

For all questions, answer choice “E) NOTA” means that none of the above answers is correct.

1. If $\frac{dy}{dx} = \cos x \cos^2 y$ and $y = \frac{\pi}{4}$ when $x = 0$, then

- A) $\tan y = \sin x + 1$
- B) $\tan y = -\sin x + 1$
- C) $\sec^2 y = -\sin x + 2$
- D) $\tan y = \frac{1}{2}(\cos^2 x + 1)$
- E) NOTA

2. A solution to $\frac{dy}{dx} = \frac{1}{xy}$ which passes through the point (1, 1) is

- A) $y = x^{-2}$
- B) $y = \sqrt{2 \ln x + 1}$
- C) $y = \sqrt{2 \ln x + 1}$
- D) $y = \sqrt{\ln x + 1}$
- E) NOTA

3. A population of wolves grows according to the differential equation $\frac{dP}{dt} = \frac{P}{20} - \frac{P^2}{2000}$ where P is the population and t is measured in years. What is the population where P is growing the fastest?

- A) 50
- B) 75
- C) 100
- D) 150
- E) NOTA

4. What is the order of the differential equation $x^2 y''' + y^4 y'' - 8x^5 y' = e^x$?

- A) 2
- B) 3
- C) 4
- D) 5
- E) NOTA

5. Which of the following functions is a solution to $y'' - y = 0$?

- I. $y = \sin x$
- II. $y = 4e^{-x}$
- III. $y = 3e^x$

- A) I only
- B) III only
- C) I and II only
- D) II and III only
- E) NOTA

6. Which of the following functions are solutions to $yy' = x(y')^2 + 1$?

I. $y = x + 1$

II. $y = \frac{1}{3}x + 3$

III. $y = 2\sqrt{x}$

IV. $y = -2\sqrt{x}$

- A) I, II only B) III, IV only C) I, II, III only D) I, II, III, IV E) NOTA

7. To estimate the height of a bridge, Pat drops a stone into the water below. How high, in feet, is the bridge if the stone hits the water 3 seconds later? (Assume 32 feet per second per second is the acceleration due to gravity.)

- A) 44.1 B) 96 C) 128 D) 144 E) NOTA

8. Which one of the following differential equations has solution $y = \sqrt{2x}$?

- A) $\frac{dy}{dx} = \frac{1}{y}$ B) $\frac{dy}{dx} = \frac{x^2}{y}$ C) $\frac{dy}{dx} = y\left(1 - \frac{1}{y}\right)$ D) $\frac{d^2y}{dx^2} = \frac{dy}{dx} + y$ E) NOTA

9. Suppose that $\frac{dP}{dt} = \frac{2P}{t-1}$ and $P(0) = 0$. When Euler's method is used to approximate $P(3)$ with step size 0.1, the method fails! Why?

- A) dP/dt is undefined at $t = 0$.
 B) dP/dt is undefined at $t = 1$.
 C) The value $t = 0.5$ lies outside the domain of $P(t)$.
 D) Non-integer step sizes cannot be used with Euler's method.
 E) NOTA

10. Find the general solution of $\sin x \sin y \, dx + \cos x \cos y \, dy = 0$.

- A) $\sec y = C \tan x$ B) $\cos y = C \sin x$ C) $\tan y = C \cot x$ D) $\sin y = C \cos x$ E) NOTA

11. A bacterial population grows according to Malthus' Law. Between noon and 2 PM the population triples. At what time will the population be 100 times what it was at noon?

- A) between 4 PM and 5 PM
- B) between 5 PM and 6 PM
- C) between 6 PM and 7 PM
- D) between 7 PM and 8 PM
- E) NOTA

12. A slope field for $\frac{dy}{dx} = 5 - y$ will exhibit which one of the properties below?

- A) a vertical asymptote at $x = 5$
- B) a horizontal asymptote at $y = 5$
- C) a family of parabolas opening upward
- D) a family of parabolas opening downward
- E) NOTA

13. Which of the following is **not** a solution to $y = xy' - 2\left(\frac{y'}{3}\right)^{3/2}$?

- A) $y = 3x - 2$
- B) $y = 75x - 50$
- C) $y = 768x - 8192$
- D) $y = x^3$
- E) NOTA

14. Thorium-234 decays at a rate proportional to the amount present. Suppose 10 grams of thorium-234 is reduced to 8 grams in one week. How many grams are left in 3 weeks, rounded to the nearest gram?

