## #0 Mu Bowl
## MAΘ National Convention 2017

Solution:

A = -6; Plugging in 0 for $y$ and solving for $x$ gives solutions of -7 and 1.

B = $\frac{27}{8}$ This is a horizontal ellipse centered on the x-axis, so plugging in $\theta = 0, \pi$ yields $r = \frac{9}{8}, \frac{9}{4}$, so the length of the major axis is $\frac{9}{8} - \left(-\frac{9}{4}\right) = \frac{27}{8}$

$\frac{A}{B} = -\frac{16}{9}$

## #1 Mu Bowl
## MAΘ National Convention 2017

$A = \frac{7}{18}$ when 10 is plugged in.

$B = \frac{1}{3a^2}$ after applying L'Hopital's rule to the limit and evaluating.

$C = \frac{1}{2}$; Using L'Hopital's rule give the limit $\lim_{x \to 0} \frac{2^x \ln 2}{4^x \ln 4}$ which can be evaluated to $\frac{\ln 2}{\ln 4} = \log_4 2 = \frac{1}{2}$. Then $B(C) = \frac{1}{3\left(\frac{1}{2}\right)^2} = \frac{4}{3}$ and $B(C) + A = \frac{4}{3} + \frac{7}{18} = \frac{31}{18}$.

## #2 Mu Bowl
## MAΘ National Convention 2017

$A = 3$; $\frac{d\left(f^{-1}(x)\right)}{dx} = \frac{1}{f'(f^{-1}(x)}$. $f^{-1}(0) = 1$ and $f'(x) = \frac{3}{(x+2)^2}$ then $\frac{1}{f'(1)} = \frac{1}{3}$

$B = \frac{-6}{121}$; Using implicit differentiation the first derivative is $y' = \frac{-3y-2}{3x+8}$ then a second derivative of $y'' = \frac{(3x+8)(-3y') - (-3y-2)3}{(3x+8)^2}$ and evaluating this at $x = 1$ gives $\frac{-6}{121}$

$C = 5e^{-2p}$; $y(p) = \cos p + e^{-2p} = -1 + e^{-2p}$ and $y''(p) = -\cos p + 4e^{-2p} = 1 + 4e^{-2p}$ adding these together results in $5e^{-2p}$

Evaluating $B\cdot\left(\frac{1}{A}+8\right)^2-\ln\left(\frac{C}{5}\right)$ gives $-6+2\pi$

**#3 Mu Bowl**
**MAΘ National Convention 2017**

$A=3, B=0$; Evaluating the first function at the given point results in the equation $3A+B=9$. Evaluating the first function's derivative at the given point results in the equation $4B=0$.

$C=-1$; Taking two derivatives of the second function and setting it equal to zero results in the equation $12x^2+48x+36=0$ which shows a relative minimum at $x=-1$.

$D=-4$; The second derivative of the third function is $\frac{4x+16}{(2-x)^4}$ which gives an inflection point of $x=-4$.

$$e^B+\frac{DA}{C}=e^0+\frac{(-4)(3)}{-1}=1+12=13$$

**#4 Mu Bowl**
**MAΘ National Convention 2017**

$A=\frac{28}{5}$; $\frac{1}{12}\int_0^{12}\sqrt{5x+4}\,dx=\frac{1}{12}\times\frac{2}{15}(5x+4)^{\frac{3}{2}}\Big|_0^{12}=\frac{28}{5}$

$B=\frac{8}{3}$; $f'(x)=2\sqrt[3]{4x^2-8}$ and $f''(x)=\frac{16}{3}x\left(4x^2-8\right)^{-\frac{2}{3}}$

$\frac{f(4)-f(1)}{4-1}=\frac{\frac{2}{3}-\frac{1}{3}}{3}=\frac{1}{9}$ so in order to satisfy the Mean Value Theorem the derivative must also be equal to $\frac{1}{9}$. $f'(x)=\frac{2}{(x+2)^2}$ and setting the two slopes equal and solving for x gives a simplified answer of $3\sqrt{2}-2$.

$$5A+3B+C^{D+E}=5\frac{28}{5}+3\frac{8}{3}+3^{2-2}=28+8+1=37$$

## #5 Mu Bowl
## MAΘ National Convention 2017

$A = \frac{\sqrt{7}}{2}$. Writing the distance between the point and the line we get $D = \sqrt{x^2 + \left(1 + x^2 - 3\right)^2}$ Squaring both sides, simplifying, and taking a derivative results in $0 = 4x^3 - 6x$ which gives the options of $x = 0$ or $x = \sqrt{\frac{3}{2}}$. It can be verified with either a number line or the second derivative that $x = \sqrt{\frac{3}{2}}$ is when the minimum distance occurs. Evaluating the distance formula with this x-coordinate results in a minimum distance of $\frac{\sqrt{7}}{2}$.

$B = \sqrt{20}$. Writing the distance between the point and the line we get $D = \sqrt{\left(1 + y^2\right)^2 + \left(y - 5\right)^2}$. Squaring both sides, simplifying, and taking a derivative results in $4y^3 + 6y - 10 = 0$ which gives the solution $y = 1$.

