Answers:

1. A
2. B
3. D
4. E
5. C
6. D
7. B
8. E
9. C
10. C
11. E
12. B
13. C
14. A
15. A
16. D
17. A
18. E
19. A
20. A
21. A
22. C
23. C
24. B
25. B
26. A
27. D
28. D
29. D
30. C

Solutions:

1. This volume can be evaluated with a disk method. Using the formula for area of a circle we obtain $\pi \int_1^\infty \frac{1}{x^2} dx = \pi \left[-\frac{1}{x}\right]_1^\infty = \pi$. **A**

2. 99 of the 100 sine functions on the top and 99 of the x's on the bottom simplify to 1 because $\lim_{x \to 0} \frac{\sin x}{x} = 1$. Now we are only left with $\frac{\sin x}{\sin 2x}$. Using the double angle formula, this simplifies to $\frac{1}{2\cos x}$ and the limit of this expression as x goes to 0 is just $\frac{1}{2}$. **B**

3. Taking the derivative of this expression entails three rounds of chain rule. We obtain the expression $2f\big(g(x)\big)f'\big(g(x)\big)g'(x)$ after differentiation. Plugging in values defined for us we obtain $2(4)(5)(3) = 120$. **D**

4. Since we are revolving a figure about a line, Pappus's Centroid formula will give us the volume, $V = 2\pi r A$. The area is simply $\pi(3)(2) = 6\pi$ and the centroid is just the center, $(1, -2)$. Using the distance from point to line formula, we can solve for the distance: $\frac{(1)(1)+(-2)(-1)-1}{\sqrt{2}} = \sqrt{2}$. Using Pappus' formula the volume of the figure is: $V = 2\pi\big(\sqrt{2}\big)(6\pi) = 12\sqrt{(2)}\pi^2$. **E**

5. A and B can be solved for by integrating both sides of the inequality from -1 to 2. $\int_{-1}^2 x^2 = 3$ and $\int_{-1}^2 6 = 18$. **C**

6. We need to find the derivative equation and then differentiate it. In other words, we are looking for the second derivative of the function. $y' = -\frac{12x}{(x^2+3)\text{^}2}$. Differentiating again we get $y'' = \frac{(x^2+3)(36x^2-36)}{(x^2+3)^4}$. Setting equal to 0 to find the extrema, we can see that we get $x = 1, -1$ as our two solutions. Substituting 1 and -1 into our derivative function we can see that $x = 1$ gives us the minimum slope of $-\frac{3}{4}$. Using point slope form, the equation of the line tangent is: $y - \frac{3}{2} = -\frac{3}{4}(x-1) \rightarrow y = -\frac{3}{4}\,x + \frac{9}{4}$. **D**

7. The first step is to separate and solve this differential equation. $\frac{1}{C}dC = -\frac{1}{k}dt \rightarrow \int \frac{1}{C}dC = \int -\frac{1}{k}dt \rightarrow \ln|C| = -\frac{1}{k}t + B \rightarrow e^{B-\frac{1}{k}t} = C$. In our particular problem, B=0 because we start with a concentration of 1%. Hence we have to solve the following equation for t: $e^{-t/k} = \frac{1}{2} \rightarrow t = 2.5\ln 2$. **B**

8. Using the information given, $\frac{dC}{dt} = -\frac{1}{2.5} = -\frac{2}{5}$ **E**

9. The trick here is to switch the summation and the integral expression. The sum is an infinite geometric series and sums to $\frac{1}{1-\sin^2 x} = \sec^2 x$. Integrating the secant squared function gives the tangent function and evaluating from zero to $\frac{\pi}{3}$ we get $\sqrt{3}$. **C**

10. We can solve for the values of a, b and c by evaluating the following areas using cross products. Using the first area of 3, we can now create the following vectors with the points D, A, and B: $\boldsymbol{DA} = (a, 0, 0), \boldsymbol{DB} = (0, b, 0)$. Now setting up the determinant we can evaluate the area of DAB in terms of a, b, and c. $\frac{1}{2}\begin{bmatrix} i & j & k \\ a & 0 & 0 \\ 0 & b & 0 \end{bmatrix} = \frac{1}{2}ab = 3 \rightarrow ab = 6$. Following this same pattern we can see that $\frac{1}{2}bc = 14$, and$\frac{1}{2}ac = 18$. Since the volume of a tetrahedron is one-sixth the volume of a parallelepiped we need to find the value of $\frac{1}{6}\begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix} = \frac{1}{6}abc$ (this is found with vectors DA, DB, and DC). Multiplying all of the areas together we obtain: $a^2b^2c^2 = (6)(28)(36) \rightarrow abc = 12\sqrt{42} \rightarrow \frac{1}{6}abc = 2\sqrt{42}$. **C**

