

The acronym “NOTA” stands for “None Of The Above”. You may find Stirling’s approximation useful: for when  $n$  becomes large,  $n! \sim \sqrt{2\pi n} \left(\frac{n}{e}\right)^n$ . Assume all derivatives are within the function’s domain.

1. Find  $\left. \frac{d}{dx} \right|_{x=2017} \left( 2017!^{2017!} - \log\left((10^{2017!})^x\right) + \ln 2017 - e^{2017} + 1^x \right)$

- A. 0                      B. 2017                      C. 2017!                      D.  $2017!^{2017}$                       E. NOTA

2. Find  $\lim_{x \rightarrow 0} \frac{\sin^{2017!}(2017x)}{x^{2017!}}$

- A. 0                      B. 2017                      C. 2017!                      D.  $2017!^{2017}$                       E. NOTA

3. Find the slope of the tangent line at  $(2017!, 1)$  of the graph  $\frac{x-y^{2017!}}{2017!-1} = 1$

- A. 0                      B. 2017                      C. 2017!                      D.  $\frac{1}{2017!}$                       E. NOTA

4. Find  $\lim_{x \rightarrow -\infty} \frac{2017!x^{2017!} + 2016!x^{2016!} + 2015!x^{2015!} + \dots + 1!x^{1!}}{2016!x^{2017!} + 2015!x^{2016!} + 2014!x^{2015!} + \dots + 0!x^{1!}}$

- A. 0                      B. 2017                      C. 2017!                      D.  $2017!^{2017!}$                       E. NOTA

5. Evaluate  $\left. \frac{d^{2017}f(x)}{dx^{2017}} \right|_{x \rightarrow 2017!^{2017!}}$  if  $f(x) = \sum_{n=0}^{2016} n! x^n$

- A. 0                      B. 2017                      C. 2017!                      D.  $2017!^{2017!}$                       E. NOTA

6. Evaluate the  $\lim_{n \rightarrow \infty} \left[ \sum_{i=1}^{2n} \frac{3}{n} \left(\frac{2i}{n}\right)^{\frac{2i}{n}} \left(\ln\left(\frac{2i}{n}\right) + 1\right) \right]$

- A.  $\frac{9}{2}$                       B. 6                      C.  $\frac{765}{2}$                       D. 384                      E. NOTA

7. Mercy starts at objective B located at  $(30,40)$  and is traveling towards objective A located at  $(-10,70)$  at a rate of 40 units/s. Roadhog starts at  $(0,0)$  and travels towards a chokepoint located at  $(70,240)$  at a rate of 25 units/s. After 1 second what is the rate of change of the distance between Mercy and Roadhog, in units/s?

- A.  $\frac{-351}{41}$                       B.  $\frac{351}{41}$                       C.  $\frac{-39}{\sqrt{65}}$                       D.  $\frac{39}{\sqrt{65}}$                       E. NOTA

8. Let  $A = \lim_{x \rightarrow -2} f(x)$ ,  $B = \lim_{x \rightarrow 0} f(x)$ ,  $C = \lim_{x \rightarrow 2^-} f(x)$ ,  $D = \lim_{x \rightarrow 4^-} f(x)$  and

$$f(x) = \begin{cases} 2x - 4, & \text{if } x < -2 \\ 6x + 4, & \text{if } -2 \leq x < 0 \\ 5, & \text{if } x = 0 \\ x^2, & \text{if } 0 < x < 2 \\ -x^2, & \text{if } x \geq 2 \end{cases}.$$

If the limit does not exist, instead let the value of the variable that equals the limit be equal to 1.

Find  $A+B+C+D$ .

- A. -19                      B. -15                      C. -13                      D. -10                      E. NOTA

9. Evaluate  $\frac{d}{dx} \int_{-2x}^{x^2} 2e^{-t^2} \sin(t) dt$ .

- A.  $-e^{-4x^2} \sin(2x) + 2e^{-x^4} \sin(x^2)$                       B.  $e^{-4x^2} \sin(2x) + 2e^{-x^4} \sin(x^2)$   
 C.  $-4e^{-4x^2} \sin(2x) + 4xe^{-x^4} \sin(x^2)$                       D.  $4e^{-4x^2} \sin(2x) + 4xe^{-x^4} \sin(x^2)$                       E. NOTA

10. Let  $n$  be an odd integer. There are  $n$  points evenly spaced out on a circle. Three distinct points are chosen on the circle.  $P(n)$  is the probability that the points form an obtuse triangle? What is

$$\lim_{n \rightarrow \infty} P(n)?$$

- A.  $\frac{1}{8}$                       B.  $\frac{1}{2}$                       C.  $\frac{3}{4}$                       D.  $\frac{4}{5}$                       E. NOTA

11. In higher levels of math, the formula for the "directional derivative in the direction of  $\mathbf{u}$ " of  $f$  at the point  $(x,y,z)$  is given by  $D_{\mathbf{u}}f(x,y,z) = \nabla f(x,y,z) \cdot \mathbf{u}$  where  $\nabla f(x,y,z)$  is a set vector and  $\mathbf{u}$  is a unit vector. If  $\nabla f(x,y,z) = \langle a,b,c \rangle$ , which unit vector,  $\mathbf{u}$ , will always minimize  $D_{\mathbf{u}}f(x,y,z)$ ?