Evaluating the distance formula with this y-coordinate results in a minimum distance of $\sqrt{20}$.

$A^2 + B^2 = \frac{87}{4}$

## #6 Mu Bowl
## MAΘ National Convention 2017

$A = \frac{49}{900p}$; Starting with the volume equation of $V = \frac{1}{3}pr^2h$ a derivative of $V' = \frac{2}{3}prr'h + \frac{1}{3}pr^2h'$ can be obtained. The proportion $h = \frac{7r}{5}$ or $r = \frac{5h}{7}$ will be used to replace either h or r. This proportion can also be used to get $h' = \frac{7r'}{5}$ or $r' = \frac{5h'}{7}$. Plugging in $V' = 1$, $r' = \frac{5h'}{7}$, $h = 6$, and solving gives $\frac{49}{900p}$.

$B = \frac{5}{63\pi}$; Using the same equations as above but instead plugging in $h' = \frac{7r'}{5}$ and $r = 3$ gives an $r'$ of $\frac{5}{63\pi}$.

$$\frac{B}{A}=\frac{5}{63\pi}\cdot\frac{900\pi}{49}=\frac{500}{343}$$

**#7 Mu Bowl**
**MAΘ National Convention 2017**

Difference = 1.5
Bill's mistake led him to get the following points: (1,0) (1.5,0) (2.5,1) and (4,5.5)

When he re-worked it correctly he got: (1,0) (1.5,0) (2,0.5) (2.5, 1.5) (3, 3) (3.5,5) and (4, 7.5)

The true value of $f(4)$ can be solved by anti-deriving the differential equation and solving for the constant using the initial value. $y=x^2-2x+C$ then $C=1$ and evaluating at $x=4$ gives $y=9$ which means the 7.5 was closer by 1.5.

**#8 Mu Bowl**
**MAΘ National Convention 2017**

$A=-\frac{5}{2}$; The limit can be re-written as the integral $\int_0^1 \frac{x^2-5x+6}{x-2}dx$ which can be simplified to $\int_0^1 (x-3)dx$ which is $-\frac{5}{2}$.

$B=1-\frac{\pi}{4}$; Dividing the denominator into the numerator simplifies the integrand to $\int_0^1\left(1-\frac{1}{x^2+1}\right)dx=x-\tan^{-1}x+C$ and evaluates to $1-\frac{\pi}{4}$.

$C=\frac{175}{3}$; The parabola has negative y-values from 0 to 2 and positive y-values from 2 to 5 so the absolute value will only affect the first interval. The integral can be split up in to as follows: $\int_0^2\left(x^2+4x-12\right)dx+\int_2^5(x^2+4x-12)dx=\frac{40}{3}+45=\frac{175}{3}$

$D=\frac{\pi}{6}-\frac{1}{3}$; Using integration by parts and using $\sin^{-1}(3x)=u$ and $dx=dv$ the resulting substitution will lead to the new integral $x\sin^{-1}(3x)-\int_0^{\frac{1}{3}}\frac{3x}{\sqrt{1-9x^2}}dx=x\sin^{-1}(3x)+\frac{1}{3}\sqrt{1-9x^2}+C$. Evaluating this from 0 to $\frac{1}{3}$ results in $\frac{\pi}{6}-\frac{1}{3}$.

$2A+4B+3C+6D=172$

## #9 Mu Bowl
## MAΘ National Convention 2017

$A = e^{\frac{3\pi}{4}}$; First find the integrating factor to be $e^{-t}$ and multiply both sides by this integrating factor to get the equation $e^{-t}y' - e^{-t}y = 2e^{-t}\sin t$. The left side can be written as the product rule $\frac{d(e^{-t}y)}{dt}$ and the right side can be integrated with two steps of integration by parts. The resulting general solution is $y = ce^{t} - \sin t - \cos t$. Evaluating with the initial value gives $c = 1$. Given the specific solution of $y = e^{t} - \sin t - \cos t$ gives a result of $y\left(\frac{3\pi}{4}\right) = e^{\frac{3\pi}{4}}$.

$B = -11$; This can be solve using the separable method to give $y = -\frac{7x^2}{2} + c$ and then evaluated with the initial condition to get $y = -\frac{7x^2}{2} + 3$. Then $y(2) = -14 + 3 = -11$.

$C = 6$; Using the separable method gives the equation $r = \frac{1}{c - \ln|\theta|}$ then a specific solution of $r = \frac{2}{1 - 2\ln|\theta|}$. Evaluated at $\sqrt[3]{e}$ gives $r = 6$.

$$C^2\ln(A) - B = 6^2\ln\left(e^{\frac{3\pi}{4}}\right) + 11 = 36\left(\frac{3\pi}{4}\right) + 11 = 27\pi + 11$$

## #10 Mu Bowl
## MAΘ National Convention 2017

$A = 23$; The absolute value equation needs to be split up into two intervals. When $x < 1$ the equation can be written as $y = 3 + x$ and when $x > 1$ it is $y = 5 - x$. The left endpoint of the area will be the intersection of $y = 3 + x$ and $y = 3 + x$ which occurs at -2. So the two integrals used to find the area will be $\int_{-2}^{1} 3 + x - (-2x - 3)\,dx$ and $\int_{1}^{2} 5 - x - (-2x - 3)\,dx$. The solutions to those are $\frac{27}{2}$ and $\frac{19}{2}$ respectively, which gives a sum of 23.