- A) 3 B) 4 C) 5 D) 6 E) NOTA

15. Compute $y(3)$ given that $y'' = |2x - 3|$ and $y(0) = 5$.

- A) 6.5 B) 8 C) 9.5 D) 16 E) NOTA

16. If $dy/dx = x - y$, then the equation of the isocline where the slopes are 1 is

- A) $y = x - 1$ B) $y = 1$ C) $y = x$ D) $y = x + 1$ E) NOTA

17. Consider the differential equation $x^2 y' + 2xy = 1$, for $x > 0$. Given that $y(2) = 2y(1)$, compute $y(3)$.

- A) $\frac{7}{27}$ B) $\frac{1}{3}$ C) $\frac{5}{21}$ D) $\frac{22}{27}$ E) NOTA

18. Solve the initial value problem $\frac{dy}{dx} = \sin \sqrt{x}$, $y\left(\frac{\pi^2}{4}\right) = 0$ in order to compute $y(\rho^2)$.

- A) $2 - 2\rho$ B) -4 C) 4 D) $2\rho - 2$ E) NOTA

19. Which differential equation below has the general solution $y = C_1 x + C_2 e^x$ for constants C_1 and C_2 ?

- A) $y' = y'' + x$
B) $y' = x(y' - y'')$
C) $y' = y''$
D) $y' = xy' + (1 - x)y''$
E) NOTA

20. A tank initially contains 100 gallons of brine in which 50 pounds of salt are dissolved. A brine containing 2 pounds per gallon of salt runs into the tank at the rate of 5 gallons per minute. The mixture is kept uniform by stirring and flows out of the tank at a rate of 4 gallons per minute. What is the concentration of salt, in pounds per gallon, 100 minutes after this process begins?

- A) $\frac{61}{32}$ B) 2 C) $\frac{525 - \ln 4}{100}$ D) $\frac{1525}{8}$ E) NOTA

21. Given that $y = xy' + \frac{2}{x}$ and $y(1) = 7$, compute $y(0.5)$.

- A) 5 B) 5.5 C) 6 D) 6.5 E) NOTA

22. Let C and K be constants. Find the orthogonal family to the family of curves $xy = Cx - 1$.

- A) $y = \frac{K}{x^2}$
B) $y^2 - 2y - x^2 = K$
C) $3y + x^3 = K$
D) $y = \frac{x}{Kx-1}$
E) NOTA

23. Solve $2y^{\ell}y^{\ll} = 1$, given that $y(1) = 5$ and $y^{\ell}(1) = 2$.

- A) $y = \frac{2}{3}(x+3)^{3/2} - \frac{1}{3}$
B) $y = \frac{4}{3}x^{3/2} + \frac{41}{3}$
C) $y = \frac{1}{4}x^2 + \frac{3}{2}x + \frac{13}{4}$
D) $y = e^{2x-2} + 4$
E) NOTA

24. An indoor thermometer, which reads 70°F , is taken outside where the air temperature is 10°F . Three minutes later it is found that the thermometer reads 25°F . What temperature does it read at 9 minutes, rounded to the nearest degree?

- A) 10°F B) 11°F C) 12°F D) 13°F E) NOTA

25. Solve the polar initial value problem $\frac{dr}{d\theta} = r \sec \theta$, $r(\pi) = 5$ to compute $r\left(\frac{5\pi}{4}\right)$.

- A) $r = -5(1 + \sqrt{2})$ B) $r = 5(1 - \sqrt{2})$ C) $r = 5(\sqrt{2} - 1)$ D) $r = 5(1 + \sqrt{2})$ E) NOTA

26. You are driving along the highway at a steady 60 mph (88 feet per second) when you see an accident ahead and slam on the brakes. What constant deceleration, in feet per second per second, is required to stop your car in 242 feet?

- A) 22 B) 32 C) 64 D) 88 E) NOTA

27. The solution curve for the differential equation $y' = \frac{6-y}{x-4}$ passes through the point (4, 6).

Which of the following statements could be true?

- I. The slope of the solution curve is zero where $y = 6$ (where defined).
- II. The slope of the solution curve is undefined where $x = 4$ (where defined).
- III. The solution curve also passes through (0, 6).

- A) II only B) I and II only C) III only D) I, II, III E) NOTA

28. Find the value of y when $x = -\ln 5$ given that $y'' + 5y' + 6y = 12$, $y(0) = 4$, and $y'(0) = 0$.

- A) -348 B) $\frac{276}{125}$ C) 2 D) $\frac{12}{5}$ E) NOTA

29. Compute $y(-4)$ given that $y = xy' + (y')^2 + y'$ and $y(1) = -1$.

- A) $-e^3$ B) $-\sqrt{2}$ C) 3 D) 4 E) NOTA

30. Which of the following choices shows the first three nonzero terms of the Taylor series solution, centered at $x = 0$, of the differential equation $y'' = y^2 - xy$ where $y(0) = 2$?

- A) $2 + 4x + 7x^2$
B) $2 + \frac{7}{2}x^2 + \frac{307}{12}x^4$
C) $2 + 4x^2 + 7x^4$
D) $2 + \frac{7}{2}x + \frac{307}{12}x^2$
E) NOTA