

Mu Going the Distance—SOLUTIONS National Mu Alpha Theta Convention—2015

1. C—The distance needed to travel is 30 miles ($40 \times 2/3$). He needs to go 30 miles in 30 minutes. Using $r = d/t$, we get $r = 30/.5 = 60$ mph.

2. D—Since both busses each move at 20 mph, they approach each other at 40 mph. So, they cover the 60 miles between them in 1.5 hrs. The fly flies at 30 mph for 1.5 hours, so the distance travelled is 45 miles.

3. D—Let the length of the train be x meters and its speed as y m/sec. Then $\frac{x}{y} = 8 \Rightarrow x = 8y$. Also,

$$\frac{x + 264}{20} = y. \text{ Sub in for } x \text{ and solve, we get } y = 22. \text{ So, speed} = 22 \text{ m/sec} = \left(22 \cdot \frac{18}{5}\right) \text{ km/hr} = 79.2.$$

4. C—Total Distance = $\int_1^2 v(t)dt + \int_2^3 -v(t)dt + \int_3^4 v(t)dt = \frac{1}{4} + \frac{1}{4} + \frac{9}{4} = \frac{11}{4}$

5. E—2:45 PM. Plane 1 takes 45 min to fly btwn the towns. So, the pilot is in the air for the first 45 min of each hour, and the last 15 min he is refueling. The potential times for sharing a cup of coffee during refueling are then, 12:45 in Town B, 1:45 in Town A, 2:45 in Town B & 3:45 in Town A. Plane 2 takes 30 min to fly btwn the towns. The 2nd pilot's potential times for coffee are 12:30 in Town B, 1:15 in Town C, 2:00 in Town A, 2:45 in Town B and 3:30 in Town C. The only concurrent time is at 2:45 in Town B.

6. B—Call Catherine's speed k , the speed of the sidewalk s , and the length of the sidewalk d . In all cases, the distance Catherine covers is d . When walking, $d = 3k$. When standing, $d = 2s$. When Catherine walks on the moving sidewalk, $d = t(s + k)$, where t is the amount of time it takes her. Solve the 1st two

equations for s and k , then subbing into this equation gives $t = d \div \frac{5}{6}d = \frac{6}{5} = 1.2$.

7. B—There are many ways to solve this problem using simultaneous equations. However, the quick way is to note that 60m of train was behind the forward-walking person when he had walked 30m, and that 60m passed him during the time he walked 10 more meters. So, for every 10m he walked, 60m of train passed him. Since he walked 40m in all, the total length of the train that passed him is 240m.

8. B—Let S denote the time in hrs to swim a mile, so the time required to run a mile is $S/2$ & the time needed to cycle a mile is $S/3$. So, $S + S/2 + S/3 - 1/6 = 3(S/3)$. So, $S = 1/5$ and $S + S/2 + S/3 = 11/30$ hrs, which is 22 min.

9. C—Let x be the distance traveled by the runner to get to the front of the convoy and y be the distance traveled by the convoy during the same time. So, $x = y + 3$. To reach the end of the convoy when the end has advanced 6 miles from the starting point, the runner needs to travel $x - 6$ miles back (after reaching the front). As a fraction of the forward journey, we get $\frac{x-6}{x}$, thus the convoy will move $y\left(\frac{x-6}{x}\right)$ while the runner is returning to the rear. So, we get $6 = y + y\left(\frac{x-6}{x}\right) \Rightarrow x^2 + 9x + 9 = 0$ after substiting in $y = x - 3$. Hence, $x = \frac{9 + 3\sqrt{5}}{2}$ and the total distance the runner travels is $2x - 6$ or $3 + 3\sqrt{5}$.

10. A—Let x denote the number of miles Sonny had traveled in the first 40 min. Since this is $1/3$ of the way to M, it took him a total of 2 hours to get to M. Now, let y be the number of miles he had left to get to M after he had gone the additional 21 miles. So, $2x = 21 + y$ and the total trip is $3x + y = x + 2y + 21$. He can travel $\frac{3}{2}x = \frac{4}{3}y$ in 1 hour. So, $9x = 8y$. Solve to get $x = 24$ and $y = 27$ and the total dist. = 99 miles.

