1. A
2. B
3. C
4. E
5. D

6. B
7. D
8. D
9. C
10. C

11. B
12. A
13. B
14. A
15. D

16. B
17. D
18. B
19. E
20. B

21. C
22. A
23. B
24. C
25. E

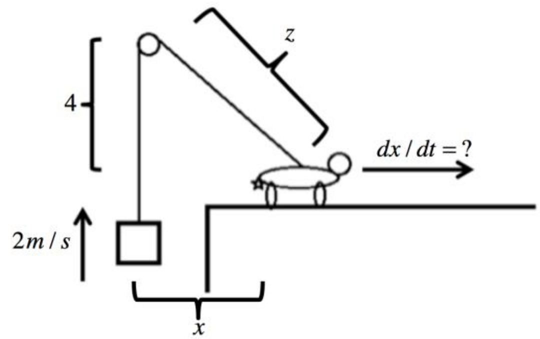
26. A
27. E
28. A
29. E
30. C

1. A

$y = 4$ for all x, so the average value of y for any interval is 4. If you don't believe me:

$$avgvalue = \frac{1}{b-a}\int_a^b f(x)\,dx = \lim_{b\to\infty}\frac{1}{b}\int_0^b 4dx = \lim_{b\to\infty}\frac{4b}{b} = 4$$

2. B



Observe that $4^2 + x^2 = z^2$.

Differentiating, $2xdx = 2zdz \to dx = \frac{zdz}{x}$

Plugging in $x = 8$, we find $z = 4\sqrt{5}$.

Thus, $dx = \frac{4\sqrt{5}\cdot 2}{8} = \sqrt{5}$

3. C

Critical points: $f'(x) = 3 - 3x^2 = 0 \to x = \pm 1 \to f(1) = 2$

Check endpoints: $f(0) = 0,\ f(3) = -18$

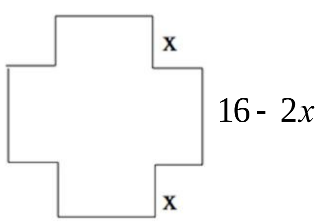
$|\max - \min| = |2 - -18| = 20$

4. E

Newton's Method: $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

We can observe that for Newton's method to yield an undefined value x_2, $f'(x_1)$ must be equal to 0. Differentiating, we find that $f'(x) = 3 - 3x^2$, which is equal to 0 when $x = \pm 1$.

5. D



$V = x(16 - 2x)^2$

$V' = (16-2x)^2 - 4x(16-2x) = (16-2x)(16-6x) = 0$

$\to x = \cancel{8}, \frac{8}{3} \to A = \frac{64}{9}$

6. B

Each interval is $15\min = \frac{1}{4}hr$

Trapezoidal rule:

$$\left(\frac{1}{2}\right)\left(\frac{1}{4}\right)\left[2.5 + 2(2.2) + 2(2.6) + 2.3\right] = \frac{19}{8}gal$$

Conversion to mi/gal:

$$\frac{57mi}{hr}\times\frac{8hr}{19gal} = \frac{24mi}{gal}$$

7. D

$passengers = 400 + \frac{1}{5}(1000 - p)$, where p represents the fare price

$revenue = price \cdot passengers = p\left(400 + \frac{1}{5}(1000 - p)\right) = 600p - \frac{1}{5}p^2$

Maximize to find that $p = \$1500$.

8. D

$f = x^2 + 2x - 8 = (x+4)(x-2) \to f < 0 \ \forall \ (-4, 2)$

$\int_{-6}^{2} \left| x^2 + 2x - 8 \right| = \int_{-6}^{-4} x^2 + 2x - 8 - \int_{-4}^{-2} x^2 + 2x - 8 = \frac{44}{3} + 36 = \frac{152}{3}$

9. C

Maximize $y = \frac{a}{a^2 + p/4}$ to find critical points $\alpha = \pm\frac{\sqrt{\pi}}{2}$, but the angle of attack must be positive to yield a positive lift to drag ratio. $\to \alpha = \frac{\sqrt{\pi}}{2}$

10. C

We know from the previous problem that this must occur when $\alpha = -\frac{\sqrt{\pi}}{2}$, yielding a lift to drag ratio of $-1/\sqrt{p}$.

