1. C
2. D
3. D
4. B
5. E
6. A
7. B
8. D
9. A
10. E
11. C
12. A
13. E
14. A
15. C
16. D
17. C
18. A
19. C
20. B
21. D
22. A
23. A
24. B
25. D
26. B
27. C
28. C
29. D
30. B

1. C Note that all 4 fractions are very close to $\frac{2}{3}$, so to make the comparisons, we can subtract $\frac{2}{3}$ from each. Note that the numbers are so close to $\frac{2}{3}$ that it makes sense to subtract $\frac{2}{3}$ the denominator from the numerator rather than getting a common denominator.

$$w-\frac{2}{3}=-\frac{\frac{1}{3}}{431},\qquad x-\frac{2}{3}=\frac{\frac{1}{3}}{460},\qquad y-\frac{2}{3}=\frac{\frac{1}{3}}{481},\qquad z-329=-\frac{\frac{1}{3}}{494}$$

So from smallest to largest is $wzyx$.

2. D Product of two numbers between 1 and 6 is 12 when they are 2 and 6 or 3 and 4. These constitute 4 of the 36 possible outcome of rolling two dice, so the probability is $\frac{1}{9}$.

3. D Take the exponents remainder 4 for computation – $i+i+i+i-i=3i$.

4. B In a palindrome, units and thousands digits are the same, so are tens and hundreds. There are 9 choices for the thousands digit (it would not be a 4-digit number if it is 0), and 10 choices for the hundreds digit. Tens and units digits are set after that, making a total of 90 4-digit palindromes.

5. E
$$x=\ln 2^2 3^2+\ln 2^3+\ln 3^5-\ln 2^3 3^6=\ln\left(\frac{2^5 3^7}{2^3 3^6}\right)=\ln 12$$

Therefore, $e^x=12$.

6. A Let $x$ and $y$ be the cost of a strawberry and a dress, respectively. Then

$$30x+7y=80x+4y$$

So $50x=3y$, or $y=\frac{50}{3}x$.

The total amount of money Kaylee has is $30x+\frac{350}{3}x=\frac{440}{3}x$. She has enough money to buy $\left\lfloor\frac{440}{3}\right\rfloor=146$ strawberries.

7. B The number of terms in this series is $\frac{398-2}{6}+1=67$.

The sum is $\frac{67}{2}(2+398)=67\cdot 200=13400$.

8. D Let $m\angle D=x$, then $m\angle A=2x$. $AB=BE$ implies $m\angle BEA=m\angle BAE=2x$. $BC\parallel AD$ implies $m\angle CBE=m\angle BEA=2x$. $CB=CE$ implies $m\angle CEB=m\angle CBE=2x$. $CE=CD$ implies $m\angle CED=m\angle CDE=x$. Finally, $\angle BEA$, $\angle CEB$, and $\angle CED$ meet at $E$ to form a straight angle, so $2x+2x+x=180$, or $x=36$. So $m\angle A=2x=72$.

9. A Add $-2$ times row 1 to row 2, and 3 times row 1 to row 3 to obtain $\begin{bmatrix}1 & -2 & 3\\ 0 & 5 & -1\\ 0 & -1 & 16\end{bmatrix}$, so the determinant is $1\big(5\cdot 16-(-1)(-1)\big)=79$.

10. E Make a quick sketch of the 5 points to ensure that they are arranged in clockwise (or counterclockwise) order, then apply shoelace theorem to find the area of the pentagon.
$\pm\frac{1}{2}\begin{vmatrix} 4 & 3 & 0 & -2 & 1 & 4 \\ 0 & -4 & -3 & 1 & 3 & 0 \end{vmatrix} = \pm\frac{1}{2}(-16-0-9-0+0-6-6-1+0-12)$
$= -\frac{1}{2}(-50) = 25.$

11. C $C$ is $\frac{2}{3}$ of the way from $A$ to $B$, so it must be the same for both $x$ and $y$ coordinates. The difference in coordinates from $A$ to $B$ is $(15,-9)$, so the difference in coordinates from $A$ to $C$ must be $(10,-6)$, making the coordinate of $C$ $(12,1)$.

