Answers:

1. B
2. A
3. C
4. D
5. B
6. B
7. E
8. B
9. C
10. B
11. C
12. D
13. A
14. E
15. E
16. D
17. D
18. B
19. E
20. B
21. C
22. C
23. A
24. C
25. E
26. A
27. B
28. A
29. D
30. C

Solutions:

1. Let the inside of the quantity also be equal to y. We know have $y = \sqrt{x+y} \rightarrow y^2 - y = x$. Deriving both sides and we obtain $2y\frac{dy}{dx} - \frac{dy}{dx} = 1 \rightarrow \frac{dy}{dx} = \frac{1}{2y-1}$. **B**

2. $y' = -\frac{\frac{2}{x^2}}{1+\frac{4}{x^2}} = -\frac{2}{x^2+4}$. **A**

3. The triangle we are observing is with the diagonal of a face, the side, and the diagonal of the cube. These sides have lengths of $s\sqrt{2}, s,$ and $s\sqrt{3}$ respectively. We know that the diagonal of the cube is increasing at a rate of 3 therefore $\sqrt{3}\frac{ds}{dt} = 3 \rightarrow \frac{ds}{dt} = \sqrt{3}$. **D**

4. The derivative of the function defined is $f'(x) - 2f'(2x)$. Therefore the information sets up two equations that can be solved simultaneously. $f'(1) - 2f'(2) = 5$ and $f'(2) - 2f'(4) = 7$. Adding two times the second equation to the first equation we obtain what we desire which is equal to 19. **D**

5. Evaluating the integral and plugging in values of k it is evident that after the first two terms the series will continue to decrease (or increase but by less than it decreased in the previous partial sum). Therefore the maximum of this function will be attained only with the first two terms. The partial sum of the first two terms is 3. **B**

6. This is the definition of e. Therefore our desired limit is $e^{-\frac{3}{2}*2} = e^{-3}$. **B**

7. Recognize that this is the product rule. We can now use the product rule in reverse. $\frac{d}{dx}(xy) = x$. Integrating both sides yields $xy = \frac{1}{2}x^2 + C \rightarrow y = \frac{1}{2}x + \frac{C}{x}$. $C = \frac{1}{2}$. Therefore $y(2) = \frac{5}{4}$. **E**

8. $\frac{dy}{dx} = -\frac{x}{y} \rightarrow \frac{d^2y}{dx^2} = \frac{-x^2-y^2}{y^3} = -\frac{9}{y^3}$. **B**

9. Since we have indeterminate form we can apply L'Hopital's rule. Our limit becomes $\frac{e^{x\cos x}(\cos x - x\sin x)-1}{2x\cos x^2}$. Since this is indeterminate, we apply it again. $\lim_{x\to 0}\frac{e^{x\cos x}(\cos x - x\sin x)^2 + e^{x\cos x}(-2\sin x - x\cos x)}{2\cos x^2 - 4x^2\sin x^2} = \frac{1}{2}$. **C**

10. Taking the derivative of f'' function twice we obtain: $f^{(4)} = 4f'''(t) - 3f''(t)$. Now we can insert our equations for f''' and f'' into our new fourth derivative equation. Our result is $40f'(t) - 39f(t) + 13 \rightarrow 40f'(0) - 39f(0) + 13 = 54$. **B**

11. $f'(c) = \frac{f(b)-f(a)}{b-a} = 3a_n^2 = \frac{\left(\left(\frac{1}{2}\right)^{3n} - 0\right)}{\left(\frac{1}{2}\right)^n} \rightarrow a_n = \frac{\left(\frac{1}{2}\right)^n}{\sqrt{3}}$. The sum is geometric, so the sum is equal to $\frac{\sqrt{3}}{3}$. **C**

12. Make common denominators to avoid indeterminate form. $\lim_{x \to 2} \frac{(x+2-4)}{x^2-4} = \frac{1}{x+2} = \frac{1}{4}$. **D**

13. $f' = 3x^2 + a$. Since they are equal $3a^2 + a = 3b^2 + a \rightarrow a^2 = b^2 \rightarrow a = \pm b$. Since a cannot equal b, $a = -b$. $f(1) = 1 + a + b$, therefore $f(1) = 1$. **A**

14. Divide by the term of the highest exponent and take the limit $\lim_{x \to -\infty} \frac{4 - \frac{1}{x}}{\sqrt{1 + \frac{2}{x^2}}} = -4$. **E**

15. The product of the roots of this equation is $(a^2 - 1)/2$. We can see that the minimum is $a = 0$ and the product increases with increasing values of a. The restraint is where the roots are no longer real. Using the discriminate value we can find this max. $a^2 - 4(2)(a^2 - 1) = 8 - 7a^2$. $a = \frac{2\sqrt{2}}{\sqrt{7}}$. **E**

16. Perform L'Hopital's rule with respect to h on the expression that was given. Now we only have to evaluate $2f'(x)$. This is simply $\ln 16 = 4 \ln 2$. **D**

