Answers:

0. 5
1. -1
2. 315
3. 16
4. -4
5. $(-2,94)$
6. C
7. 8
8. 45
9. $\frac{3+\sqrt{93}}{6}$
10. 22
11. $\frac{1}{6}$
12. 13
13. $\frac{16\pi}{3}$
14. 262

Solutions:

0. $\lim_{x\to 4}\frac{2x^2-7x+6}{x^2-5x+6}=\frac{2\cdot 4^2-7\cdot 4+6}{4^2-5\cdot 4+6}=\frac{10}{2}=5$

1. $\lim_{x\to 2}\frac{2x^2-7x+6}{x^2-5x+6}=\lim_{x\to 2}\frac{(2x-3)\cancel{(x-2)}}{(x-3)\cancel{(x-2)}}=\frac{2\cdot 2-3}{2-3}=\frac{1}{-1}=-1$

2. Using the stars and bars method, there are $\binom{7+5-1}{7}=\binom{11}{7}=330$ ways to distribute the candy, which includes ways in which each person receives at least one piece, which we don't want to count. The number of ways to distribute the candy so that each person gets one piece is $\binom{2+5-1}{2}=\binom{6}{2}=15$ (essentially giving each person one piece and finding the number of ways to distribute the remaining two pieces), so the number of ways to distribute the candy so that at least one person receives no pieces is $330-15=315$.

3. Setting $k=\frac{n-71}{n-5}$ and solving for *n*, $n=\frac{5k-71}{k-1}=5-\frac{66}{k-1}$, so as long as $k-1$ divides 66, *n* will be an integer. Since $66=2^1\cdot 3^1\cdot 11^1$, there are $(1+1)^3=8$ positive integral factors of 66, meaning there are 16 total (positive or negative) integral factors of 66 . Each factor corresponds to a value of *k* and thus a value of *n*, so there are 16 such values of *n*.

4. Differentiating implicitly, $3y^2\frac{dy}{dx}+4xy\frac{dy}{dx}+2y^2+4x\frac{dy}{dx}+4y+2x=0$. Plugging $x=3$ into the original equation yields $0=y^3+6y^2+12y+9=(y+3)(y^2+3y+3)$ $\Rightarrow y=-3$ (since *y* must be real). Therefore, plugging $(x,y)=(3,-3)$ into the result of the implicit differentiation yields $27\left.\frac{dy}{dx}\right|_{x=3}-36\left.\frac{dy}{dx}\right|_{x=3}+18+12\left.\frac{dy}{dx}\right|_{x=3}-12+6=0$ $\Rightarrow\left.\frac{dy}{dx}\right|_{x=3}=-4$, so this is the slope of the tangent at that point.

5. The tangent line slope is given by $y'=3x^2+12x-36$, so we are trying to minimize this. Therefore, $y''=6x+12$, which changes signs from negative to positive only once, at

$x=-2$. Therefore, this is where the absolute minimum tangent slope occurs. The *y*-value at the point is therefore $(-2)^3+6(-2)^2-36(-2)+6=94$; the point is $(-2,94)$.

6. Every time Mary walks into a point on a road, she must have another road on which to exit (except for the point at which she begins and the point at which she ends). Each point has an even number of roads emanating from it, except for A (which is the point at which she begins) and C (which must be the point at which she ends). So her home is at point C. One such path Mary could take would be ABGACAFCGEHFHDHDEC.

7. $$\int_0^1 \frac{2x+1}{x^2+1}dx=\int_0^1 \frac{2x}{x^2+1}dx+\int_0^1 \frac{1}{x^2+1}dx=\left(\ln\left(x^2+1\right)+\arctan x\right)\Big|_0^1=\ln 2+\frac{\pi}{4}=\frac{4\ln 2+\pi}{4}$$
$$=\frac{\ln 16+\pi}{4}\Rightarrow a=16 \text{ and } b=4\Rightarrow \frac{5ab}{2a+2b}=\frac{5(16)(4)}{2(16)+2(4)}=\frac{320}{40}=8.$$

8. Substituting $\Delta x=\frac{3}{n}$ and $x_{i-1}=1+(i-1)\Delta x=1+\frac{3(i-1)}{n}$, this limit becomes

$\lim\limits_{n\to\infty}\left(\Delta x\sum_{i=1}^{n}\left(1+2\left(x_{i-1}\right)^2\right)\right)$, where $x_0=1$ and $x_n=4$, so using left endpoints, this is equal to

$$\int_1^4\left(1+2x^2\right)dx=\left(x+\frac{2}{3}x^3\right)\Big|_1^4=\left(4+\frac{128}{3}\right)-\left(1+\frac{2}{3}\right)=45.$$