12. A Note that all radicands are powers of 2. Rewriting the original expression using rational exponents, we have

$$2^{\frac{5}{6}} \cdot 2^{\frac{7}{10}} \cdot 2^{\frac{11}{15}} = 2^{\frac{25}{30}+\frac{21}{30}+\frac{22}{30}} = 2^{\frac{34}{15}} = 4\sqrt[15]{16}$$

Thus $a+b+n = 4+15+16 = 35$.

13. E To solve this equation, we need to remove the absolute values. To do so, 3 intervals of $x$ must be considered:
$(-\infty,-2]$, where $2x-4 \le 0$ and $3x+6 \le 0$:

$$-2x+4-3x-6 = 8 \to x = -2$$

$[-2,2]$, where $2x-4 \le 0$ and $3x+6 \ge 0$:

$$-2x+4+3x+6 = 8 \to x = -2$$

$[2,\infty)$, where $2x-4 \ge 0$ and $3x+6 \ge 0$:
$2x-4+3x+6 = 8 \to x = \frac{6}{5}$ (but this is not in the appropriate interval)
So there is 1 value of $x$ that satisfies the equation, namely $x = -2$.

14. A $XM = 7$, since $M$ is the midpoint.
Let $XA = x$, then $AY = (14-x)$, then $ZA$ is a leg of both right triangles $ZAX$ and $ZAY$.
By Pythagorean Theorem, $ZA^2 = 13^2 - x^2 = 15^2 - (14-x)^2$.
Expanding to get $169 - x^2 = 225 - 196 + 28x - x^2$
Collecting and solving, $140 = 28x \to x = 5$.
By angle bisector theorem, $XB:BY = XZ:ZY = 13:15$, so $XB = \frac{13}{13+15} \cdot 14 = 6.5$.
Since $XA < XB < XM$, the proper arrangement is $ABM$.

15. C To find $f^{-1}(3) = a$, it is equivalent to finding $f(a) = 3$.
$x^3 + x - 7 = 3$ when $x = 2$ (by inspection or rational root theorem)
So $f(3x+2) = 3$ when $x = 2$, thus $f(8) = 3$.

16. D To produce the $x^8$ term, we need $2a - (13-a) = 8$, where $a$ is the exponent on $x^2$.
Solving to get $a = 6$. So the coefficient is $\binom{13}{6} = 1716$.

17. C If replacing $y$ with $-y$ does not change the relation, then the graph is symmetric with respect to x axis. Similarly, replace $x$ with $-x$ for y-axis; replace $x$ with $-x$ and $y$ with $-y$ for origin; swap $x$ and $y$ for $y = x$. By inspection, only III and IV are satisfied.

18. A
$$a^3 + \frac{1}{a^3} = \left(a + \frac{1}{a}\right)\left(a^2 - 1 + \frac{1}{a^2}\right)$$
$$a^2 + \frac{1}{a^2} = \left(a + \frac{1}{a}\right)^2 - 2 = 34$$
Thus $a^3 + \frac{1}{a^3} = 6(33) = 198$.

