Answers:

0. -5
1. -2
2. $y=46x-52$ (must be in slope-intercept form)
3. 1372
4. $[-1,2]$ (must be in interval notation)
5. 112
6. $\sqrt{2}$
7. 20,195
8. -3
9. $y=\ln\left(e^x+e-1\right)$
10. 5
11. $(3,6)$
12. e^2-2

Solutions:

0. $\lim_{x\to -1}\frac{-3x^2+7x-5}{2x^2+8x+9}=\frac{-3(-1)^2+7(-1)-5}{2(-1)^2+8(-1)+9}=\frac{-15}{3}=-5$

1. $\lim_{x\to -\infty}\frac{-4x^2+7x-5}{\sqrt{4x^4+5x^3-1}}=\lim_{x\to -\infty}\frac{(-4x^2+7x-5)/x^2}{\left(\sqrt{4x^4+5x^3-1}\right)/x^2}=\lim_{x\to -\infty}\frac{-4+\frac{7}{x}-\frac{5}{x^2}}{\sqrt{4+\frac{5}{x}-\frac{1}{x^4}}}=\frac{-4}{\sqrt{4}}=-2$

2. Plugging in $x=2$ yields $y=40$, so the point $(2,40)$ is on the tangent. Further, $y'=12x^2-2$, so the slope of the tangent is $12\cdot 2^2-2=46$. Therefore, the tangent equation is $y-40=46(x-2)$, which in the requested form is $y=46x-52$.

3. Letting *x* be the length of the horizontal side of the rectangle from the *y*-axis to the right corner, the total length will be $2x$, and the height will be $147-x^2$. This makes the area enclosed by the rectangle $A=2x(147-x^2)=294x-2x^3$. $A'=294-6x^2$, and the only positive solution to $A'=0$ is $x=7$. Sign analysis has A' change from positive to negative there, indicating an absolute maximum since this is the only sign change for A'. Therefore, the maximum area is $A=294\cdot 7-2\cdot 7^3=1372$.

4. Any argument of a square root must be nonnegative, so $1-\sqrt{2-\sqrt{3-x}}\ge 0$ $\Rightarrow 0\le\sqrt{2-\sqrt{3-x}}\le 1$. Squaring all three quantities yields $0\le 2-\sqrt{3-x}\le 1$ $\Rightarrow -2\le-\sqrt{3-x}\le -1\Rightarrow 1\le\sqrt{3-x}\le 2$. Again, squaring all three quantities yields $1\le 3-x\le 4\Rightarrow -2\le -x\le 1\Rightarrow -1\le x\le 2$, so the domain is $[-1,2]$.

5. $\int_0^2 f(x)dx=8\Rightarrow\int_0^2 f(3x)dx=5\int_0^2 f(x)dx=40\Rightarrow 3\int_0^2 f(3x)dx=120$. Making the substitutions $u=3x$ and $du=3dx$, along with substituting the limits of integration yields $\int_0^6 f(u)du=120\Rightarrow\int_0^6 f(x)dx=120$. Therefore, $\int_2^6 f(x)dx=\int_0^6 f(x)dx-\int_0^2 f(x)dx$ $=120-8=112$.

6. We know that $\pi\int_0^a (f(x))^2\,dx=a^2$. Differentiating both sides with respect to *a* yields $\pi(f(a))^2=2a$, and plugging in $a=\pi$ yields $\pi(f(\pi))^2=2\pi\Rightarrow(f(\pi))^2=2$. Since *f* was a positive function, $f(\pi)=\sqrt{2}$.

7. You could just churn out the numbers of the sequence, which ends up being 2, 5, 13, 35, 97, 275, 793, 2315, 6817, 20195,…

OR

Use the ansatz $a_n = r^n$, which means that $r^n = 5r^{n-1} - 6r^{n-2}$, and since clearly $r \neq 0$, divide by r^{n-2} and rearrange terms to get $0 = r^2 - 5r + 6 = (r-2)(r-3) \Rightarrow r = 2$ or $r = 3$, so $a_n = C \cdot 2^n + D \cdot 3^n$ for some real *C* and *D*. Plugging in the given terms yields $a_n = \frac{1}{2} \cdot 2^n + \frac{1}{3} \cdot 3^n = 2^{n-1} + 3^{n-1}$. Therefore, $a_{10} = 2^9 + 3^9 = 512 + 19{,}683 = 20{,}195$.