11. B

Weight deceases by 600N steadily over 4m, so the rate of sand loss is 150N/m

$F(x) = 1200 - 150x \to W = \int_0^4 (1200 - 150x)\, dx = 3600J$

12. A

$\frac{dx}{dt} = 1 - \cos t, \ \frac{dy}{dt} = \sin t$

$Speed = \sqrt{\frac{dx}{dt}^2 + \frac{dy}{dt}^2} = \sqrt{(1 - \cos t)^2 + \sin^2 t} = \sqrt{2 - 2\cos t}$

13. B

$arclength = \int_a^b speed = \int_0^p \sqrt{2 - 2\cos t}\, dt = 2\int_0^p \sin(t/2)\, dt = 4$

14. A

Separate variables: $\frac{dy}{y^2+1}=dt$

Integrate: $\arctan\left(y\right)=t+C\rightarrow$ using initial condition, $C=\frac{\pi}{4}$

$y=\tan\left(t+\frac{\pi}{4}\right)$, which is undefined at $t=\frac{\pi}{4}+k\pi$

15. D

$$\left.\begin{matrix}x=r\cos\theta\\ y=r\sin\theta\end{matrix}\right\}\rightarrow\frac{dy}{dx}=\frac{\frac{dr}{d\theta}r\sin\theta}{\frac{dr}{d\theta}r\cos\theta}=\frac{\frac{dr}{d\theta}\sin\theta+r\cos\theta}{\frac{dr}{d\theta}\cos\theta-r\sin\theta}$$

$$r=2\sin\left(2\theta\right)\rightarrow\frac{dr}{d\theta}=4\cos\left(2\theta\right)$$

$$\frac{dy}{dx}=\frac{4\cos\left(2\theta\right)\sin\theta+2\sin\left(2\theta\right)\cos\theta}{4\cos\left(2\theta\right)\cos\theta-2\sin\left(2\theta\right)\sin\theta}\rightarrow\left.\frac{dy}{dx}\right|_{\theta=\frac{\pi}{6}}=\frac{4\cdot\frac{1}{2}\cdot\frac{1}{2}+2\cdot\frac{\sqrt{3}}{2}\cdot\frac{\sqrt{3}}{2}}{4\cdot\frac{1}{2}\cdot\frac{\sqrt{3}}{2}-2\cdot\frac{\sqrt{3}}{2}\cdot\frac{1}{2}}=\frac{\frac{5}{2}}{\frac{\sqrt{3}}{2}}=\frac{5\sqrt{3}}{3}$$

16. B

Use shell method, where y-axis is axis of symmetry of shells. Each shell has a radius x, height $\sin\left(x^2\right)$, and thickness dx.

$$dV=2px\sin\left(x^2\right)dx\rightarrow V=\int dV\rightarrow V=\int_0^{\sqrt{p}}2px\sin\left(x^2\right)dx=2p$$

17. D

$$z=x^x\rightarrow\ln z=x\ln x\rightarrow\frac{z'}{z}=\frac{x}{x}+\ln x\rightarrow z'=x^x\left(1+\ln x\right)$$

$$y=x^x\ln x\rightarrow y'=\frac{x^x}{x}+\ln x\left(x^x\right)\left(1+\ln x\right)$$

$$\left.y'\right|_{x=e}=e^{e-1}+2e^e=e^{e-1}\left(1+2e\right)$$

18. B

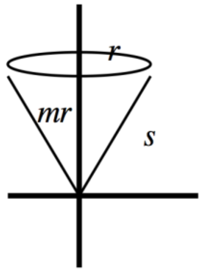
By Pappus' Theorem: $V=2\pi\left(\text{distance from centroid to axis of revolution}\right)\left(\text{cross-sectional area}\right)$

Area of ellipse $=\pi ab=\pi\cdot2\cdot3=6\pi$

By inspection, distance from center of ellipse to $y+x=1$ is $4\sqrt{2}$

Thus, $V=2\pi\cdot4\sqrt{2}\cdot6\pi=48\sqrt{2}\pi^2$

19. E



$$A = \pi r s = \pi r^2 \sqrt{1+m^2} \to r = \sqrt{\frac{A}{\pi\sqrt{1+m^2}}}$$

$$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi r^3 m \to V = \frac{1}{3}\pi m \cdot \frac{A^{3/2}}{\pi^{3/2}\left(1+m^2\right)^{3/4}} = \frac{Cm}{\left(1+m^2\right)^{3/4}}$$

Maximizing V… $m = \pm\sqrt{2} \to m = \sqrt{2}$

20. B

This is a simple first-order differential equation.

$$\frac{dy}{y} = -\frac{3}{5}dt \to \ln y = -\frac{3}{5}t + C \to y = Ce^{-\frac{3}{5}t}$$

Plugging in initial condition, $y = 100e^{-\frac{3}{5}t} \to y\left(\frac{5}{3}\right) = \frac{100}{e}$

21. C

Observe from inspection that $\frac{1}{4}th$ of the area is removed at each step. Thus, the problem reduces to an infinite geometric series with common ratio $\frac{3}{4}$.

