Mu Bowl 2015 Answers and Solutions

0. -2

1. $113\sqrt{15}$

   A. $(3)^3 = 27$
   $\lim_{x\to 3^-} \frac{2|x-3|}{x-3} = -2$ =-54
   B. $\sqrt{15}$
   C. 5
   D. -2

2. 3
   A. 128
   B. 16
   C. 96
   D. 144

3. 1
   A. $\frac{1}{2e}$
   B. $\ln 2 + 1$

4. 51/19
   A. 1
   B. $\frac{32p}{35}$
   C. $\frac{19p}{35}$

5. -3
   A. Diverges
   B. Converges Conditionally
   C. Diverges
   D. Diverges

6. 79/90
   A. 0.8
   B. 9
   C. 7
   D. 8

7. $2p$
   A. -1
   B. 0
   C. $16p$
   D. 8

8. 2

A. -4

B. $\frac{-8}{3}$

C. $\frac{-4}{3}$

D. 33

9. 0

A. $\frac{1}{2}$

B. 0

C. 1

D. 0

10. 10

A. -256

B. $\frac{517}{2}$

11. 3001.917

A. 1000.5

B. 1000.667

C. 1000.75

12. -33

13. $e^{12}+9$

A. 9

B. $13e^{12}$

C. $\frac{4}{7}$

D. $\frac{-5}{32}$

14. $\frac{1}{9}$

A. $\frac{1}{p}$

B. $\frac{1}{9p}$

Solutions

0	There are 4 points on the curve: (2,-5), (2,3), (14, -15) and (14,1). Differentiating implicity, $2yy'+xy'+y=0 \Rightarrow y'=\frac{-y}{2y+x}$. Evaluating this expression at each of the points yields slopes of $-\frac{5}{8}$, $-\frac{3}{8}$, $-\frac{15}{16}$, and $-\frac{1}{16}$, respectively. The sum of these slopes is -2.	
1	$(3)^3=27$ A = $\lim_{x\to 3^-}\frac{2\lvert x-3\rvert}{x-3}=-2$ =-54	Using Ln and L'Hopital's B = $\lim_{x\to 0}\frac{\ln\left(\frac{3^x+5^x}{2}\right)}{x} \Rightarrow \lim_{x\to 0}\frac{\left(\frac{(\ln 3)3^x+(\ln 5)5^x}{2}\right)}{\frac{3^x+5^x}{2}}$ $\lim_{x\to 0}\frac{\left(\frac{(\ln 3)3^x+(\ln 5)5^x}{5^x}\right)}{\frac{3^x+5^x}{5^x}}=\lim_{x\to 0}\frac{(\ln 3)\left(\frac{3}{5}\right)^x+(\ln 5)1^x}{\left(\frac{3}{5}\right)^x+1^x}=$ $\frac{\ln 3+\ln 5}{2}=\ln\sqrt{15}$
	Same as B except $\left(\frac{3}{5}\right)^x$ will go to 0. C = 5	L'Hopital's twice remembering that $\frac{d(x^x)}{dy}=x^x\ln x+x^x$ D = -2
2	$0=-16(4)^2+32(4)+A$ A= $0=-256+128+A$ $A=128$	B = $\frac{\int_0^1 -32t+32dt}{1-0}=16$
	C= $v(4)=-32(4)+32=96$	$-32t+32=0$ D= $t=1$ $f(1)=-16+32+128=144$
3	$Area=xy=xe^{-2x}$ $Area'=e^{-2x}-2xe^{-2x}=e^{-2x}(1-2x)=0$ A= $x=\frac{1}{2}$ $Area=\frac{1}{2e}$	$Perimeter=2x+2y=2x+2e^{-2x}$ $Perimeter'=2-4e^{-2x}=0$ B= $e^{-2x}=\frac{1}{2}$ $x=\ln\sqrt{2}$ $Perimeter=2\ln\sqrt{2}+2e^{-2\ln\sqrt{2}}=\ln 2+1$
4	A= $Area=2\int_0^1 x^{\frac{1}{3}}-x^3\,dx=1$	B= $Volume=2\pi\int_0^1\left(x^{\frac{1}{3}}\right)^2-\left(x^3\right)^2dx=\frac{32\pi}{35}$