$B = \frac{2}{3}$; The line $y = 1$ intersects the curve at $x = 3$ and is above the curve for this entire interval. Since the curve is above the x-axis so the integral used to find the area would be $2 \cdot 1 - \int_{2}^{4}\left(-x^2 + 6x - 8\right)dx$ which gives the following:

$2 - \left(-x^3 + 3x^2 - 8x\right)\Big|_{2}^{4}$ which evaluates to $2 - \frac{64}{3} + 48 - 32 - \left(-\frac{8}{3} + 12 - 16\right) = \frac{2}{3}$.

$C=\frac{8}{3}$; The two curves intersect at $y=-1$ and $y=-3$ so the integral for the area would be $\int_{-3}^{-1} -y^2+1-\left(y^2+8y+7\right)dy$ which gives a result of $\frac{8}{3}$.

$D=\pi-\frac{3\sqrt{3}}{2}$; The inner loop opens an closes when $r=0$. This occurs at $\frac{2\pi}{3}$ and $\frac{4\pi}{3}$ so the integral would be $\int_{\frac{2\pi}{3}}^{\frac{4\pi}{3}} \frac{1}{2}\left(1+2\cos\theta\right)^2 d\theta$ which can be evaluated simplified to $\int_{\frac{2\pi}{3}}^{\frac{4\pi}{3}} \left(\frac{1}{2}+2\cos\theta+1+\cos 2\theta\right)d\theta$ which gives as result of $\pi-\frac{3\sqrt{3}}{2}$.

$$A\pi+D\cdot\frac{C}{B}=27\pi-6\sqrt{3}$$

**#11 Mu Bowl**
**MAΘ National Convention 2017**

---

$A=\frac{128\pi}{3}$; Using Shell Method the integral used to find the volume would be $2\pi\int_0^4 (4-y)\left(4-\frac{y}{2}-\left(\frac{y}{2}\right)\right)dy=2\pi\int_0^4 (4-y)^2\,dy=\frac{128\pi}{3}$. If Washer Method is used then the absolute value curve has to be split up but then the interval can be halved and the volume doubled to achieve the full volume of the object. The integral for this would be $2\left[\pi\int_0^2\left(4^2-(4-2x)^2\right)dx\right]=\frac{128\pi}{3}$.

$B=32\pi$; Using the Washer Method the integral used would be $\pi\int_0^4\left(4-\frac{y}{2}\right)^2-\left(4-\left(4-\frac{y}{2}\right)\right)^2 dy=32\pi$.

$$\frac{A}{B}=\frac{4}{3}$$

**#12 Mu Bowl**
**MAΘ National Convention 2017**

---

$A$ is convergent by using the root test. A resulting ratio of $\frac{2}{9}$ is less than 1 therefore the series is convergent.
$B$ is also convergent. Using the ratio test will yield a ratio of 0.
$C$ is convergent because it satisfies the conditions for the Alternating Series Test.
$D$ can be solved using the Direct Comparison Test and comparing this series to the harmonic which is also divergent.

$E$ can be solved using the integral test which will result in an undefined value of $\infty$ which indicates the series is divergent also.

*D* and *E* are divergent.

**#13 Mu Bowl**
**MAΘ National Convention 2017**

---

$A=\sum_{n=0}^{\infty}\frac{f^n(-1)}{n!}(x+1)^n=\sum_{n=0}^{\infty}(n+1)(x+1)^n=1+2(x+1)+3(x+1)^2$ which can be re-written as the polynomial $3x^2+8x+6$.

$B=\sum_{n=0}^{\infty}\frac{f^n(3)}{n!}(x-3)^n=57+33(x-3)$ or it can be thought of as the tangent line through the point $x=3$. Either way the simplified polynomial would be $33x-42$.

The polynomials would intersect when $3x^2+8x+6=33x-42$ which simplifies to $3x^2-25x+48=(x-3)(3x-16)$. So the two x-coordinates are $3$ and $\frac{16}{3}$, and their sum is $\frac{25}{3}$.

**#14 Mu Bowl**
**MAΘ National Convention 2017**

---

$$A=1\left(\frac{35}{8}+\frac{133}{8}+\frac{351}{8}+\frac{737}{8}\right)=157$$

$$B=\frac{1}{2}\left(2+18+56+130+126\right)=166$$

$$C=\frac{2(157)+166}{3}$$

$$3C-(A+B)=3\left(\frac{2(157)+166}{3}\right)-(157+166)=157$$