{1}{2}(100 - x))dx \). So the total work is \( \int_0^{100} (2.5x + 150)dx = 27500 \) ft-lbs. foot-pounds.

3. A—\[
\lim_{x \to 0} \frac{d}{dx} \sin x = \lim_{x \to 0} \frac{x \cos x - \sin x}{x^3} = \lim_{x \to 0} \frac{\cos x - x \sin x - \cos x}{3x} = \lim_{x \to 0} -\sin x = -\frac{1}{3}.
\]

4. E—\[
\frac{4\pi}{e^2} V = \pi \int_0^2 \left(\sqrt{2x - x^2} \cdot e^{-\frac{x}{2}}\right)^2 dx = \pi \int_0^2 (2x - x^2)e^{-x} dx = \pi x^2e^{-x} \bigg|_0^2 = \frac{4\pi}{e^2}
\]

5. C—Let \( w(t) \) and \( s(t) \) denote the amounts of water and salt, respectively, in the tank at time \( t \). We can see that \( w(t) = t + 100 \). Since the tank is constantly mixed, we know that \( \frac{ds}{dt} = -\frac{s(t)}{w(t)} \Rightarrow \frac{ds}{s} = -\frac{dt}{t+100} \).

So, \( \ln(s) = -2\ln(C(t+100)) \Rightarrow s = \frac{C}{(t+100)^2} \). Since \( s(0) = 50 \), then \( C = 500000 \) and \( s(60) = \frac{625}{3} \).

6. B—\[
d = \int_0^\infty \frac{dt}{1 + t^2} = \tan^{-1}(t) \bigg|_0^\infty = \frac{\pi}{2}.
\]

7. D—Let the circular island be a circle of radius 2 centered at the origin. Let the length of the rectangular base be from \(-x\) to \(x\) and the width from \(-y\) to \(y\). By the equation of a circle, \( x^2 = 4 - y^2 \). Then,
\[
V = \frac{1}{3}(2x)^2(2y) = \frac{8}{3}x^2y = \frac{8}{3}(4y - y^3) \Rightarrow \frac{dV}{dy} = \frac{8}{3}(4 - 3y^2) = 0 \Rightarrow y = \sqrt[3]{\frac{4}{3}} \text{ and } V = \frac{128\sqrt{3}}{27}.
\]

8. A—This integral is equal to \[
\int_5^0 (x + x^2 + x^3)dx = \frac{250}{3}.
\]

9. D—The set of points satisfying the equation form a closed curve that encloses a region. This curve is preserved if we transform \( x \rightarrow -x \) and \( y \rightarrow -y \), so it is symmetric in all 4 quadrants. In particular, we can find the area in the first quadrant. In this quadrant, we can rewrite our equation as \( y = 1 - x^{2/5} \). This curve intersects the coordinate axes at \((0, 1)\) and \((1, 0)\), and it is continuous, so the area is \[
A = \int_0^1 (1 - x^{2/5})dx = \frac{2}{7} \text{. The total area is } 4A = \frac{8}{7}.
\]

10. D—The number of theorems proven is \((s + \ln c)(24 - s - \frac{c}{12})\). Differentiating with respect to \( s \) gives 
\[
24 - \frac{c}{12} - 2s - \ln c = 0, \text{ so } s = 12 - \frac{c}{24} - \frac{1}{2} \ln c. \text{ This is a maximum since the second derivative is } -2.
\]

Plugging this back in and simplifying gives \((12 - \frac{c}{24} + \frac{\ln c}{2})^2 = f(c)^2\). This differentiates to \(2f'(c)f(c)\), so the derivative will be zero when either \(f(c)\) or \(f'(c)\) is zero. \(f(c) = 0\) is too difficult to solve, but
\[ f'(c) = \frac{1}{2c} - \frac{1}{24}, \] so \( c = 12 \) is a solution. Testing shows that it is a maximum.

11. B—The graphs do not need to intersect or they could intersect. However, if they do intersect, then they will intersect no more than once because \( f(x) \) grows faster than \( g(x) \).

12. A—\( a(t) = 24t^2, v(t) = 8t^3 + c \) and \( v(0) = 0 \Rightarrow c = 0 \). The particle is always moving to the right, so the distance \( \int_{0}^{3} 8t^3 \, dt = 32 \).

13. D—\( y'' - y' - 2y = 0, y'(0) = -2, y(0) = 2 \). The characteristic equation is \( r^2 - r - 2 = 0 \Rightarrow r = -2, r = 1 \). So, the general solution to the differential equation is \( y = c_1 e^{-x} + c_2 e^{x}, \) Using the initial conditions, one can solve \( c_1 = 2, c_2 = 0 \). So, the solution is \( f(x) = 2e^{-x} \Rightarrow f(1) = 2e^{-1} \).