	C= $Volume = \pi \int_0^1 (1-x^3)^2 - \left(1-x^{\frac{1}{3}}\right)^2 dx = \frac{19\pi}{35}$	
5	A=diverges $n! > x^n$	B=Converges conditionally The sine function alternates and $\frac{1}{n}$ is harmonic.
	C=Diverges $(\ln n)^2 < n$ So you can compare this to the harmonic. Can be proven using L'hopital	D=Diverges, terms alternate between e and -e
6	A = Difference is just the 1st and last values multiplied by the width, which are 6 and 2 with width 0.2, so the difference is 0.8.	B= $\frac{1}{2}\left[f\left(-\frac{1}{2}\right)+f(0)+f\left(\frac{1}{2}\right)+f(1)\right]=$ $\frac{1}{2}\left[\frac{15}{4}+4+\frac{17}{4}+6\right]=9$
	C= $\frac{1}{2}\left[f(-1)+f\left(-\frac{1}{2}\right)+f(0)+f\left(\frac{1}{2}\right)\right]=$ $\frac{1}{2}\left[2+\frac{15}{4}+4+\frac{17}{4}\right]=7$	D= $\frac{1}{2}\left[f(-1)+2f\left(-\frac{1}{2}\right)+2f(0)+2f\left(\frac{1}{2}\right)+f(1)\right]=$ $\frac{1}{2}\left(\frac{1}{2}\right)\left[2+\frac{15}{2}+8+\frac{17}{2}+6\right]=8$
7	A= $\frac{dy}{dx}=\frac{\frac{dy}{dt}}{\frac{dx}{dt}}=\frac{-8\sin(2t)}{8\cos(2t)}=-\tan(2t)$ $-\tan\left(2\frac{\pi}{8}\right)=-1$	B= $x(0)=0, y(0)=4$ $x(2\pi)=0, y(2\pi)=4$ So there is 0 displacement.
	$\int_0^{2\pi}\sqrt{(8\cos(2t))^2+(-8\sin(2t))^2}\,dt=$ $\int_0^{2\pi}\sqrt{64\left[\cos^2(2t)+\sin^2(2t)\right]}\,dt=$ C= $\int_0^{2\pi}\sqrt{64}\,dt=$ $\int_0^{2\pi}8\,dt=$ 16π	D= $speed=\sqrt{(8\cos(2\pi))^2+(-8\sin(2\pi))^2}=8$
8	$f(x)=x^3+4x^2+5x+20$ A= $f(x)=(x+4)(x^2+5)$ $x=-4$	$f'(x)=3x^2+8x+5$ $f'(x)=(3x+5)(x+1)=0$ B= $x=\frac{-5}{3},-1$ $sum=-\frac{8}{3}$