17. The model of decay is simply $y = y_0 e^{kt}$. In order to find the average value of substance over the first half life we will have to integrate our equation (average value formula). $\frac{1}{T}\int_0^T y_0 e^{kt} dt = \frac{y_o}{kt}(e^{kt} - 1)$. We know that half-life's occur when half the substance has been used so now we can solve for the time. $\frac{1}{2} y_0 = y_0 e^{kt} \rightarrow kt = -\ln 2$. Plugging this in and the initial amount we get $\frac{5}{\ln 2}$. **D**

18. Length of a curve in polar can be derived from our basic relation $dx^2 + dy^2 = ds^2$. Changing to polar gives the formula: $\int_{\theta_1}^{\theta_2} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}$. Plugging in we get $\int_0^{\sqrt{5}} \sqrt{\theta^4 + \theta^2} d\theta$, Using a u-sub of $u = \theta^2 + 4$ we simplify to $19/3$. **B**

19. Taking the natural logarithm of both sides we obtain $\ln y = \frac{\ln\frac{\sin x}{x}}{(1-\cos x)}$. Applying L-hopital's rule, gives us: $\lim_{x\to 0}\frac{x\cos x-\sin x}{x\sin^2 x}$. Another round of L'hopital's gives us $\lim_{x\to 0}\frac{-x\sin x}{x\sin 2x+\sin^2 x}$, which is still indeterminant. A third round of L'hopital's gives us $\lim_{x\to 0}\frac{-x\cos x-\sin x}{2x\cos 2x+2\sin 2x}$, which is still indeterminant. A fourth and final round of L'hopital's gives us $\lim_{x\to 0}\frac{-x\sin x-2\cos x}{6\cos 2x-4x\sin 2x}$, which equals -1/3. But we have to raise each side to an exponential so our answer is $e^{-1/3}$. **E**

20. Begin with Boyle's law of PV=k. Deriving both sides gives us $\frac{dP}{dt}V+\frac{dV}{dt}P=0$. Plugging our values into the equation and solving leaves us with $\frac{dP}{dt}=1/3$. **B**

21. First it is evident that this function is simply $\frac{1-x}{1-x^3}$. This now is simply two Maclaurin series. The 36th term of our Maclaurin series will help us find the 36th derivative since that term is in the form of $\frac{f^{(36)}(0)x^{36}}{36!}$. Since the first series is the only one that has multiples of 3 it is obvious that the coefficient is 1. Therefore equating the two forms of the 36th term allows us to solving for the 36th derivative and it is 36!. **C**

22. Simply take the first two derivatives and match answer choices. The first three derivatives are $' = \frac{1}{\sqrt{2x-1}}, y'' = -\frac{1}{(2x-1)^{-\frac{3}{2}}}, y''' = \frac{3}{(2x-1)^{\frac{5}{2}}}$. This matches choice **C.**

23. The distance between the curves is found by subtraction $2+\sin x-\cos x$. Deriving and setting equal to 0 we can solve for our values of x. $\cos x+\sin x=0 \to \tan x=-1 \to x = \frac{3\pi}{4}, \frac{7\pi}{4}$. plugging in, we can see that $\frac{7\pi}{4}$ gives us the smallest distance of $2-\sqrt{2}$. **A**

24. Upon observation it is evident that the function f must be of the form ax^2+bx+c. Therefore plugging this in for f and f' we obtain the equation $ax^2+bx+c-2ax-b=x^2+2x+1$. Equating each power of x together we solve for a=1,b=4, and c=5. $f(5)=50$. **C**

25. The Maclaurin series for $\ln x+x^2 = x-\frac{x^2}{2}+\frac{x^3}{3}+\cdots$. Plugging in for x^2, it now becomes $x^2-\frac{x^4}{2}+\frac{x^6}{3}+\cdots$. This does not match any of the choices. **E**

26. Let the leading term be of the form ax^n. This must be equal to the first term of the first and second derivative multiplied together. Therefore, $ax^n = n^2a^2(n-1)x^{2n-3}$. Setting the exponents equal gives us $n=3$ and solving for a given n we obtain $1/18$. **A**

27. Take the natural log of both sides and apply L'Hopitals rule. With both of these operations applied we obtain $\frac{1+\ln x}{-\cos(1-x)}$. Taking the limit as x goes to 0 we get -1. Exponenting both sides results in e^{-1}. **B**

28. Start with the equality $A = \pi r^2 \to \frac{dA}{dt} = 2\pi r \frac{dr}{dt} \to 2\frac{dr}{dt} = 2\pi r \frac{dr}{dt} \to r = \frac{1}{\pi}$. **A**

29. Plug in and multiply the whole function by a factor of 2. We obtain $f'\left(\frac{\pi}{6}\right) = 2/\tan\left(\frac{\pi}{3}\right) + 2 = \frac{2\sqrt{3}}{3} + 2$. **D**

30. Derive with respect to theta. $\frac{dx}{d\theta} = 2\cos\theta$ and $\frac{dy}{d\theta} = -2\sin 2\theta$. Divide the two to obtain $\frac{dy}{dx} = -2\sin\theta$. Derive again with respect to theta to obtain $\frac{d^2y}{dx^2} = -2\cos\theta * \frac{d\theta}{dx} = -1$. **C**