(1) Find \( \lim_{x \to 3} \frac{x^2-x-6}{x^2-4x+3} \).

(A) \( \frac{2}{3} \)  
(B) \( \frac{5}{2} \)  
(C) \( \infty \)  
(D) 0  
(E) NOTA

(2) Find \( \lim_{x \to \infty} \left( 1 + \frac{2017}{x} \right)^{2017x} \).

(A) 2017  
(B) \( e^{2017} \)  
(C) \( e^{2017^2} \)  
(D) \( e^{\frac{1}{2017}} \)  
(E) NOTA

(3) Find the equation of the line tangent to \( y = \frac{\sin(x)}{x} \) at \( x = \pi \).

(A) \( y = -\frac{1}{\pi} x + 1 \)  
(B) \( y = \frac{1}{\pi} x + 1 \)  
(C) \( y = -\frac{1}{\pi} x - 1 \)  
(D) \( y = \frac{1}{\pi} x - 1 \)  
(E) NOTA

(4) Find the slope of the line normal to \( y = x \cdot \ln(x^2 + 1) \) at \( x = 1 \).

(A) \( \frac{1}{\ln(2)+1} \)  
(B) \(-\frac{1}{\ln(2)+1} \)  
(C) \( \ln(2) + 1 \)  
(D) \(-\ln(2) - 1 \)  
(E) NOTA

(5) Find \( \frac{dy}{dx} \) at the point \((1,2)\) if \( y^3 + xy^2 + x^2 = 13 \).

(A) \( \frac{3}{4} \)  
(B) \(-\frac{3}{4} \)  
(C) \( \frac{3}{8} \)  
(D) \(-\frac{3}{8} \)  
(E) NOTA

(6) Approximate the area between \( y = x^2 + x + 1 \) and the x-axis from \( x = 1 \) to 3 using a left-handed Riemann Sum with four rectangles of equal width.

(A) \( \frac{49}{2} \)  
(B) \( \frac{49}{4} \)  
(C) \( \frac{69}{2} \)  
(D) \( \frac{69}{4} \)  
(E) NOTA

(7) Evaluate: \( \int_1^3 (x^3 - 4x + 3) \, dx \)

(A) 10  
(B) \( \frac{49}{4} \)  
(C) 15  
(D) 20  
(E) NOTA

(8) Find the area in the first quadrant of the finite region bounded by the curves \( y = x^3 \) and \( y = \sqrt[3]{x} \).

(A) 1  
(B) \( \frac{1}{4} \)  
(C) \( \frac{1}{2} \)  
(D) \( \frac{13}{12} \)  
(E) NOTA
(9) Evaluate: \( \int_{0}^{\sqrt{\ln(2)}} x e^{x^2} dx \)

(A) 1 \hspace{1cm} (B) 1 - e \hspace{1cm} (C) \frac{1}{2} \hspace{1cm} (D) 2 - \frac{1}{2} e \hspace{1cm} (E) NOTA

(10) Find \( f'(1) \) if \( f(x) = \int_{\sqrt{\arctan(x)}}^{\frac{\sqrt{\pi} x^4}{\sqrt{\pi}}} \cos(t^2) \, dt \).

(A) \( \frac{\sqrt{\pi}}{2\sqrt{\pi}} \) \hspace{1cm} (B) \( \frac{\sqrt{\pi} + 16\pi}{4\sqrt{\pi}} \) \hspace{1cm} (C) \( \frac{-7\sqrt{\pi}}{2} \) \hspace{1cm} (D) \( \frac{\sqrt{\pi} - 16\pi}{4\sqrt{\pi}} \) \hspace{1cm} (E) NOTA

(11) A rectangle is drawn such that one side is on the x-axis, one side is on the y-axis, and one vertex is on the curve \( y = \frac{2}{1+x^2} \) for \( x > 0 \). Find the maximum area the rectangle could enclose.

(A) 1 \hspace{1cm} (B) 2 \hspace{1cm} (C) \( \frac{3}{2} \) \hspace{1cm} (D) \( \frac{1}{2} \) \hspace{1cm} (E) NOTA

(12) Find the average value of \( f(x) = \sec(x) \) for \( x \in [-\frac{\pi}{4}, \frac{\pi}{4}] \).