19. C We will consider the space in this question in 4 regions:
Region 1 – inside the box:
An easy way to compute the volume of this region is to find the volume of the box, then remove the $2 \times 4 \times 6$ region that is more than 1 inch from the faces. So the volume for this region is $(4)(6)(8) - (2)(4)(6) = 144$.
Region 2 – rectangular prisms of height 1 protruding from the faces
The volume of this region is the surface area of the box times the constant 1 inch height. That is, $2(4 \cdot 6 + 6 \cdot 8 + 8 \cdot 4) \cdot 1 = 208$.
Region 3 – quarter circular cylinders attaching the small prisms from region 2 together on each of the edges.
There are 12 of these, one for each edge of the box. Each of the 3 sets of 4 cylinders can be combined to form a right circular cylinder with radius of 1. So the total volume is $\pi \cdot 1^2 \cdot (4 + 6 + 8) = 18\pi$
Region 4 – rounded corners at each of the vertices of the box connecting the cylinders of region 3.
Each of these corners is one eighth of a sphere of radius 1. There are 8 vertices, so they combine to make a sphere, with volume $\frac{4}{3}\pi \cdot 1^3 = \frac{4}{3}\pi$.
Approximating $\pi$ as 3, the total volume is approximately $144 + 208 + 54 + 4 = 410$. This is an underestimate, but clearly not enough to make the volume closer to 450 than 400.

20. B For $f(x)$ to be defined, we first need $\log_2(\log_3(\log_4(x))) \geq 0$,
So $\log_3(\log_4(x)) \geq 1$, then $\log_4(x) \geq 3$, or $x \geq 4^3 = 2^6$.

21. D Let the radius of the larger circle be $R$, the radius of the smaller circle be $r$, and the distance between the centers of the 2 circles be $d$. Then
$$(R + r)^2 + 6^2 = d^2$$
$$(R - r)^2 + 10^2 = d^2$$
Setting the two left sides equal to each other, we have
$$R^2 + 2Rr + r^2 + 36 = R^2 - 2Rr + r^2 + 100$$
or $4Rr = 64$. Therefore, there are 3 possibilities for $(R, r)$ – $(16, 1), (8, 2), (4, 4)$.

The total area of the possibilities is $\pi(16^2 + 1^2 + 8^2 + 2^2 + 4^2 + 4^2) = 357\pi$

22. A Partially simplified, $\sqrt{44 + 16\sqrt{6}}$ can be expressed as $\sqrt{x} + \sqrt{y}$. To more easily solve for $x$ and $y$, square both quantities to remove the layered square roots:

$$x + y + 2\sqrt{xy} = 44 + 16\sqrt{6} = 44 + 2\sqrt{8^2 \cdot 6}$$

In other words, $x + y = 44$ and $xy = 2^7 \cdot 3$. Since 44 is a multiple of 4 but not 8, and $x$ and $y$ have a combined 7 2's in their prime factorization, one of $x$ and $y$ must be a multiple of 4 but not 8. It can be seen that they must be 12 and 32.

Therefore, $\sqrt{44 + 16\sqrt{6}} = \sqrt{12} + \sqrt{32} = 2\sqrt{3} + 4\sqrt{2}$, and $2 + 3 + 4 + 2 = 11$.

23. A Sum of the digits leaves the same remainder as the number when divided by 9. As the process continues until a single digit, the final result is at most 9. Therefore, $d(n)$ is simply the remainder when $n$ is divided by 9, except it produces 9 instead of 0 when $n$ is divisible by 9. So every block of 9 consecutive integers must contain exactly one each of 1 through 9. Since $2016 = 9 \cdot 224$

$$\sum_{n=1}^{2017} d(n) = 224 \sum_{i=1}^{9} i + d(2017) = 224 \cdot 45 + 1 = 224 \cdot 9 \cdot 5 + 1 = 10081$$

24. B First, collect everything to one side of the inequality:

$$\frac{x+1}{(x-1)(x-5)} - \frac{2x}{(x+6)(x-1)} - \frac{x+3}{(x+6)(x-5)} \leq 0$$

Then get a common denominator:

$$\frac{(x+1)(x+6) - 2x(x-5) - (x+3)(x-1)}{(x-1)(x-5)(x+6)} \leq 0$$

Expanding on top and collecting to get

$$\frac{-(2x^2 - 15x - 9)}{(x-1)(x-5)(x+6)} \leq 0$$

Using quadratic formula, the roots on top are $\frac{15 \pm \sqrt{297}}{4}$.

