Solutions

1. **C** Limit does not go to infinity so you can evaluate it at x=2 to get $-\frac{33}{28}$.

2. **B** $g(x)=3+2(x-1)$ so the x-intercept is $-\frac{1}{2}$ and the y-intercept is 1 so the difference is $\frac{3}{2}$

3. **A** The expression decomposes to $\frac{A}{2x+1}+\frac{Bx+C}{x^2+1}$. Then using partial fractions A=1, C=2 and B=0. So new integral $\int \frac{1}{2x+1}+\frac{2}{x^2+1}dx$ integrates to $\frac{1}{2}\ln|2x+1|+2\tan^{-1}(x)+c$.

4. **C** The integral needs to be split up since the parabola has negative values from -2 to 1 and positive values from 1 to 3. $\int_{-2}^{1}(x^2+x-2)dx=-\frac{9}{2}$ and $\int_{1}^{3}(x^2+x-2)dx=\frac{26}{3}$ so $\left|\frac{-9}{2}\right|+\frac{26}{3}=\frac{79}{6}$

5. **A** Since the tangent line goes through an x-value that is positive the absolute values are not necessary so the equation can be reduced to $y=4x^2+1$. The tangent slope would be 4 so the normal slope is $-\frac{1}{4}$. The y-value is 2 so the equation is $y-2=-\frac{1}{4}\left(x-\frac{1}{2}\right)$, which reduces to $y=-\frac{1}{4}x+\frac{17}{8}$.

6. **C** $f'(x)=3x^2-18x-48$ which gives critical points of 8 and -2. $x=8$ is the only relative and absolute minimum on the interval, and $f(8)=-396$.

7. **E (2.2)** $f'(x)=3x^2$ so $f'(2.5)=\frac{75}{4}$ and $f(2.5)=\frac{45}{8}$. Using the Newton's Method formula or solving the tangent line equation for the x-intercept gives $x_1=\frac{11}{5}=2.2$.

8. **A** $\left(\frac{n}{n-1}\right)^n=\left(1-\frac{1}{n}\right)^n$ and $\lim_{n\to\infty}\left(1-\frac{1}{n}\right)^{-n}=e^{-1}$. This means the n^{th} term never gets to zero and therefore the sum diverges.

9. **B** $3B_6=18+B$, $10_7=7$ and $20_{1B}=20+2B$ so the equation can be rewritten in decimal as $18+B+7=20+2B$. Solving this gives B=5.

10. **D.** Evaluating at x=1 gives 0/0 so L'Hopital's Rule can be used to give a new limit of $\lim_{x\to 1}\frac{4^x\ln 4-2\cdot 2^x\ln 2}{2x}$ which can be evaluated and simplified to $\ln 4$.

11. **B.** $\begin{vmatrix} 2 & 0 \\ 1 & 6 \end{vmatrix}^{-1}=\begin{vmatrix} \frac{1}{2} & 0 \\ -\frac{1}{12} & \frac{1}{6} \end{vmatrix}$ so $\frac{1}{2}+\frac{1}{6}-\frac{1}{12}=\frac{7}{12}$

12. **D** $5\sqrt{6+5\sqrt{6+5\sqrt{6+\ldots}}}$ can be rewritten as $y=5\sqrt{6+y}$. Squaring both sides gives $y^2=25(6+y)$ and solves to 30 and -5, but -5 is not possible.

13. **A** $V=4\pi h$ since the radius is not variable. $V'=4\pi h'$ and substituting $h'=0.4$ gives 1.6π

14. **B** $i\ln\left(\frac{1}{\sqrt{i}}\right)=\ln(i^{-i/2})=\ln(e^{\pi/4})=\frac{\pi}{4}$ then $\tan\left(\frac{\pi}{4}\right)=1$

15. **B** $A_{one\_petal}=\frac{1}{2}\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}}(2\cos 3\theta)^2\,d\theta=\frac{1}{2}\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}}2+2\cos 6\theta\,d\theta$ which can be integrated and evaluated to $\frac{\pi}{3}$. The area of all three petals is π.

16. **D** The arc length can be written as $\int_0^{\pi}\sqrt{(x')^2+(y')^2}\,dt$ where $x'=-4\cos\theta\sin\theta=-2\sin 2\theta$ and $y'=2\cos 2\theta$. $\sqrt{(x')^2+(y')^2}=\sqrt{4\sin^2 2\theta+4\cos^2 2\theta}=\sqrt{4}=2$. So the integral will evaluate to 2π

17. **C** Differentiating both sides gives $(xy'+y)\cos xy=1+y'$ which can be simplified to $-\frac{1-y\cos(xy)}{1-x\cos(xy)}$

18. **E** This first order linear differential equation can be solved using the integrating factor of e^{-5x} giving the new equation $e^{-5x}y'-5e^{-5x}y=xe^{-5x}$ which can be rewritten as $\frac{d(e^{-5x}y)}{dx}=xe^{-5x}$ and integrated to get $y=-\frac{x}{5}-\frac{1}{25}+c$. Solving for c results in 0. Evaluating for $x=\frac{1}{5}$ yields $y=-\frac{2}{25}$.