(A) \( \frac{2}{\pi} \ln(\sqrt{2} - 1) \) \hspace{1cm} (B) \( \frac{2}{\pi} \ln(\sqrt{2} + 1) \) \hspace{1cm} (C) \( \frac{4}{\pi} \ln(\sqrt{2} - 1) \) \hspace{1cm} (D) \( \frac{4}{\pi} \ln(\sqrt{2} + 1) \) \hspace{1cm} (E) NOTA

(13) Evaluate: \( \int_{4}^{49} \sqrt{2 + \sqrt{x}} \, dx \).

(A) 116 \hspace{1cm} (B) 118 \hspace{1cm} (C) \( \frac{1772}{15} \) \hspace{1cm} (D) \( \frac{1672}{15} \) \hspace{1cm} (E) NOTA

(14) Approximate \( \sqrt[3]{e} \) using differentials and the fact that \( e^0 = 1 \).

(A) 1 \hspace{1cm} (B) 1.05 \hspace{1cm} (C) 1.1 \hspace{1cm} (D) 1.2 \hspace{1cm} (E) NOTA

(15) \( \int_{0}^{4} \frac{x^2 - 2x}{x^3 - 3x^2 + 4} \, dx = ? \)

(A) \( \frac{1}{3} \ln(5) \) \hspace{1cm} (B) \( \frac{1}{3} \ln(20) \) \hspace{1cm} (C) \ln(20) \hspace{1cm} (D) \ln(4) \hspace{1cm} (E) NOTA
(16) The base of a certain solid corresponds to the finite region in the first quadrant bounded by the 
\(x\)-axis and \(y = \sin(x)\) for \(x \in [0, \pi]\). The solid’s cross-sections perpendicular to the \(x\)-axis are 
semi-circles whose diameter lies on the base. Find the volume of this solid.

(A) \(\frac{\pi^2}{16}\)  (B) \(\frac{\pi^2}{32}\)  (C) \(\frac{\pi}{16}\)  (D) \(\frac{\pi}{32}\)

(E) NOTA

(17) Consider the unbounded region in the first quadrant below the curve \(y = e^{-x}\). If this region is 
revolved around the \(y\)-axis, what is the resulting volume?

(A) \(e\)  (B) \(\pi\)  (C) \(2e\)  (D) \(2\pi\)

(E) NOTA

(18) A wheel corresponds to a circle of radius 1 centered at the origin. The wheel is rotating at a rate 
of 4 radians per minute counterclockwise, and has a light at one point on its edge. What is the 
rate of change, in units per minute, of the distance from the light to an observer at the point 
\((3,3)\) when the light is at the point \((1,0)\)? Assume only the two dimensions described in this 
problem.

(A) \(-\frac{5\sqrt{3}}{13}\)  (B) \(-\frac{12\sqrt{3}}{13}\)  (C) \(\frac{5\sqrt{3}}{13}\)  (D) \(\frac{12\sqrt{3}}{13}\)

(E) NOTA

(19) Consider the function \(f(x) = x + \frac{d}{dx} \left(x + \frac{d}{dx} \left(x + \frac{d}{dx} (x + \cdots)\right)\right)\). If \(f(1) = 5\), what is \(f''(1)\)?

(A) 3  (B) 2  (C) 1  (D) 0

(E) NOTA

(20) Evaluate: \(\int_{-1}^{0} \frac{x^2 + 4x + 5}{x^2 + 2x + 2} \, dx\).

(A) 1  (B) \(1 + \frac{\pi}{4}\)  (C) \(1 + \frac{\pi}{4} + \ln(2)\)  (D) \(1 + \frac{\pi}{4} + \ln(2) + e^2\)

(E) NOTA

(21) If \(\int_{0}^{-5} f(x) \, dx = 7\), \(\int_{-4}^{5} f(x) \, dx = 3\), and \(f(x)\) is odd, what is \(\int_{-4}^{4} f(x) \, dx\)?

(A) 3  (B) 4  (C) 7  (D) 10

(E) NOTA

(22) Find \(\frac{d}{du} \left(f^{-1}(u)\right)\) at \(u = 8\) if \(f(x) = x^3 + 2x^2 + 3x + 2\).

