1. B. Consider the octagon split up into eight isosceles triangles with vertex angle 45° and base angles 135°/2. We want to calculate the apothem using the tangent half-angle formula and a right triangle with base 4/2=2.

$$\tan\frac{135^\circ}{2}=\frac{a}{2}$$

$$a=2\frac{1-\cos 135^\circ}{\sin 135^\circ}$$

$$a=2\frac{1+\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}}=2\left(\sqrt{2}+1\right)$$

$$A=\frac{asn}{2}=\frac{2\left(\sqrt{2}+1\right)\cdot 4\cdot 8}{2}=32\left(\sqrt{2}+1\right)$$

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2. C.

Given: $\frac{dA}{dt}=9, A=\frac{s^2\sqrt{3}}{4}$, at $s=6$

$$\frac{dA}{dt}=2s\frac{\sqrt{3}}{4}\frac{ds}{dt}$$

$$9=2(6)\frac{\sqrt{3}}{4}\frac{ds}{dt}\Leftrightarrow\frac{ds}{dt}=\sqrt{3}$$

$$r_{in}=\frac{1}{3}\frac{s\sqrt{3}}{2}$$

$$\frac{dr_{in}}{dt}=\frac{\sqrt{3}}{6}\frac{ds}{dt}=\frac{1}{2}$$

$$A_{in}=pr_{in}^{\ 2}$$

$$\frac{dA_{in}}{dt}=2pr_{in}\frac{dr_{in}}{dt}=2p\sqrt{3}\left(1/2\right)=p\sqrt{3}$$

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3. D. Since we want the total area, we give all areas a positive sign. For each period of the sine wave, the area is 4, so 1.5 periods gives a total area of 6.

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4. C. $V=\frac{1}{3}pR^2H=\frac{1}{3}p(12/2)^2(8)=96p$

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5. C. $S=pR^2+pRl$, where l is the lateral length, calculated by Pythagorean theorem:
$l=\sqrt{R^2+H^2}=10$. $S=36p+60p=96p$

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6. C. The volume of a frustum is given by $V=\frac{p}{3}\left(R^2H-r^2h\right)$, where R and H are the fixed lengths of the full cone, and r and h are the radius and height of the deteriorating cone. Through similar triangles, we have the relation $\frac{r}{h}=\frac{R}{H}\Leftrightarrow r=\frac{R}{H}h$, which can be substituted into the volume equation to give $V=\frac{p}{3}\left(R^2H-\frac{R^2}{H^2}h^3\right)$ since we know how h changes. Taking the time derivative and assuming R and H are constant gives $\frac{dV}{dt}=-\frac{\pi R^2}{3H^2}3h^2\frac{dh}{dt}=-\frac{\pi R^2}{H^2}h^2\frac{dh}{dt}$. At 20 seconds, or 1/3 minute, the height has dropped 1/6 units, giving us a volume decrease of

$\frac{dV}{dt} = -\frac{\pi 6^2}{8^2}\left(\frac{1}{6}\right)^2\left(\frac{1}{2}\right) = -\frac{\pi}{128}$ units^3/min, and then taking the opposite answer since we asked for negative rate of change.

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7. B. The two curves change at x=$\frac{\pi}{4}$, so we need to make two integrations. Using the shell/cylinder method:

$$V = 2\pi\int_0^{\frac{\pi}{4}} t\left(\cos^2 t - \sin^2 t\right)dt + 2\pi\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} t\left(\sin^2 t - \cos^2 t\right)dt$$

$$V = 2\pi\int_0^{\frac{\pi}{4}} t\cos 2t\,dt - 2\pi\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} t\cos 2t\,dt$$

$$V = 2\pi\left[\frac{t}{2}\sin 2t + \frac{1}{4}\cos 2t\right]_0^{\frac{\pi}{4}} - 2\pi\left[\frac{t}{2}\sin 2t + \frac{1}{4}\cos 2t\right]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$$

$$V = 2\pi\left[\frac{\pi}{8} - \frac{1}{4}\right] - 2\pi\left[\frac{1}{4} - \frac{\pi}{8}\right] = \frac{\pi^2}{4}$$

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8. B. We set up the integration and solve for $B(\alpha,\beta)$

$$\int_0^1 \frac{1}{B(\alpha,\beta)} x^{4-1}(1-x)^{3-1}\,dx = 1$$

$$\int_0^1 x^3(1-x)^2\,dx = B(\alpha,\beta)$$

Using integration by parts...