11. Looking at this fraction, we can see that the easiest way to attack it is to use product rule. Grouping the first radical on the top and the first radical on the bottom we can use the product rule. We begin with: $\frac{d}{dx}\left(\frac{\sqrt{1+2x}}{\sqrt[3]{1+3x}}\right) * \frac{\sqrt[4]{1+4x}}{\sqrt[5]{1+5x}} \dots + \frac{\sqrt{1+2x}}{\sqrt[3]{1+3x}} * \frac{d}{dx}\left(\frac{\sqrt[4]{1+4x}}{\sqrt[5]{1+5x}}\right) * \dots + \cdots$. Evaluating the first derivative we get $\frac{x}{\sqrt{1+2x}*(1+3x)}$. Plugging in 0, we get this derivative to be equal to 0. Repeating this procedure with the next derivative it is evident that we get zero for every derivative. As a result, our entire product rule expansion goes to zero. **E**

12. Plugging in our function into the given inequality, we obtain the equation: $(3x^2 - 10x - 7)e^{x^3-5x^2-7x+9} < e^{x^3-5x^2-7x+9} \rightarrow 3x^2 - 10x - 7 < 1$. Solving this equation, we obtain the roots $-2/3$, and 4. Since the equation has a less than sign we know that the integer solutions must be between -2/3 and 4. Hence, there are 4 integral solutions. **B**

13. Using differentials, we know that $dy = f'(x)dx \rightarrow dy = \sqrt{x^2+5}dx$. Hence, $dy = \sqrt{9}(.05) = .15$. Now adding our newly found dy to our original value we see that our approximation is -3.85. **C**

14. Integrating a derivative leaves the function. Therefore we simply have to evaluate the original function form zero to infinity without any integration. $\left[e^{1+x-x^2}\right]_0^\infty = -e$. **A**

15. The only possible way for this limit to ever equal a finite number is for the limit to be in indeterminate form. Hence, applying L'Hopital's rule we attain $\lim_{x \to 0} \frac{\sin 2x}{ae^{ax} - b} = \frac{1}{2}$. Again this also must be an indeterminate form hence we see that $a = b$. Applying another round of L'Hopital's rule we obtain $\lim_{x \to 0} \frac{2\cos 2x}{a^2 e^{ax}} = \frac{1}{2}$. Hence, $a^2 = 4 \to a = 2, -2$. Therefore the two ordered pairs are (2,2) and (-2,-2). The sum is 0. **A**

16. Let θ be the viewing angle, β be the whole angle (from the ground to the top of the screen), and α be the angle from the ground to the bottom of the screen. Hence, we can obtain the following equations form the information provided: $\theta = \beta - \alpha, \tan\alpha = \frac{10}{x}, \tan\beta = \frac{30}{x}$. Hence, $\theta = \arctan\frac{30}{x} - \arctan\frac{10}{x}$. Differentiating with respect to x we and setting the derivative of theta equal to 0 we obtain: $\frac{30}{x^2+30^2} = \frac{10}{x^2+10^2} \to x = 10\sqrt{3}$. **D**

17. This sum is a known sum in a different form. Noticing the reverse change of base simplification in the logarithms simplifies the sum to $\sum_{n=0}^{\infty} \frac{(-\ln 2)^n}{n!}$. This is the Maclaurin series for e^x with $x = -\ln 2$ inserted for x. This simplifies to $\frac{1}{2}$. **A**

18. Solving for y will give us half of one of the sides of our square: $y = \sqrt{1-x^2}$. Therefore the area the square is $\left(2\sqrt{1-x^2}\right)^2$. Hence the volume of an infinitely small piece of the solid is $\left(2\sqrt{1-x^2}\right)^2 dx$. Integrating from -1 to 1 will give us the volume of the whole solid. $\int_{-1}^{1} \left(2\sqrt{1-x^2}\right)^2 dx = \frac{16}{3}$. **E**