(A) 10  (B) \(-10\)  (C) \(\frac{1}{10}\)  (D) \(-\frac{1}{10}\)

(E) NOTA
(23) Which of the following statements is/are true concerning the series \( \sum_{n=1}^{\infty} \frac{n}{n^2 + 1} ? \)

I. This series converges based on the \( n \)th term test.
II. This series diverges based on the limit comparison test when compared to \( \sum_{n=1}^{\infty} \frac{1}{n} \).
III. This series diverges based on the integral test.
IV. This series diverges based on the ratio test.
V. This series diverges based on the root test.

(A) I only  (B) II, III, IV, & V only  (C) II, III, & V only  
(D) II & III only  (E) NOTA

(24) \( \int_{\pi/6}^{\pi/4} \frac{\cos(x) - \sin(x)}{\cos(x) + \sin(x)} \, dx = ? \)

(A) \( \ln \left( \frac{2\sqrt{2}}{\sqrt{3} + 1} \right) \)  (B) \( \ln \left( \frac{1}{\sqrt{3} + \sqrt{2}} \right) \)  (C) \( \pi \)  (D) \( 2\pi \)

(E) NOTA

(25) Let \( f(x) \) be a continuous, differentiable function such that \( \lim_{x \to \infty} f(x) = 0 \) and, for any real \( \alpha > 0 \), \( \int_{1}^{\infty} \frac{f'(ax)}{x} \, dx = \frac{\alpha}{\alpha^4 + 1} \). Find \( \int_{1}^{\infty} \frac{f'(x)}{x} \, dx \).

(A) \( \frac{\pi}{8} \)  (B) \( -\frac{\pi}{8} \)  (C) \( -\frac{\pi}{4} \)  (D) \( \frac{\pi}{4} \)

(E) NOTA

(26) The finite region in the first quadrant bounded by the x-axis and \( y = \frac{4}{m^3} x (m - x) \) is revolved around the line \( y = \frac{1}{m} x + 1 \). If \( V(m) \) is the resulting volume, find \( \lim_{m \to \infty} V(m) \).

(A) \( \frac{4\pi}{3} \)  (B) \( \frac{3\pi}{2} \)  (C) \( \pi \)  (D) \( 2\pi \)

(E) NOTA

(27) Jackson and Max are playing a game with two coins. Max’s coin is fair. Jackson’s coin, however, is a magic coin that remembers how many times Jackson has flipped the coin during the game. The first time Jackson flips the coin, the probability of Heads is \( \frac{1}{2} \). The second time it is \( \frac{2}{3} \), and in general the probability of Heads on the \( n \)th flip is \( \frac{n}{n+1} \). With Jackson going first, they alternate flipping their coins until one of them flips Heads; that player wins. Find the probability that Jackson wins.

(A) \( 2 - \sqrt{e} \)  (B) \( \frac{e}{e+1} \)  (C) \( 4 - 2\sqrt{e} \)  (D) \( \frac{1}{\sqrt{e}} \)

(E) NOTA
(28) Find, in terms of positive integer $m$, \( \lim_{n \to \infty} \sum_{k=0}^{n-1} \frac{k}{n} \left( \sqrt[2]{\frac{k+1}{n}} - \sqrt[2]{\frac{k}{n}} \right) \).

(A) \( \frac{1}{m+1} \) \hspace{1cm} (B) \( \frac{2}{m} \) \hspace{1cm} (C) \( m \) \hspace{1cm} (D) \( \frac{m}{\sqrt{m}} \)

(E) NOTA

(29) Let $f(x)$ be a continuous, even function such that \( \int_{0}^{a} f(x) \, dx = K \). Let $g(x)$ be a continuous, positive function such that $g(x)g(-x) = 1$ and \( \int_{0}^{a} g(x) \, dx = L \). Find \( \int_{-a}^{a} \frac{f(x)}{1+g(x)} \, dx \).

(A) \( \frac{K}{1+L} \) \hspace{1cm} (B) \( \frac{K}{L} \) \hspace{1cm} (C) \( K \) \hspace{1cm} (D) \( 2K \)

(E) NOTA

(30) Find $f'(1)$ if $f(x) = 2x^2 + 3x - 4$.

(A) \( 2 \) \hspace{1cm} (B) \( 0 \) \hspace{1cm} (C) \( 1 \) \hspace{1cm} (D) \( 7 \)

(E) NOTA