14. D—The ratio test shows that the series is convergent for any value of \( x \) that makes \( |x + 1| < 1 \). This gives you \(-2 < x < 0 \). Checking endpoints shows that both series are convergent, so the interval is \(-2 \leq x \leq 0 \).

15. D—For \( x \) in the interval \((-1, 1)\), \( g(x) = \left|x^2 - 1\right| = -(x^2 - 1) \) and so \( y = \ln g(x) = \ln(-(x^2 - 1)) \). So,
\[
y' = \frac{2x}{x^2 - 1} \Rightarrow y'' = \frac{-2x^2 - 2}{(x^2 - 1)^2} < 0 \Rightarrow \text{concave down}
\]

16. C—\( A = \frac{1}{2} \int_{0}^{2\pi} (1 - \cos \theta)^2 \, d\theta = \frac{3\pi}{2} \).

17. E—\( F'(x) = xg'(x) \) with \( x \geq 0 \) and \( g'(x) < 0 \Rightarrow F'(x) \leq 0 \Rightarrow F \) is not increasing, \( F \) is differentiable (therefore, continuous, so a, b, and d are true). It is easy to check that c works as well.

18. B—\( \int_{0}^{2} (4x - 2xf(x)) \, dx = -3 \).

19. E—\( \frac{dy}{dt} = ky(1 - y) \)

20. A—\( A = \pi r^2 \) and \( A = 64\pi \) when \( r = 8 \). \( \frac{dA}{dr} = 2\pi \frac{dr}{dr} \Rightarrow 96\pi = 2\pi(8) \frac{dr}{dt} \Rightarrow \frac{dr}{dt} = 6 \).

21. D—The solution is of the form \( y = y_h + y_p \) where \( y_h \) is the solution to \( y' - y = 0 \) and the form of \( y_p \) is \( Ax^2 + Bx + K \). So, \( y_h = Ce^x \). Substitute \( y_p \) into the original differential equation to determine \( A, B, \) and \( K \).

22. D—\( \int_{-4}^{4} f(x) \, dx - 2\int_{-1}^{1} f(x) \, dx = (A_1 - A_2) - 2(-A_2) = A_1 + A_2 \).

23. D—Recall from Taylor series that if \( f(0) = 0 \), then \( f(x) \approx f'(0)x \) when \( x \) is small. So, this means that
\[
\lim_{x \to 0} \frac{\sin^7(5x) \tan^3(4x)}{(\ln(2x + 1))^5} = \lim_{x \to 0} \frac{(5x)^7(4x)^3}{(2x)^5} = 50.
\]
24. C—Profit \( (P) \) is revenue – cost. So, \( P = n \cdot D - (600 + 10n + n^2)0 = -3n^2 + 100n - 600 \). The critical point is \( 16 \cdot \frac{2}{3} \), which means \( n = 16 \) or \( n = 17 \) is the largest profit. Checking yields \( n = 17 \).

25. B—In general, for even functions, \( \int_{-a}^{a} \frac{f(x)\,dx}{1+b^x} = \frac{1}{2} \int_{-a}^{a} f(x)\,dx \). So, \( \int_{-2}^{2} \frac{1+x^2\,dx}{1+2^x} = \frac{1}{2} \int_{-2}^{2} (1+x^2)\,dx \). 

26. E—II and III only \( \Rightarrow \) I is FALSE (max could be at a cusp). II is TRUE (there is a critical pt at \( x = c \) where \( f'(c) \) exists). III is TRUE (if 2nd deriv is \( > 0 \), then there would be a relative min, not max).

27. D—If \( f(x) = \sum_{n=0}^{\infty} a_n \cdot x^n \), then \( f'(x) = \sum_{n=1}^{\infty} na_n \cdot x^{n-1} = \sum_{n=1}^{\infty} na_n \cdot x^{n-1} \). So, \( f'(1) = \sum_{n=1}^{\infty} na_n \cdot 1^{n-1} = \sum_{n=1}^{\infty} na_n \).

28. B—\( \int_{3}^{5} [f(x) + g(x)]\,dx = \int_{3}^{5} [2g(x) + 7]\,dx = 2 \int_{3}^{5} g(x)\,dx + 7(2) = 2 \int_{3}^{5} g(x)\,dx + 14 \).

29. D—I is TRUE... \( \frac{f(3) - F(1)}{3-1} = \frac{5}{2} \). II is FALSE...there is not enough info to determine the average value. III is TRUE...the average value of \( f' \) is the average rate of change of \( f \).

30. A—\( s_n = \frac{1}{5} \left( \frac{5+n}{4+n} \right)^{100} \), \( \lim_{n \to \infty} s_n = \frac{1}{5} (1) = \frac{1}{5} \).