$$\cdot\ \frac{1}{4}x^4(1-x)^2\Big|_0^1 - \int_0^1 -\frac{1}{2}x^4(1-x)\,dx = B(\alpha,\beta)$$

$$0 + \frac{1}{2}\left[\frac{1}{5}x^5 - \frac{1}{6}x^6\right]_0^1 = B(\alpha,\beta)$$

$$B(\alpha,\beta) = \frac{1}{60}$$

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9. A. We need to calculate the first derivative of the pdf, set equal to 0 and solve for x.

$$f_X'(x;3,6) = 336x(1-x)^5 - 840x^2(1-x)^4 = 0$$

$$x(1-x)^4[336(1-x) - 840x] = 0$$

$$x = \frac{336}{336+840} = \frac{2}{7}$$

10. B. Set up the integral and evaluate:

$$m = \int_0^1 168x^3(1-x)^5 dx$$

$$m = \int_0^1 168x^3(1-5x+10x^2-10x^3+5x^4-x^5)dx$$

$$m = \int_0^1 168x^3 - 840x^4 + 1680x^5 - 1680x^6 + 840x^7 - 168x^8 dx$$

$$m = \frac{168}{4} - \frac{840}{5} + \frac{1680}{6} - \frac{1680}{7} + \frac{840}{8} - \frac{168}{9}$$

$$m = 42 - 168 + 280 - 240 + 105 - \frac{56}{3} = \frac{1}{3}$$

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11. C. The first three terms of sin x are $P_3(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!}$. The integral is then

$$\int_0^1 x - \frac{x^3}{3!} + \frac{x^5}{5!} dx = \frac{x^2}{2} - \frac{x^4}{4!} + \frac{x^6}{6!}\Big|_0^1 = \frac{1}{2} - \frac{1}{24} + \frac{1}{720} = \frac{331}{720}.$$

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12. A. We need to do a chain rule rate of change on the integral:

$$\frac{dA}{dt} = \frac{dA}{dx}\frac{dx}{dt} = 2x\left(\left(x^2\right)^2 e^{x^2} - x^2 \sin x^2\right)\frac{1}{2} = x^5 e^{x^2} - x^3 \sin x^2$$

$$@x = 3 \Rightarrow \frac{dA}{dt} = 243e^9 - 27\sin 9$$

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13. E. If we define one of the vertices of the rectangle as (x,y), then the perimeter of the rectangle is $P = 4x + 4y$. From the equation of the ellipse, we know that differentiating wrt x gives $\frac{2x}{a^2} + \frac{2y}{b^2}\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -\frac{b^2 x}{a^2 y}$. We want to maximize P, so $\frac{dP}{dx} = 4 + 4\frac{dy}{dx}$. Substituting gives $\frac{dP}{dx} = 4 - \frac{4b^2 x}{a^2 y} = 0$. Solving for x gives $x = \frac{a^2}{b^2}y$. We can substitute this back into the equation of the ellipse to get that $y = \frac{b^2}{\sqrt{a^2+b^2}}$ and also $x = \frac{a^2}{\sqrt{a^2+b^2}}$. The area is then $A = 4xy = \frac{\left(2ab\right)^2}{a^2+b^2}$.

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14. C. The area enclosed by a chord and a parabola is given by the formula $A = \frac{2}{3}bh$, essentially 2/3 of the area formed by the parallelogram. The height is the focal length and the base is the length of the latus rectum. Converting the equation of the parabola to standard form gives $(y-3)^2 = 12(x+1)$. The focal length is 12/4=3, the height must be 3 and width 12, 2/3(36)=24.