8. Substituting $\Delta x = -\frac{1}{n}$, $x_i = 1 + i\Delta x = 1 - \frac{i}{n}$, and $x_{i+1} = 1 + (i+1)\Delta x = 1 - \frac{i+1}{n}$, this limit becomes $\lim_{n\to\infty}\left(\Delta x \sum_{i=1}^{n}\left(5 - 8\left(\frac{x_i + x_{i+1}}{2}\right)^3\right)\right)$, where $x_0 = 1$ and $x_n = 0$, so using midpoints, this is equal to $\int_1^0 (5 - 8x^3)\,dx = (5x - 2x^4)\Big|_1^0 = (0-0) - (5-2) = -3$.

OR (if you didn't realize the above)

$$\lim_{n\to\infty}\left(-\frac{1}{n}\sum_{i=0}^{n-1}\left(5 - 8\left(\frac{\left(1-\frac{i}{n}\right) + \left(1 - \frac{i+1}{n}\right)}{2}\right)^3\right)\right) = \lim_{n\to\infty}\left(-\frac{1}{n}\sum_{i=0}^{n-1}\left(5 - 8\left(1 - \frac{i}{n} - \frac{1}{2n}\right)^3\right)\right)$$

$$= \lim_{n\to\infty}\left(-\frac{1}{n}\sum_{i=0}^{n-1}\left(\frac{8i^3}{n^3} + \frac{12i^2}{n^3} - \frac{24i^2}{n^2} + \frac{6i}{n^3} - \frac{24i}{n^2} + \frac{24i}{n} + \frac{1}{n^3} - \frac{6}{n^2} + \frac{12}{n} - 3\right)\right)$$

$$= \lim_{n\to\infty}\left(-\frac{8(n-1)^2 n^2}{4n^4} - \frac{12(n-1)n(2n-1)}{6n^4} + \frac{24(n-1)n(2n-1)}{6n^3} - \frac{6(n-1)n}{2n^4} + \frac{24(n-1)n}{2n^3}\right.$$
$$\left. - \frac{24(n-1)n}{2n^2} - \frac{n}{n^4} + \frac{6n}{n^3} - \frac{12n}{n^2} + \frac{3n}{n}\right) = -\frac{8}{4} + 0 + \frac{48}{6} - 0 + 0 - \frac{24}{2} - 0 + 0 - 0 + 3 = -3$$

9. $\ln\left(\frac{dy}{dx}\right) = x - y \Rightarrow \frac{dy}{dx} = e^{x-y} = \frac{e^x}{e^y} \Rightarrow e^y dy = e^x dx \Rightarrow e^y = e^x + C$. Substituting the initial condition yields $e = e^1 = e^0 + c = 1 + C \Rightarrow C = e - 1$. Therefore, $e^y = e^x + e - 1$, and solving for *y*, $y = \ln(e^x + e - 1)$.

10. Using partial fraction decomposition (which the hint should have suggested to you), $\int_0^{\sqrt{2}} \frac{1-x^2}{1+x^4}dx = \frac{1}{2}\int_0^{\sqrt{2}}\left(\frac{\sqrt{2}x+1}{x^2+\sqrt{2}x+1} - \frac{\sqrt{2}x-1}{x^2-\sqrt{2}x+1}\right)dx$, and both of these fractions can be

integrated using a simple substitution for the denominator. Therefore,

$$\int_0^{\sqrt{2}} \frac{1-x^2}{1+x^4}dx = \frac{1}{2}\cdot\frac{\sqrt{2}}{2}\left(\ln\left|x^2+\sqrt{2}x+1\right|-\ln\left|x^2-\sqrt{2}x+1\right|\right)\Big|_0^{\sqrt{2}} = \frac{\sqrt{2}}{4}\left(\left(\ln 5-\ln 1\right)-\left(\ln 1-\ln 1\right)\right)$$

$= \frac{\sqrt{2}}{4}\ln 5$, so $b=5$.

11. $y=3x^5+5x^4-80x^3-360x^2+1400x+72 \Rightarrow y'=15x^4+20x^3-240x^2-720x+1400$ $\Rightarrow y''=60x^3+60x^2-480x-720=60(x+2)^2(x-3)$. Sign analysis confirms that the only sign change for y'' occurs at $x=3$, so this is where the inflection point occurs. Plugging that value back into the function yields $y=6$, so the inflection point is $(3,6)$.

12. Based on the graphs of the given equations, the volume can be written as

$$\pi\int_0^1\left(\left(\sqrt{2}e^x\right)^2-\left(\sqrt{3}x\right)^2\right)dx = \pi\int_0^1\left(2e^{2x}-3x^2\right)dx = \pi\left(e^{2x}-x^3\right)\Big|_0^1 = \pi\left(\left(e^2-1\right)-\left(1-0\right)\right)$$

$= \pi\left(e^2-2\right)$, so $A=e^2-2$.