OR (if you didn't realize the above)

$$\lim_{n\to\infty}\left(\frac{3}{n}\sum_{i=1}^{n}\left(1+2\left(1+\frac{3(i-1)}{n}\right)^2\right)\right)=\lim_{n\to\infty}\left(\frac{3}{n}\sum_{i=1}^{n}\left(3+\frac{12(i-1)}{n}+\frac{18(i-1)^2}{n^2}\right)\right)$$
$$\lim_{n\to\infty}\left(\frac{3}{n}\cdot 3n+\frac{36}{n^2}\cdot\frac{(n-1)n}{2}+\frac{54}{n^3}\cdot\frac{(n-1)n(2n-1)}{6}\right)=9+18+18=45.$$

9. According to the Mean Value Theorem, there exists a number *c* in the interval $(1,3)$ such that $6c^2-6c=f'(c)=\frac{f(3)-f(1)}{3-1}=\frac{30-2}{2}=14\Rightarrow 0=3c^2-3c-7$, and using the quadratic formula, $c=\frac{-(-3)\pm\sqrt{(-3)^2-4(3)(-7)}}{2(3)}=\frac{3\pm\sqrt{93}}{6}$. However, $\frac{3+\sqrt{93}}{6}$ is the only one of these two numbers that is in the interval, so it is the only value guaranteed by the Mean Value Theorem.

10. Let K be the area enclosed by the triangle. The length of the radius of the inscribed circle is $\frac{2K}{9+11+16}=\frac{K}{18}$, and the length of the radius of the circumscribed circle is $\frac{9\cdot 11\cdot 16}{4K}=\frac{396}{K}$. Therefore, the product of the lengths is $\frac{K}{18}\cdot\frac{396}{K}=22$.

11. $\lim\limits_{x\to 0}\left(\frac{60\sin x+10x^3-60x}{3x^5}\right)$ (which is of $\frac{0}{0}$ type) $=\lim\limits_{x\to 0}\left(\frac{60\cos x+30x^2-60}{15x^4}\right)$ (which is of $\frac{0}{0}$ type) $=\lim\limits_{x\to 0}\left(\frac{-60\sin x+60x}{60x^3}\right)$ (which is of $\frac{0}{0}$ type) $=\lim\limits_{x\to 0}\left(\frac{-60\cos x+60}{180x^2}\right)$ (which is of $\frac{0}{0}$ type) $=\lim\limits_{x\to 0}\left(\frac{60\sin x}{360x}\right)$ (which is of $\frac{0}{0}$ type) $=\lim\limits_{x\to 0}\left(\frac{60\cos x}{360}\right)=\frac{60}{360}=\frac{1}{6}$

12. Let x be the distance Owen has covered since passing underneath the balloon, and let y be the distance the balloon is above the road. If z is the distance between Owen and the balloon, then $x^2+y^2=z^2 \Rightarrow 2x\frac{dx}{dt}+2y\frac{dy}{dt}=2z\frac{dz}{dt}$. Three seconds after passing under the balloon, Owen has traveled 45 feet while the balloon has risen 15 feet (on top of the 45 feet it had already risen). Therefore, $2(45)(15)+2(60)(5)=2\left(\sqrt{45^2+60^2}\right)\frac{dz}{dt}$ $\Rightarrow 1950=150\frac{dz}{dt}\Rightarrow\frac{dz}{dt}=13$ feet per second.

13. The two functions intersect when $x=2$ and $x=4$, so using the shell method, the volume is $2\pi\int_2^4(x-2)\left(-(x-2)(x-6)-(x-2)^2\right)dx=2\pi\int_2^4-(x-2)^2(2x-8)dx$ $=4\pi\int_2^4-(x-2)^2(x-4)dx$, and making the substitution $u=x-2$, $=4\pi\int_0^2-u^2(u-2)du$ $=4\pi\int_0^2\left(2u^2-u^3\right)du=4\pi\left(\frac{2}{3}u^3-\frac{1}{4}u^4\right)\Big|_0^2=4\pi\left(\frac{16}{3}-4\right)=\frac{16\pi}{3}$.

14. Since each factor of 10 consists of one factor of 2 and one factor of 5, and that there are many more factors of 2 in $50!^*$ than factors of 5, so we need to determine how many factors of 5 are in the expansion of $50!^*$. Since this is the case for each factorial as well, we just find the number of factors of 5 in each factorial from $1!$ to $50!$. There are no factors of 5 in 1! to 4!, one factor of 5 in each of 5! to 9!, two factors of 5 in each of 10! to 14!, three factors of 5 in each of 15! to 19!, and four factors of 5 in each of 20! to 24!. Once we reach 25!, we jump to six factors of 5 in each of 25! to 29!, seven factors of 5 in each of 30! to 34!, eight factors of 5 in each of 35! to 39!, nine factors of 5 in each of 40! to 44!, ten factors of 5 in each of 45! to 49!, then 12 factors of 5 in 50! (an additional one in the factor of 50). Therefore, the total number of

factors of 5 in $50!^{*}$, and therefore the total number of consecutive zeros at the end of it, is $5(1+2+3+4+6+7+8+9+10)+12=262$.