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15. A. Simpson's rule applied to sec x on the given subinterval gives the formula

$$A=\frac{\frac{\pi/4-(-\pi/4)}{4}}{3}\left[\sec(-\tfrac{\pi}{4})+4\sec(-\tfrac{\pi}{8})+2\sec 0+4\sec(\tfrac{\pi}{8})+\sec(\tfrac{\pi}{4})\right]$$
$$=\frac{\pi}{24}\left[\sqrt{2}+4\sqrt{2\left(2-\sqrt{2}\right)}+2+4\sqrt{2\left(2-\sqrt{2}\right)}+\sqrt{2}\right]$$
$$=\frac{\pi}{24}\left[2+2\sqrt{2}+8\sqrt{2\left(2-\sqrt{2}\right)}\right]$$

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16. D. If the bases are $\sqrt{x}$ and the legs are $2\sqrt{x}$, then the height of the triangles must be given by $h=\sqrt{\left(2\sqrt{x}\right)^2-\left(\frac{1}{2}\sqrt{x}\right)^2}=\frac{1}{2}\sqrt{15x}$. The integral becomes

$$V=\int_0^9\frac{1}{2}\left(\sqrt{x}\right)\left(\frac{1}{2}\sqrt{15x}\right)dx=\int_0^9\frac{\sqrt{15}}{4}x\,dx$$
$$V=\frac{81\sqrt{15}}{8}$$

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17. C. First we need to determine the intersections of the two curves:

$$x^2+6x-22=4x-7$$
$$x^2+2x-15=0$$
$$(x+5)(x-3)=0$$
$$x=\{-5,3\}$$
$$\int_{-5}^{3}4x-7-x^2-6x+22\,dx=\frac{256}{3}$$

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18. B. With the projection into the xy-plane, we know that *a=2* and *b=3.* Keeping the x-component, we know that the xz-projection must be $\frac{x^2}{4}+\frac{z^2}{c^2}=1$ and that c>2. Given the eccentricity, we can calculate c to be $e=\frac{\sqrt{c^2-4}}{c}=\frac{2\sqrt{5}}{5}\quad\therefore\ c^2=20\Rightarrow c=2\sqrt{5}$. The volume of an ellipsoid is given by $V=\frac{4\pi}{3}abc=\frac{4\pi}{3}(2)(3)(2\sqrt{5})=16\pi\sqrt{5}$

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19. D. Shoelace theorem.

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20. B. The centroid is calculated by finding the intersection of the line segments connecting the centroids of the four triangles made by the vertices of the quadrilateral choosing three at a time for the vertices of those triangles. The opposite centroids for two triangles are (2,-1) and (13/3,5/3), while the other set of opposite centroids are (7/3,7/3) and (10/3,0). Connecting these two pairs of points and finding the intersection yields the point (697/219,77/219), so $\bar{y}=77/219$.

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21. D. Pappus' centroid theorem allows us to calculate the centroid easily. We know that the volume of a revolved solid is $V=2\pi rA$, where *r* is the distance from the axis to the centroid. The area of the lamina is $\frac{1}{2}\pi r^2$ and the volume of the sphere is $\frac{4}{3}\pi r^3$. Equating and solving for *r* gives $\frac{4r}{3\pi}$.

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22. A.

$$V=\pi\int_{\pi/4}^{\pi/4}(\sec x+1)^2-(\tan x+1)^2\,dx$$

$$V=\pi\int_{\pi/4}^{\pi/4}\sec^2 x+2\sec x+1-\tan^2 x-2\tan x-1\,dx$$

$$V=\pi\int_{\pi/4}^{\pi/4}1+2\sec x-2\tan x\,dx \text{ by trigonometric identities}$$

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23. A. The right endpoint Riemann sum approximates the integral to

$\ln 2+\ln 3+\ln 4+\ln 5=\ln 120$. The true value of the integral is

$\int_1^5 \ln x\,dx = x\ln x - x\Big|_1^5 = 5\ln 5-5-1\ln 1+1=\ln 5^5-4.$ The right Riemann sum will overestimate the integral, giving us an absolute difference of $\ln 120-\ln 5^5+4=\ln\frac{120e^4}{5^5}=\ln\frac{24e^4}{625}$.

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24. D. We need to calculate the area of the parallelograms formed by each combination of vectors, multiplied by 2 to account for the parallel faces. For each pair, the magnitude is $\left\|[3,2,1]\times[-1,3,0]\right\|=\sqrt{(-3)^2+1^2+11^2}=\sqrt{131}$, $\left\|[3,2,1]\times[2,2,5]\right\|=\sqrt{8^2+(-13)^2+2^2}=\sqrt{237}$, and $\left\|[-1,3,0]\times[2,2,5]\right\|=\sqrt{15^2+5^2+(-8)^2}=\sqrt{314}$. The sum of each times two is the answer.

