Where applicable, "E) NOTA" indicates that none of the above answers is correct.

1. A parallelogram has sides of length 4 and 7 and longer diagonal of length 9. Find the length of the shorter diagonal.

A) 6 B) 7 C) $\sqrt{53}$ D) $\sqrt{65}$ E) NOTA

2. An arithmetic sequence has third term 2 and seventy-fifth term 38. Find the 2015^{th} term of this sequence.

A) 1006 B) 1007 C) 1008 D) 1009 E) NOTA

3. If $y^2 + xy = 15$, find the value of $\left.\frac{dy}{dx}\right|_{(x,y)=(2,3)}$.

A) $1/2$ B) $-3/8$ C) 0 D) $-1/2$ E) NOTA

4. Find the probability that when drawing exactly two cards from a standard, 52-card deck of playing cards, both cards are the same color or same rank (2, 3, 4, …, Q, K, A), but not both.

A) $26/51$ B) $9/17$ C) $7/13$ D) $5/9$ E) NOTA

5. Define a recursive function D_n on non-negative integers n in the following way: $D_0 = 1$ and $D_n = n! - \sum_{k=1}^{n}\left(\binom{n}{k} D_{n-k}\right)$ for integers $n \geq 1$. Find the value of D_6.

A) 176 B) 216 C) 265 D) 275 E) NOTA

6. Use the tangent line approximation to $y = \sin^{-1} x$ at the origin to approximate the value of $\sin^{-1}(0.2)$.

A) 0.05 B) 0.1 C) 0.15 D) 0.2 E) NOTA

7. Evaluate: $\lim_{x\to-\infty}\left(e^{8-5x+x^3} + e^{\frac{2x-6x^2}{4+x+3x^2}}\right)$

A) e^{-2} B) ∞ C) $1+e^{-2}$ D) $-\infty$ E) NOTA

8. The hyperbolic sine function, $\sinh x$, is defined as $\sinh x = \frac{e^x - e^{-x}}{2}$, while its derivative is defined as the hyperbolic cosine function, $\cosh x$. Which of the following is equivalent to $\cosh x - \sinh x$?

A) e^x B) e^{-x} C) $\frac{e^x}{2}$ D) $\frac{e^{-x}}{2}$ E) NOTA

9. Find the positive difference between the maximum and minimum values of the function $f(x) = 3x^4 + 8x^3 - 30x^2 - 72x + 24$ on the interval $[-2,1]$.

A) 99 B) 125 C) 128 D) 189 E) NOTA

10. For the hyperbola with equation $4x^2 - 3y^2 - 32x - 6y + 109 = 0$, find the length of its latus rectum.

A) 3 B) 4 C) 5 D) 6 E) NOTA

11. Find the value of $\sum_{n=1}^{\infty}\left(\left(2n^2 + n + 1\right)\left(\frac{3}{4}\right)^n\right)$.

A) 181 B) 183 C) 185 D) 187 E) NOTA

12. Find the point on the graph of $y = x^2 + 1$ with non-negative x-coordinate that is closest to the point $(0,2)$.

A) $(0,1)$ B) $\left(\frac{\sqrt{2}}{2}, \frac{3}{2}\right)$ C) $\left(\frac{1}{2}, \frac{5}{4}\right)$ D) $(1,2)$ E) NOTA

13. Find the number of subsets of the set $\{x \in \mathbb{Z} \mid 1 \le x \le 10\}$ that contain the number 1 or the number 2, but not both.

A) 512 B) 640 C) 768 D) 896 E) NOTA

14. If $y = x^{\ln x}$, and if $z = 2^{\ln x}$, find the value of $\left.\frac{dy}{dz}\right|_{x=2}$.

A) $2^{\ln 2}\ln 2$ B) $2^{\ln 2}$ C) 1 D) 2 E) NOTA

15. Evaluate: $\lim_{n\to\infty}\sum_{i=1}^{n}\frac{i}{i^2+n^2}$

A) $\frac{1}{2}\ln 2$ B) $\frac{\pi}{4}$ C) $\frac{e-1}{2}$ D) 1 E) NOTA

16. Let v and a be the velocity and acceleration functions, respectively, for a particle moving along the x-axis. Suppose $a=t-2$, where $t\geq 0$ is measured in seconds, and suppose that $v(2)=3$ m/s. Find the displacement, in meters, of the particle over the interval $[1,3]$.