Thus, $S = \frac{a}{1-r} = \frac{\frac{b^2\sqrt{3}}{4}}{1-\frac{3}{4}} = b^2\sqrt{3}$

22. A

$$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{2n+2} = \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{2n} = \left(\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{n}\right)^2 = e^2$$, by definition of e

23. B

Let $y = \sqrt{1+x^2}$ and $w = \frac{x^2}{\sqrt{1+x^2}}$

$$\frac{dy}{dw} = \frac{\frac{dy}{dx}}{\frac{dw}{dx}} = \frac{\frac{x}{\left(1+x^2\right)^{\frac{1}{2}}}}{\frac{x\left(x^2+2\right)}{\left(1+x^2\right)^{\frac{3}{2}}}} = \frac{x\left(1+x^2\right)}{x\left(x^2+2\right)} = \frac{1+x^2}{2+x^2} \to \left.\frac{dy}{dw}\right|_{x=0} = \frac{1}{2}$$

24. C

$$\lim_{n\to\infty}\sum_{i=1}^{n}\frac{e}{i+n}=\lim_{n\to\infty}\sum_{i=1}^{n}\left(\frac{e}{\frac{i}{n}+1}\right)\left(\frac{1}{n}\right)=\int_0^1\frac{e}{x+1}\,dx=\left[e\ln|x+1|\right]_0^1=e\ln 2\to b=2$$

$$x=\frac{i}{n},\ \Delta x=\frac{1}{n}$$

$$\lceil e\rceil+\lfloor 2\rfloor=3+2=5$$

25. E

$$V=\pi\int_a^b N(x)^2\,dx=\pi\left(b^2-ab\right)$$

$$\to V=\pi\int_a^x N(x)^2\,dx=\pi\left(x^2-ax\right)$$

$$\to V'=\pi N(x)^2=2\pi x-\pi a,\ \text{for all } x>a$$

$$\to N(x)=\pm\sqrt{2x-a}\to N(x)=\sqrt{2x-a}$$

$$\to N\left(\frac{a+1}{2}\right)=1$$

26. A

Solve using 2 iterations of Improved Euler's Method:

Identify: $y_0=3, x_0=0, x_1=\frac{1}{2}, x_2=1$

$$z_1=3+2(3)(1)\left(\frac{1}{2}\right)=6,\ y_1=3+\left(\frac{2(3)(1)+2(6)\left(\frac{3}{2}\right)}{2}\right)\left(\frac{1}{2}\right)=9$$

$$z_2=9+2(9)\left(\frac{3}{2}\right)\left(\frac{1}{2}\right)=\frac{45}{2},\ y_2=9+\left(\frac{2(9)\left(\frac{3}{2}\right)+2\left(\frac{45}{2}\right)(2)}{2}\right)\left(\frac{1}{2}\right)=38.25$$

27. E

$$\int u\,dv=uv-\int v\,du$$

$$u=t^x,\ du=xt^{x-1};\ dv=e^{-t}dt,\ v=-e^{-t}$$

$$\Gamma(x+1)=\int_0^\infty t^x e^{-t}\,dt=\lim_{b\to\infty}\left[-t^x e^{-t}\right]_0^b+x\int_0^\infty t^{x-1}e^{-t}\,dt=\lim_{b\to\infty}\left(-\frac{b^x}{e^b}+0^x e^0\right)+x\Gamma(x)=x\Gamma(x)=x!$$

28. A

$$\lim_{x\to 0}\frac{\sin(ax)-\sin x-x}{x^3}=\lim_{x\to 0}\frac{\left(ax+\frac{a^3x^3}{3!}+...\right)-\left(x-\frac{x^3}{3!}+...\right)-x}{x^3}=\lim_{x\to 0}\left[\frac{a-2}{x^2}-\frac{a^3}{3!}+\frac{1}{3!}-\left(\frac{a^5}{5!}-\frac{1}{5!}\right)x^2\right]$$

is finite if $a-2=0\to a=2$

$$\lim_{x\to 0}\frac{\sin(2x)-\sin x-x}{x^3}=-\frac{2^3}{3!}+\frac{1}{3!}=-\frac{7}{6}$$

29. E

$$\frac{1}{1-x}=1+\sum_{n=1}^{\infty}x^n,\text{ where }|x|<1\to\frac{d}{dx}\left(\frac{1}{1-x}\right)=\frac{1}{(1-x)^2}=\sum_{n=1}^{\infty}nx^{n-1}$$

When $x=\frac{1}{2}$, we have $1+\frac{1}{2}\bullet 2+\frac{1}{4}\bullet 3+\cdots+\frac{n}{2^{n-1}}+\cdots=\frac{1}{\left(1-\frac{1}{2}\right)^2}=4$

30. C

$$\int_3^{\infty}\left(\frac{1}{x-2}-\frac{1}{x}\right)dx\neq\int_3^{\infty}\frac{1}{x-2}dx-\int_3^{\infty}\frac{1}{x}dx$$

The left hand integral converges but both of the right hand integrals diverge.
Therefore, Line 7 is where the error occurs.