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25. C. To determine the bounds for the inner loop, we set the equation *r=0* and solve for angles: $0=2+4\cos\theta \Leftrightarrow -\frac{1}{2}=\cos\theta \Rightarrow \theta=\left\{\frac{2\pi}{3},\frac{4\pi}{3}\right\}$. Setting up the integral on polar area:

$$A=\frac{1}{2}\int_{2\pi/3}^{4\pi/3}(2+4\cos\theta)^2\,d\theta=\int_{2\pi/3}^{4\pi/3}2+8\cos\theta+8\cos^2\theta\,d\theta=4\pi-6\sqrt{3}.$$

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26. D.

$$\Delta x_i=\frac{3}{n}=\frac{b-a}{n}\Rightarrow b-a=3 \qquad 1+\frac{3i}{n}=a+i\Delta x_i\Rightarrow a=1,b=4$$

$$2\left(1+\frac{3i}{n}\right)=2x$$

$$\therefore \int_1^4(2x+1)\,dx=x^2+x\Big]_1^4=20-2=18$$

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27. C. If $V=\frac{4}{3}pR^3$, then $\frac{dV}{dt}=4pR^2\frac{dR}{dt}$. Likewise, $S=4pR^2$ Þ $\frac{dS}{dt}=8pR\frac{dR}{dt}$. Setting up the ratio gives $\frac{dV}{dt}=4\pi R^2\frac{1}{8\pi R}\frac{dS}{dt}\Leftrightarrow\frac{dV/dt}{dS/dt}=\frac{R}{2}$.

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28. A. The formula for the annular sector is given by

$$A=\frac{1}{2}\left(R^2-r^2\right)\theta=\frac{1}{2}R^2\theta-\frac{1}{2}r^2\theta$$

$$\frac{dA}{dt}=\frac{1}{2}R^2\frac{d\theta}{dt}-\frac{1}{2}(2r)\frac{dr}{dt}\theta-\frac{1}{2}r^2\frac{d\theta}{dt}=\frac{1}{2}R^2\frac{d\theta}{dt}-r\theta\frac{dr}{dt}-\frac{1}{2}r^2\frac{d\theta}{dt}$$

$$\frac{dr}{dt}=\sqrt{t}\Rightarrow r=\frac{2}{3}t^{3/2}\qquad\frac{d\theta}{dt}=\frac{2\pi}{100}\Rightarrow\theta=\frac{2\pi t}{100}$$

$$\frac{dA}{dt}=\frac{1}{2}(10)^2\frac{2\pi}{100}-\left(\frac{2}{3}t^{3/2}\right)\left(\frac{2\pi t}{100}\right)\sqrt{t}-\frac{1}{2}\left(\frac{2}{3}t^{3/2}\right)^2\left(\frac{2\pi}{100}\right)$$

$$\frac{dA}{dt}=\pi-\frac{\pi}{75}t^3-\frac{\pi}{225}t^3$$

$$0=\pi-\left(\frac{\pi}{75}-\frac{\pi}{225}\right)t^3\Rightarrow t=\sqrt[3]{225/4}=\sqrt[3]{56.25}\approx 4\text{ sec}$$

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29. C. $V=p\int_1^{\infty}\frac{1}{x^2}dx=-\frac{p}{x}\Big|_1^{\infty}=p$

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30. B.

$$A=\frac{1}{2}\int_0^{2\pi}a\cos^3 t\,d(a\sin^3 t)-a\sin^3 t\,d(a\cos^3 t)$$

$$A=\frac{1}{2}\int_0^{2\pi}3a^2\cos^4 t\sin^2 t+3a^2\sin^4 t\cos^2 t\,dt$$

$$A=\frac{3a^2}{2}\int_0^{2\pi}\sin^2 t\cos^2 t\left(\sin^2 t+\cos^2 t\right)dt$$

$$A=\frac{3a^2}{2}\int_0^{2\pi}\left(\frac{1-\cos 2t}{2}\right)\left(\frac{1+\cos 2t}{2}\right)dt$$

$$A=\frac{3a^2}{8}\int_0^{2\pi}1-\cos^2 2t\,dt=\frac{3a^2}{8}\int_0^{2\pi}1-\frac{1+\cos 4t}{2}dt$$

$$A=\frac{3a^2}{8}\left[\frac{t}{2}+\frac{1}{4}\sin 4t\right]_0^{2\pi}=\frac{3\pi a^2}{8}$$