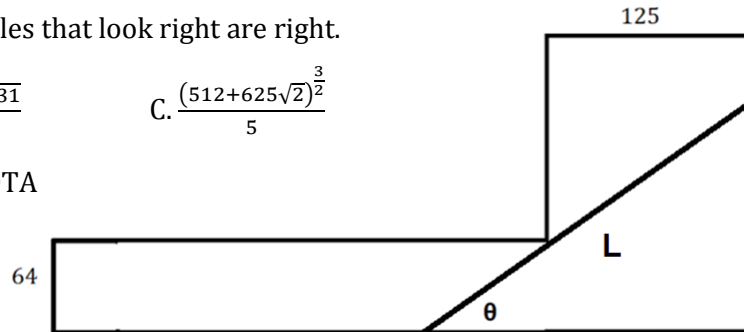
- A.  $-\frac{\langle a,b,c \rangle}{|\langle a,b,c \rangle|}$                       B.  $\frac{\langle a,b,c \rangle}{|\langle a,b,c \rangle|}$                       C.  $\frac{\langle -bc,-ac,2ab \rangle}{|\langle -bc,-ac,2ab \rangle|}$                       D.  $\frac{\langle bc,ac,-2ab \rangle}{|\langle bc,ac,-2ab \rangle|}$                       E. NOTA

12. The drain current of a NFET,  $I$ , is represented by  $I(V_T) = \frac{k}{2} \left(\frac{W}{L}\right) (V_{GS} - V_T)^2 (1 + \lambda V_{DS})$  where  $k, W, L, V_{GS}, V_{DS}$ , and  $\lambda$  are all constants. What is  $\frac{dI}{d\sqrt{V_T}}$ ?

- A.  $\frac{-k\left(\frac{W}{L}\right)(V_{GS}-V_T)(1+\lambda V_{DS})}{2\sqrt{V_T}}$                       B.  $\frac{k\left(\frac{W}{L}\right)(V_{GS}-V_T)(1+\lambda V_{DS})}{2\sqrt{V_T}}$   
 C.  $k\left(\frac{W}{L}\right)2\sqrt{V_T}(V_{GS}-V_T)(1+\lambda V_{DS})$                       D.  $-k\left(\frac{W}{L}\right)2\sqrt{V_T}(V_{GS}-V_T)(1+\lambda V_{DS})$                       E. NOTA

13. A  $90^\circ$  bend in a hallway is shown below with dimensions. The **length** of the longest ladder that can fit through the bend in the hallway is " $L$ " and the **tan**( $\theta$ ) the ladder makes with the bottom wall is given as " $T$ ". What is  $LT$ ? Assume the ladder is always parallel to the ground and has negligible width. All angles that look right are right.

- A.  $\frac{205\sqrt{41}}{4}$       B.  $\frac{31\sqrt{31}}{4}$       C.  $\frac{(512+625\sqrt{2})^{\frac{3}{2}}}{5}$   
 D.  $\frac{164\sqrt{41}}{5}$       E. NOTA



For questions 14-16 refer to the following function:  $f(x) = \begin{cases} 0, & x \leq 0 \\ e^{-\frac{1}{x}}, & x > 0 \end{cases}$

14. What is the  $\lim_{x \rightarrow 0} f(x)$ ?  
 A. 0      B. 1      C.  $e$       D. Does not Exist      E. NOTA

15. What is  $f''(0)$ ?  
 A. 0      B. 1      C.  $e$       D. Does not Exist      E. NOTA

16. What is  $\lim_{n \rightarrow \infty} f^{(n)}(0)$ ?  
 A. 0      B. 1      C.  $e$       D. Does not Exist      E. NOTA

17. Using the Delta-Epsilon definition of a limit for  $\lim_{x \rightarrow 1} x^{\frac{1}{3}} - 4 = -3$ . If  $\epsilon = 0.1$ , what is the maximum value of  $\delta$  that would satisfy the definition?  
 A. 0.729      B. 0.602      C. 0.331      D. 0.271      E. NOTA

18. Find  $\lim_{x \rightarrow 2} \frac{\sqrt{2+x}-2}{2-x}$   
 A. 0      B.  $\frac{1}{4}$       C.  $\frac{1}{2}$       D. Does Not Exist      E. NOTA

19. Find  $y'$  if  $y = x^{x^x}$ .  
 A.  $y \cdot x^{y-1}$       B.  $\frac{y \ln(x^x) + y^2}{x}$       C.  $\frac{y^2}{x(1-y \ln(x))}$       D.  $\frac{y^2}{x(1+y \ln(x))}$       E. NOTA