11. C—The right triangle formed by the trunk of the tree, the path of the feather, and its projection on the ground, has hypotenuse 18, & legs 14 and D . So, $14^2 + D^2 = 18^2$ and $D^2 = 128$, which is between 11 & 12.

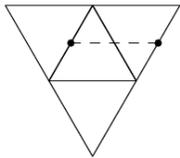
12. D—The ants can only avoid a collision if they all decide to move in the same direction (either clockwise or anti-clockwise). If the ants do not pick the same direction, there will definitely be a collision. Each ant has the option to either move clockwise or anti-clockwise. There is a one in two chance that an ant decides to pick a particular direction. Using simple probability calculations, we can determine the probability of no collision. $P(\text{No collision}) = P(\text{All ants go in a clockwise direction}) + P(\text{All ants go in an anti-clockwise direction}) = 0.5 * 0.5 * 0.5 + 0.5 * 0.5 * 0.5 = 0.25$

13. A—Reflect $(5, 3)$ about the x -axis to get $(5, -3)$. Now the shortest distance from $(7, 4)$ is the same as for the original problem. Imagine shrinking the vertical segment to zero by lowering the point $(7, 4)$ down to $(7, 2)$. The line between $(7, 2)$ and $(5, -3)$ goes through the x -axis at $x = 31/5$ and the length of this segment is $\sqrt{29}$. So, the shortest distance is $2 + \sqrt{29}$.

14. D—The dog is at home for the 1st time after 18 min. Then he runs back to Donna at double speed. It takes $18/3$ or 6 min to run back to Donna. Hence, Donna and the dog meet for the 1st time after $18 + 6 = 24$ min. Since $24 = (2/3) 36$, at that moment Donna still has a third of the way to walk and the dog begins its 2nd leg. The 2nd leg of the dog, forward to home and back to Donna, takes $1/3$ of the time of the 1st leg, hence $24/3 = 8$ min. So, Donna and the dog meet for the 2nd time after $24 + 8 = 32$ minutes.

15. E—48. Let $3d$ be the length of the entire route. Then the time, t , in hours for the cyclist to cover the entire route is $t = \frac{d}{16} + \frac{d}{24} + \frac{d}{r}$, where r is the rate in mph over the downhill part of the course. So, the average rate in mph of the entire route is $\frac{3d}{t} = 3\left(\frac{48r}{5r + 48}\right)$. Setting this equal to 24, gives $r = 48$.

16. B—Given any path on a surface, we can unfold the surface into a plane to get a path of the same length in the plane. A pair of opposite points is marked by dots in the picture below. It is obvious that in the plane the shortest path is just a segment that connects these 2 points. Its length is the same length as



the tetrahedron's edge...hence it is 1.

17. D—The 1st point can be anywhere on the circle; it doesn't matter where it is chosen. The 2nd point must lie within 60 degrees of arc on either side (total of 120 degrees possible) giving a 1/3 total chance. The last point must lie within 60 degrees of both. The minimum area of freedom we have to place the 3rd point is a 60 degrees arc (if the 1st two are 60 degrees apart), with a 1/6 probability. The maximum amount of freedom we have to place the 3rd point is a 120 degrees arc (if the 1st two are the same point), with a 1/3 probability. As the 2nd point moves farther away from the 1st point, up to a max of 60 degrees, the probability changes linearly (every degree it moves, it adds 1 degree to where the 3rd point could be). So, we can average the probabilities at each end to get 1/4. To find the average probability, we can place the 3rd point based on a varying 2nd point. So, the total probability is $(1)\left(\frac{1}{3}\right)\left(\frac{1}{4}\right) = \frac{1}{12}$.

18. C— $v(t) = \langle 12e^{3(t-1)}, -\sin(t-1) \rangle$ and speed = $\sqrt{(12e^{3(t-1)})^2 + (-\sin(t-1))^2}$. At $t=1$, speed = 12.