19. **C** $e^x=1+x+\frac{x^2}{2!}+\ldots$ and $\sin x=x-\frac{x^3}{3!}+\frac{x^5}{5!}-\ldots$ so $e^{\sin x}=1+\left(x-\frac{x^3}{3!}+\frac{x^5}{5!}-\ldots\right)+\frac{1}{2!}\left(x-\frac{x^3}{3!}+\frac{x^5}{5!}-\ldots\right)^2+\ldots$ and the second degree polynomial would only have the terms $1+x+\frac{x^2}{2!}$ so $T(4)=1+4+\frac{4^2}{2}=13$

20. **D** First divided out the $\frac{1}{n}$ which will represent our dx. Then let $x=\frac{i-1}{n}$ and rewrite the summation as the integral $\int\frac{x+2}{x+1}dx$. Since $i=1,2,3,\ldots n$ the interval must be $[0,1]$.
$\int_0^1\frac{x+2}{x+1}dx=\int_0^1 1+\frac{1}{x+1}dx=x+\ln(x+1)\Big|_0^1=1+\ln 2$

21. **E** Since the two graphs intersect and swap at $\frac{\pi}{4}$ the integral must be split up.
$2\int_0^{\frac{\pi}{4}}(\sin y-\cos y)dy=2\sqrt{2}-2$

22. **B** First solve for the gravity constant with $v(30)=g(30)+12.6=0$ since the rock should have no velocity halfway through its journey. This gives $g=-0.42$ which gives the position equation to be $s(t)=-0.21t^2+12.6t$. Evaluated at t=30 gives 189 meters.

23. **A** Letting $x = r\cos\theta$ and $y = r\sin\theta$ the equation can be rewritten and reduced to $\frac{x^2}{25}+\frac{y^2}{16}=1$. This gives us $a=5$ and $c=3$ and an eccentricity of $\frac{3}{5}$.

24. **A** Using Newton's Heating/Cooling equation we can set up the first situation as $60-20=(100-20)e^{rt}$ which can be solved for r to be $r=-\frac{\ln 2}{t}$. The second situation can be expressed as $-2+4=(60+4)e^{15r}$. Plugging in *r* and solving for *t* gives 3 minutes.

25. **D** Using either tabular or by parts the antiderivative becomes $e^x(x^4-4x^3+12x^2-24x+24)$ which then evaluates to $8e^2$ at x=2 and 24 at x-0.

26. **B** The upper Riemann sum can be written as $\frac{1}{n}\left(1+\frac{1}{1+1/n}+\frac{1}{1+2/n}+\frac{1}{1+3/n}+\ldots\frac{1}{1+(n-1)/n}\right)$ while the lower Riemann sum can be written as $\frac{1}{n}\left(\frac{1}{1+1/n}+\frac{1}{1+2/n}+\frac{1}{1+3/n}+\ldots+\frac{1}{2}\right)$. The difference between these two is $\frac{1}{n}\left(1-\frac{1}{2}\right)=\frac{1}{2n}$

27. **A** Using the Shells method we can set up the integral as $2\pi\int_0^1 x(-x^2+1)\,dx$ which evaluates to $\frac{\pi}{2}$.

28. **D** The sum can be rewritten as $\sum_2^\infty \frac{ne^n}{n!}-\sum_2^\infty \frac{e^n}{n!}$ and $\sum_2^\infty \frac{e^n}{n!}=e^e-1-e$ using the Taylor series for e^x and subtracting the first two terms of the series. The same can be done with $\sum_2^\infty \frac{ne^n}{n!}$ which can be written as $\sum_2^\infty \frac{e^n}{(n-1)!}=\sum_1^\infty \frac{e^{n+1}}{n!}=e\sum_1^\infty \frac{e^n}{n!}=e\cdot(e^e-1)$. Subtracting the two sums and factoring out the common terms leaves $e^e(e-1)+1$

29. **E** $\frac{d}{dx}\left(\frac{x^2}{4}-2\right)^{10}=5x\left(\frac{x^2}{4}-2\right)^9$ then we solve for *k* since $x^7=x\cdot x^{2k}=x^{2k+1}$ so $k=3$. Then $5x\frac{9!}{3!6!}\left(\frac{x^2}{4}\right)^3(-2)^6=420x^7$

30. **D** $\frac{d}{dx}\left(\int_{\tan x}^{\pi}\sqrt{1+t^2}\,dt\right)=\frac{d(\pi)}{dx}\sqrt{1+\pi^2}-\frac{d(\tan x)}{dx}\sqrt{1+\tan^2 x}=0-\sec^2 x\sqrt{\sec^2 x}=-\sec^3 x$