19. Making the substitution $u = \frac{\ln x}{\ln 3}$, the integral becomes: $\int_{\log_3 \sin 1}^{\log_3 \csc 1} \ln 3 * \sin^3 u \, du$. Now it is evident that we are integrating an odd function over an integral of –a to a. Hence, the integral is 0 upon evaluation. **A**

20. This function is routinely simplified by identifying the spot in the pattern where the function repeats itself. This makes the function equal to: $y = \frac{9}{x+\sqrt{x+y}} \to xy + y\sqrt{x+y} = 9$. Deriving both sides we can solve for dy/dx. $y + x\frac{dy}{dx} + \frac{dy}{dx}\sqrt{x+y} + \frac{y\left(1+\frac{dy}{dx}\right)}{2\sqrt{x+y}} = 0$. Immediately plugging in values of x and y simplifies the calculations. $\frac{15}{4}\frac{dy}{dx} + \frac{15}{4} = 0 \to \frac{dy}{dx} = -1$. **A**

21. The volume of the sand pile takes the form: $V = \frac{1}{3}\pi r^2 h$. From the given information, we know that $r = h$, hence the new volume formula is $V = \frac{1}{3}\pi h^3$. Upon differentiation we get: $\frac{dV}{dt} = \pi h^2 \frac{dh}{dt} \rightarrow 1 = 4\pi \frac{dh}{dt} \rightarrow \frac{dh}{dt} = \frac{1}{4\pi}$. **A**

22. Area in polar is equal to $\frac{1}{2}\int r^2 d\theta$. $\frac{1}{2}\int_0^{2\pi} e^{2\theta}\, d\theta$=$\frac{1}{4}\left[e^{2\theta}\right]_0^{2\pi} = \frac{1}{4}(e^{4\pi} - 1)$ **C**

23. The denominator of each of these expressions is equal to the sum of the squares of the roots minus the value in the numerator. The sum of the squares is equal to 0 hence each fraction is equal to -1. Hence, the whole expression is equal to -4. **C**

24. If these functions are continuous then the values of the left must be equal to values on the right. Hence we must plug in for a in both equations and set them equal to each other. $a^2 - a^2 + b = a^2 - ab + a \rightarrow a^2 - ab + a - b = 0 \rightarrow (a+1)(a-b) = 0 \rightarrow a = -1 \text{ or } a = b$. Note that a cannot equal b from the problem. Therefore a must equal -1. **B**

25. This limit can be solved by transforming it into the Riemann sum definition of the integral. Expanding the factorial our limit becomes: $\lim_{n\to\infty} \frac{1}{n} \ln \frac{n(n-1)(n-2)(n-3)\ldots 3*2*1}{n*n*n*n\ldots n*n*n}$. Using log laws to split the limit it becomes $\frac{1}{n}\left(\ln 1 + \ln\left(1 - \frac{1}{n}\right) + \ln\left(1 - \frac{2}{n}\right) + \ln\left(1 - \frac{3}{n}\right) + \cdots \ln\left(\frac{1}{n}\right)\right)$. Now it is evident that our integral is $\int_0^1 \ln(1-x)dx = -1$. **B**

26. From Geometry it is known that squares maximize area. Hence only having a square would maximize the area. Therefore, the value of x that would maximize the total area is 0. **A**

27. We need to find out what we need to insert into $f(3x-6)$ as the $2x-1$ factor does not matter. $0 \leq 3x - 6 \leq 12 \Longrightarrow 2 \leq x \leq 4$. Hence our desired answer is **D**.

28. $g'(x) = e^{-x}(F'(x) - F(x)) = e^{-x}\left(f^{(2009)}(x) - f(x)\right)$, so $g'(1) = e^{-1}\left(f^{(2009)}(1) - f(1)\right) = e^{-1}(0 - 11) = -\frac{11}{e}$. **D**

29. The third degree Maclaurin polynomial for $\sin x = x - \frac{x^3}{3!}$. Hence we need to integrate $\int_0^1 \left(1 - \frac{x^2}{6}\right) = \frac{17}{18}$. **D**

30. This is simply a square with side length $4\sqrt{2}$. Hence, the area is 32. **C**