$\sqrt{297}$ is slightly larger than $\sqrt{289} = 17$, so the roots on top are approximately $-\frac{1}{2}$ and 8

Examining the number line, the solution to the inequality, within the domain, is

$$\left(-6, \frac{15 - \sqrt{297}}{4}\right] \cup (1, 5) \cup \left(\frac{15 + \sqrt{297}}{4}, 100\right]$$

In the first interval, there are 5 integers. In the second interval, there are 3. In the third interval, there are 92. In total, there are 100 integers.

25. D A fraction is a terminating decimal if and only if its reduced version takes on the form of $\frac{n'}{2^a 5^b}$ for some non-negative integers $a$ and $b$. Since $840 = 2^3 \cdot 3 \cdot 5 \cdot 7$, as long as $n$ is a multiple of 21, $\frac{n}{840}$ is a terminating decimal. From 1 to 2017, the number of multiples of 21 is $\left\lfloor \frac{2017}{21} \right\rfloor = 96$.

26. B $(0,0,0)$ is a trivial solution to the system. Further, if any of the 3 variables is 0, the other 2 must be as well. Thus, any other solution is purely non-zero, and for rest of the solution of this problem, it is safe to assume $x, y, z \neq 0$.
Substitute the first equation into the second, we have $xy^2 = 2x \rightarrow y^2 = 2 \rightarrow y = \pm\sqrt{2}$.
Similarly, $\frac{yz^2}{2} = 3y \rightarrow z^2 = 6 \rightarrow z = \pm\sqrt{6}$, and $\frac{zx^2}{3} = z \rightarrow x^2 = 3 \rightarrow x = \pm\sqrt{3}$.
However, $x, y, z$ cannot just arbitrarily take on positive and negative values due to the original inequality. They can be all positive. Alternatively, there must be 2 negatives and 1 positive among $x, y, z$.
That means there are 5 solutions.

27. C The water level rises to 4 cm, so the bottom 4 cm of the cone displaces 1cm of water. The bottom 4 cm of the cone contains $\frac{7}{8}$ of the volume of the entire cone, so the entire cone would only displace $\frac{8}{7}$ cm of water. So for the cone to be entirely submerged, there must be at least $8 - \frac{8}{7} = \frac{48}{7}$ cm of water in the container initially.

28. C The length of the latus rectum for a parabola is $4p$, where $p$ is the distance from the focus to the vertex. The distance from the focus to the directrix is $2p$, as the vertex is equidistance from focus and directrix, like any other point on the parabola. So

$$2p = \frac{|3(5) + 4(3) - 12|}{\sqrt{3^2 + 4^2}} = 3$$

The length of the latus rectum is 6.

29. D Any point on the parabola is equidistance to the focus and the directrix, so the square of the two distances must also be the same for any point $(x, y)$ on the parabola:

$$(x-5)^2 + (y-3)^2 = \left(\frac{3x + 4y - 12}{\sqrt{3^2 + 4^2}}\right)^2$$

This is quite messy to work out. Fortunately, the sum of the coefficients for a polynomial (of 2 variables in this case) is simply the polynomial evaluated for variable(s) equaling to 1. We simply have to manipulate the equation to have integer coefficients, and ensure that those coefficients cannot be reduced. So multiply both sides by 25 (the denominator on the right side), then collecting everything on the left:

$$25(x-5)^2 + 25(y-3)^2 - (3x + 4y - 12)^2 = 0$$

An inspection on the coefficients of $x^2$ and $y^2$ show that they are 16 and 9, respectively. 16 and 9 are already relatively prime, so there is no need to worry about the exact coefficients for the other 4 terms. So the sum of the coefficients is

$$25(1-5)^2 + 25(1-3)^2 - (3 + 4 - 12)^2 = 400 + 100 - 25 = 475$$

30. B $$(8 + 8 \times 11) \div 4 = 24$$