	C= $f''(x)=6x+8=0$ $x=-\frac{4}{3}$	D= $f(0)=0+4(0)+5(0)+20=20$ $f'(0)=3(0)+8(0)+5=5$ $f''(0)=6(0)+8=8$ $sum=33$
9	A= $f'(x)=\frac{-F_x}{F_y}=\frac{-(4x^3-10xy^2)}{-10x^2y+16y^3}$ $f'(2)=\frac{-(32-20)}{-40+16}=\frac{-12}{-24}=\frac{1}{2}$	B= $f'(2)=\frac{1}{2}$ $y-1=\frac{1}{2}(x-2)$ $y-1=\frac{1}{2}(0-2)$ $y=0$
	C= $f'(x)=\frac{-F_x}{F_y}=\frac{-(4x^3-10xy^2)}{-10x^2y+16y^3}$ $f'(-2)=\frac{-(-32+80)}{80-128}=\frac{-48}{-48}=1$	D= $f'(-2)=1$ $y+2=1(x+2)$ $y+2=1(0+2)$ $y=0$
10	A= $\int_{-4}^{4}(x^3+3x^2-16x-48)dx=$ $\frac{x^4}{4}+x^3-8x^2-48x\Big\vert_{-4}^{4}=$ $(16+64-128-192)-(16-64-128+192)=$ -256	B= $x^3+3x^2-16x-48=(x+4)(x+3)(x-4)$ $\int_{-4}^{4}\left\vert x^3+3x^2-16x-48\right\vert dx=$ $\int_{-4}^{-3}x^3+3x^2-16x-48dx-\int_{-3}^{4}x^3+3x^2-16x-48dx$ $\frac{x^4}{4}+x^3-8x^2-48x\Big\vert_{-4}^{-3}=\frac{5}{4}$ $\frac{x^4}{4}+x^3-8x^2-48x\Big\vert_{-3}^{4}=-\frac{1029}{4}$ $\frac{5}{4}-\left(-\frac{1029}{4}\right)=\frac{1034}{4}=\frac{517}{2}$
11	A= $y(1)=0(1-0)+1000=1000$ $y(2)=\frac{1}{2}(2-1)+1000=1000.5$	B= $y\left(\frac{2}{3}\right)=0\left(\frac{2}{3}\right)+1000=1000$ $y\left(\frac{4}{3}\right)=\frac{1}{3}\left(\frac{2}{3}\right)+1000=1000\frac{2}{9}=1000.222$ $y(2)=\frac{2}{3}\left(\frac{2}{3}\right)+1000\frac{2}{9}=1000\frac{6}{9}=1000.667$

	C= $y\left(\frac{1}{2}\right) = 0\left(\frac{1}{2}\right) + 1000 = 1000$ $y(1) = \frac{1}{4}\left(\frac{1}{2}\right) + 1000 = 1000.125$ $y\left(\frac{3}{2}\right) = \frac{1}{2}\left(\frac{1}{2}\right) + 1000.125 = 1000.375$ $y(2) = \frac{3}{4}\left(\frac{1}{2}\right) + 1000.375 = 1000.75$	
12	$\ln y = 2\ln(2x+7) + 3\ln(x+2) - 4\ln(1+2x)$ $\frac{y'}{y} = \frac{4}{2x+7} + \frac{3}{x+2} - \frac{8}{1+2x}$ $\frac{y'(1)}{y(1)} = \frac{4}{9} + 1 - \frac{8}{3} = \frac{-11}{9}$ $y(1) = \frac{9^2 3^3}{3^4} = 27$ $y' = \left(\frac{-11}{9}\right)27 = -33$	
13	A= $h'(x) = f'(g(g(x)))g'(g(x))g'(x)$ $h'(4) = (1)(3)(3) = 9$	$h(x) = e^{f(x) \cdot g(x)}$ B= $h'(x) = \left[f'(x)g(x) + f(x)g'(x)\right]e^{f(x) \cdot g(x)}$ $h'(4) = \left[(1)(4) + (3)(3)\right]e^{(3)(4)} = 13e^{12}$
	C= $h'(x) = \frac{f'(x) + g'(x)}{f(x) + g(x)}$ $h'(4) = \frac{1+3}{3+4} = \frac{4}{7}$	$h(x) = \frac{f(x)}{2g(x)}$ D= $h'(x) = \left[\frac{1}{2}\right]\frac{g(x)f'(x) - g'(x)f(x)}{(g(x))^2}$ $h'(4) = \left[\frac{1}{2}\right]\frac{(4)(1) - (3)(3)}{(4)^2} = -\frac{5}{32}$
14	$V = \frac{\pi r^2 h}{3} = \frac{4\pi}{3}$ $V = \frac{\pi(2h)^2 h}{3} = \frac{\pi 4h^3}{3}$ A= $h = \frac{4}{r^2}, r = 2h$ $V' = \frac{12\pi h^2 h'}{3} = 4$ $h = 1, r = 2$ $h = 1, h' = \frac{1}{\pi}$	$V = \frac{\pi(2h)^2 h}{3} = 36\pi$ $V = \frac{\pi(2h)^2 h}{3} = \frac{\pi 4h^3}{3}$ B= $\frac{108}{4} = h^3 = 27$ $V' = \frac{12\pi h^2 h'}{3} = 4$ $h = 3, r = 6$ $h = 3, h' = \frac{1}{9\pi}$