A) $-11/3$ B) $+11/3$ C) $-19/3$ D) $+19/3$ E) NOTA

17. Which of the following differential equations has as one of its solutions a line that is both non-vertical and non-horizontal?

A) $\frac{dy}{dx}=2x+y$ B) $\frac{dy}{dx}=2xy$ C) $\frac{dy}{dx}=\frac{2x}{y}$ D) $\frac{d^2y}{dx^2}=1$ E) NOTA

18. Estimate the value of $\frac{1}{\sqrt{2\pi}}\int_{-1}^{1}e^{-\frac{x^2}{2}}dx$ to two decimal places.

A) 0.34 B) 0.48 C) 0.68 D) 0.95 E) NOTA

19. Two cars leave an intersection at the same time, one headed west and the other south. The westbound car is moving at 30 miles/hour while the southbound car is moving at 60 miles/hour. Twenty minutes later, what is the rate of change, in miles/hour, of the perimeter of the right triangle formed using the two cars and the intersection as its vertices?

A) $90+10\sqrt{5}$ B) $90+30\sqrt{5}$ C) $90+10\sqrt{30}$ D) $90+30\sqrt{30}$ E) NOTA

20. Find the area enclosed by the graphs of $y=x^3-3x^2+4x$ and $y=2x$.

A) $1/4$ B) $1/3$ C) $1/2$ D) 1 E) NOTA

21. Find the slope of the tangent to the curve $r^2=2\cos(3\theta)$ at the polar point $\left(\sqrt{2},2\pi/3\right)$.

A) $\sqrt{3}/3$ B) $-\sqrt{3}/3$ C) $\sqrt{3}$ D) $-\sqrt{3}$ E) NOTA

22. Find the volume when the region enclosed by the graphs of $y = x^2 - 2x + 2$ and $y = -x^2 + 4x - 2$ is revolved about the line $x = -1$.

A) $2\pi/3$ B) π C) $4\pi/3$ D) $5\pi/3$ E) NOTA

23. Find the length of the curve $y = \ln(\cos x)$ between the points where $x = 0$ and $x = \pi/4$.

A) $\ln(\sqrt{3}+2)$ B) $\ln(\sqrt{3}+1)$ C) $\ln(2)$ D) $\ln(\sqrt{2}+1)$ E) NOTA

24. Find the average value of the function $f(x) = 3x^2 - 2x + 1$ on the interval $[1, e]$.

A) $e^2 - 1$ B) $e^2 - e$ C) $e^2 + 1$ D) $e^2 + e$ E) NOTA

25. Which of the following adjectives describe the differential equation $\frac{dy}{dx} = 3x + 2y$?

I. first-order II. homogeneous III. ordinary
IV. separable V. linear VI. autonomous

A) I, II, & III only B) I & III only C) I, III, & V only D) I, II, III, & V only E) NOTA

26. Evaluate, if possible: $\int_{1.5}^{2} \frac{1}{\sqrt{2x - x^2}}\,dx$

A) $\pi/2$ B) $\pi/4$ C) $\pi/3$ D) unintegrable due to discontinuity
E) NOTA

27. Evaluate, if possible: $\int_{1}^{\infty} \frac{\sqrt{x}}{1 + x^3}\,dx$

A) $\pi/6$ B) $2\pi/3$ C) $\pi/3$ D) $4\pi/3$ E) NOTA

28. Find the sum of the series: $\sum_{i=2}^{\infty} \left(\frac{2^i + 3^i}{5^i} \right)$

A) $31/6$ B) $25/6$ C) $13/6$ D) $7/6$ E) NOTA

29. Which of the following convergent series can be show to absolutely converge by the Ratio Test?

A) $\sum_{n=1}^{\infty}\frac{\sqrt{n}}{1+n^3}$ B) $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}n}{n^2+1}$ C) $\sum_{n=1}^{\infty}\frac{2}{(n+1)(n+3)}$ D) $\sum_{n=2}^{\infty}\frac{1}{n(\ln n)^2}$ E) NOTA

30. The number 84 has the property that the sum of the *n* consecutive least positive integers greater than 84 is equal to the sum of the least 84 positive integers (i.e., $1+2+3+...+84 = 85+86+87+...+m$ for some integer m—in this case, $m=119$). Find the least integer x greater than 84 that also has this property (i.e., find the least integer x greater than 84 such that $1+2+3+...+x=(x+1)+(x+2)+(x+3)+...+(x+p)$ for some integer p).

A) 492 B) 408 C) 660 D) 576 E) NOTA