For questions 20-21 refer to the following table:

x	F(x)	G(x)	H(x)	F'(x)	G'(x)	H'(x)
0	1	2	2	-2	1	4
1	2	-3	2	1	2	-1
2	0	1	2	1	0	-1
3	0	-1	1	-3	2	1

20. If  $P(x) = F(G(H(x) - x))$ , what is  $P'(2)$ ?  
 A. -4                      B. -2                      C. 2                      D. 4                      E. NOTA

21. If  $Q(x) = \frac{F(x^2)G(x-1)}{H(x)}$ , what is  $Q'(1)$ ?  
 A. -4                      B. -2                      C. 2                      D. 4                      E. NOTA

22. Find  $\frac{d^3y}{dx^3}$  if  $y = \sin(t)$  and  $x = e^t$   
 A.  $-e^{-3t} \sin(t) - 3e^{-3t} \cos(t)$                       B.  $3e^{-3t} \sin(t) + e^{-3t} \cos(t)$   
 C.  $-e^{-3t} \sin(t) + 3e^{-3t} \cos(t)$                       D.  $2e^{-t} \sin(t)$                       E. NOTA

23. Evaluate  $\lim_{x \rightarrow \infty} \sqrt{\frac{5}{2} \sqrt{2x^{\frac{5}{2}} + 5x^{\frac{3}{2}} + 1}} - \sqrt{\frac{5}{2} \sqrt{2x^{\frac{5}{2}} - 2x^{\frac{3}{2}} + 3x}}$   
 A.  $\frac{7}{2\sqrt{2}}$                       B.  $\frac{7 \cdot 2^{\frac{2}{5}}}{5}$                       C.  $\frac{7 \cdot 2^{\frac{3}{5}}}{5}$                       D.  $\frac{7 \cdot 2^{-\frac{3}{5}}}{5}$                       E. NOTA

24. Find line perpendicular to the graph  $xy + x^2y^3 = 10$  at the point (1,2).  
 A.  $-23 - 13x + 18y = 0$                       B.  $-49 + 13x + 18y = 0$   
 C.  $-44 + 18x + 13y = 0$                       D.  $-8 - 18x + 13y = 0$                       E. NOTA

For questions 25-26 refer to the following description: Let  $\alpha$  be a fixed positive integer and  $n$  be a non-negative integer. The  $n^{\text{th}}$  derivative of  $\frac{1}{x^{\alpha-1}}$  is written in the form  $\frac{P_n(x)}{(x^{\alpha-1})^{n+1}}$  where  $P_n(x)$  is a polynomial.

25. Find  $P_n(1)$ .  
 A.  $(-1)^n(a)^n 2^n$                       B.  $(-1)^n(a)^n n^n$                       C.  $(-1)^n a^n (n+1)!$                       D.  $(-1)^n a^n (n-1)!$                       E. NOTA

26. Find the  $\lim_{n \rightarrow \infty} \frac{P_{n+1}(1)}{(-a)^{n+2} \left(\frac{n+1}{e^{n+1}}\right) \sqrt{2\pi(n+1)}}$   
 A.  $\frac{e}{a}$                       B.  $-\frac{e}{a}$                       C.  $\frac{1}{a}$                       D.  $-\frac{1}{a}$                       E. NOTA

27. Give the Maclaurin series of  $\cosh(x)$ .  
 A.  $\sum_{k=0}^{\infty} \frac{(-1)^k x^{(2k)}}{(2k)!}$                       B.  $\sum_{k=0}^{\infty} \frac{(-1)^k x^{1+2k}}{(1+2k)!}$                       C.  $\sum_{k=0}^{\infty} \frac{x^{2k}}{(2k)!}$                       D.  $\sum_{k=0}^{\infty} \frac{x^{1+2k}}{(1+2k)!}$                       E. NOTA

28. Find  $\lim_{b \rightarrow \infty} \int_{-1}^b \left[ \left( \frac{x^{1513}}{1+x^{2018}} \right)^2 \right] dx$

A.  $\frac{2+3\pi}{8072}$

B.  $\frac{-2+3\pi}{8072}$

C.  $\frac{1+3\pi}{8072}$

D.  $\frac{-1+3\pi}{8072}$

E. NOTA

29. Find **an** implicit solution to the differential equation:

$$e^{-x} \sin(y) + y + (-e^{-x} \cos(y) + x + 1)y' = 0$$

A.  $e^{-x} \sin(y) + xy = 1$

B.  $-e^{-x} \cos(y) + xy + y = 3$

C.  $-e^{-x} \sin(y) + xy + y = 10$

D.  $e^{-x} \cos(y) + xy = -1$

E. NOTA

30. Find  $\lim_{x \rightarrow 0} \sqrt{\arctan(x)}$

A. 0

B.  $\pi$

C.  $-\pi$

D.  $\infty$

E. NOTA