19. B—Length = $\sqrt{(x-0)^2 + (y-0)^2} = \sqrt{x^2 + (2x+7)^2} \Rightarrow L^2 = 5x^2 + 28x + 49 \Rightarrow$

$$2L \frac{dL}{dx} = 10x + 28 = 0 @ x = \frac{-14}{5}. \text{ So, distance} = \sqrt{\left(\frac{-14}{5}\right)^2 + \left(\frac{-28}{5} + 7\right)^2} = \frac{7\sqrt{5}}{5}$$

20. A—There is 1 way to get from A to one of the 5 vertices which are 1-edge length away from A (call those vertices C). There are 2 ways to get from ea C to ea vertex D, 1-edge length away. From each vertex D, there is 1 way to get to the opp vertex, B. So, the total number of shortest paths from A to B is $5 \times 2 = 10$.

21. E—29. The eqn of the parabola is $y = \frac{x^2}{4}$ and x must be even, so let $x = 2k$. The dist from Q $(2k, k^2)$ to $(0, 1) = \sqrt{(2k-0)^2 + (k^2-1)^2} = k^2 + 1 \leq 197$. So, $k^2 \leq 196$ and $k \in \{-14, -13, \dots, 13, 14\}$, which is 29 points.

22. D—Given opposite vertices (15, 9) and (15, 5), then the center of the decagon is (15, 7). By congruent triangles, note that for any vertex (15 + x, 7 + y), there is a corresponding opposite vertex (15 - x, 7 - y). The sum of all twelve x-coordinates is 15 x 12 and then the sum of all y-coordinates is 7 x 12. The difference is (15 - 7) x 12 = 96.

23. C—The right hand edge of the pile is at the following distance from the left hand edge:

$$2 + \left(2 - \frac{1}{1000}\right) + \left(2 - \frac{2}{1000}\right) + \dots + \left(2 - \frac{1998}{1000}\right) + \left(2 - \frac{1999}{1000}\right) = 2(2000) - \left(\frac{1 + 2 + \dots + 1999}{1000}\right) = 2001.$$

24. B—If he finished at x min past noon, then he walked for $16 + x$ mins. Setting his average (in min/mile) = to the time he stopped expressed as minutes, gives $\frac{16 + x}{5} = 12 + \frac{x}{60} \Rightarrow x = 48$. So, he stopped at 12:48.

25. D—Since the domain of \sin^{-1} is $[-1, 1]$, then $-1 \leq 2 \sin x \leq 1 \Rightarrow \frac{-1}{2} \leq \sin x \leq \frac{1}{2}$. This means, for example,

that in $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$, f is only defined on the closed interval $\left[\frac{-\pi}{6}, \frac{\pi}{6}\right]$. The endpoints of the graph of f on

that interval are $\left(\frac{-\pi}{6}, \frac{-\pi}{2}\right)$ and $\left(\frac{\pi}{6}, \frac{\pi}{2}\right)$. The distance is $\sqrt{\left(\frac{\pi}{3}\right)^2 + \pi^2} = \frac{\pi\sqrt{10}}{3}$.

26. C—Since the distances are the same, you can then set the distance expressions equal and get:
 $1100t = 16,500(t - 6)$ and $t = 45/7$. Multiply that by 1100 to get the answer.

27. D— Distance = Rate \times Time = 20 ft/s \times 60s/min \times 4 min \times 12in/ft = 57600 in

28. D—Time = Distance \div Rate = (1200 in \times 1 ft/12in) \div (20 ft/hr) = 5 hr

29. C—Let v denote the canoeist's speed in still water and w the speed of the current. Let x denote the distance the canoeist paddles downstream. Since the canoeist is paddling perpendicular to the current, the crossing time in either direction is the same. While crossing the river, the canoeist will drift a distance of $10w$ downstream. The net speed of the canoeist going upstream is $v - w$ and the total distance traveled is $10w + z + 10w$, hence $20w + x = 50(v - w)$. The net speed going downstream is $v + w$ and the distance traveled is x , so $x = 20(v + w)$. If the ratio of the canoeist's speed to that of the river current is L , then $v = Lw$. Substituting the 2nd equation into the 1st equation and replacing v by Lw you get, $20w + 20(Lw + w) = 50(Lw - w)$ and $L = 3$.

30. C—The actual elevation is 6900 feet.