\[ A = \int_{-2016}^{2016} x^2 \sin x^5 \, dx \quad B = \int_{-\infty}^{\infty} e^{-x^2} \, dx \quad C = \sum_{n=1}^{\infty} \frac{1}{n^2} \]

Compute \( ABC \).
Let $R$ be the region bound by the graphs of $y = \sqrt{x}$ and $x = 2y$.

A = the volume when $R$ is revolved about the x-axis.

B = the volume when $R$ is revolved about the y-axis.

C = the volume when $R$ is revolved about $y = -1$.

D = the volume when $R$ is revolved about $x = 4$.

Compute $A + B + C + D$. 
Compute $A B C D \ln E$.

$$A = \lim_{x \to 1} \frac{3x^5 - 9x^3 + 13x^2 - 14x + 7}{5x^4 - 3x^3 - 13x^2 + 15x - 4}$$

$$B = \lim_{x \to 2} \frac{\sqrt{x + 2} - 2}{\sqrt{3x + 3} - 3}$$

$$C = \lim_{h \to 0} \frac{\sin\left(\frac{\pi}{3} + h\right) - \sin\left(\frac{\pi}{3}\right)}{h}$$

$$D = \lim_{x \to \infty} \left(\sqrt{x^2 + 5x} - \sqrt{x^2 - 3x}\right)$$

$$E = \lim_{x \to \infty} \left(1 - \frac{3}{x}\right)^{2x}$$

Compute $A B C D \ln E$.
Let \( f(x) = x^2 \). \( S(a, b) \) is defined to be the sum of the slope(s) of the line(s) tangent to \( f \) that pass through the point \((a, b)\). If no tangent line to \( f \) pass through \((a, b)\), then \( S(a, b) = 0 \). For example:

\[
S(1, 1) = 2, \text{ since } y = 2x - 1 \text{ is the only line tangent to } f \text{ that passes through } (1, 1);
\]

\[
S(0, 1) = 0, \text{ since no tangent line to } f \text{ pass through } (0, 1);
\]

\[
S(0, -1) = 0, \text{ since both } y = 2x - 1 \text{ and } y = -2x - 1 \text{ are tangent to } f \text{ and pass through } (0, -1).
\]

Compute

\[
\sum_{b=0}^{100} \sum_{a=0}^{10} S(a, b)
\]
A = the area enclosed by the rectangular equation $4x^2 + 9y^2 - 16x - 54y + 61 = 0$.

B = the area enclosed by the polar equation $r = 3 \sin 3\theta$.

C = the area enclosed by the parametric equation $x = \sin t, y = 3 \sin 2t$.

Compute $\frac{A}{B} + C$. 
A. Let \( f_1(x) = \sin x + \cos x \), approximate \( f_1(0.2) \) using the line tangent to \( f_1(x) \) at \( x = 0 \).

B. Let \( f_2(x) = \sqrt{25 - x^2} \), approximate \( f_2(3.1) \) using the line tangent to \( f_2(x) \) at \( x = 3 \).

C. Let \( f_3(x) = \frac{4}{x^2 + 3} \), approximate \( f_3(-1.1) \) using the line tangent to \( f_3(x) \) at \( x = -1 \).

D. Let \( f_4(x) = x^3 - 6x^2 + 12x \), approximate \( f_4(0.9) \) using the line tangent to \( f_4(x) \) at \( x = 1 \).

Define \( E(X) \) as a function of part \( X \) of this problem. The value of \( E(X) \) is the value of the approximation if part \( X \) results in an overestimate, or the negation of the approximation if part \( X \) results in an underestimate. Compute \( E(A) + E(B) + E(C) + E(D) \).
Compute $A + B + C$.
A = \int_0^6 (36x - x^3) \, dx

B = approximation of A using midpoint Riemann sum with 3 equal intervals.

C = approximation of A using trapezoidal rule with 3 equal intervals.


Compute \( A + B - C - D \).
A 200mL container is lined with a membrane that allows water to seep through at a rate proportional to the concentration of water in the container. The container is initially full of a 20% solution of alcohol in water. As water seeps out of the container, it is instantly replaced with a 20% solution of alcohol in water and remixed evenly. If 400mL of water seeps through in 2 hours, how many hours does it take in total for the concentration of alcohol inside the container to reach at least 90%?
Let $f(x) = \frac{1}{2}x^2 + bx + c$, where $b = 3, c = -8$.

A = the greater of two zeros of $f(x)$.

B = the rate of change of the greater of two zeros of $f(x)$ if $b$ is changing at 2 per second and $c$ is constant at the moment when $b = 3, c = -8$.

C = the rate of change of the greater of two zeros of $f(x)$ if $b$ is constant and $c$ is changing at 2 per second at the moment when $b = 3, c = -8$.

D = the rate of change of the greater of two zeros of $f(x)$ if $b$ and $c$ are changing at 2 per second at the moment when $b = 3, c = -8$.

Compute $A + B + C + D$. 
\( f \) is a twice differentiable function over all real numbers. The table below shows the value of \( f \) and \( f' \) at select values on the interval \([0, 10]\).

<table>
<thead>
<tr>
<th>( x )</th>
<th>0</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) )</td>
<td>-3</td>
<td>2</td>
<td>4</td>
<td>-4</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>( f'(x) )</td>
<td>1</td>
<td>-1</td>
<td>-2</td>
<td>2</td>
<td>-1</td>
<td>4</td>
</tr>
</tbody>
</table>

A = the minimum number of zeros of \( f \) on \((0, 10)\).
B = the minimum number of local maxima of \( f \) on \((0, 10)\).
C = the minimum number of local minima of \( f \) on \((0, 10)\).
D = the minimum number of points of inflection of \( f \) on \((0, 10)\).
Compute \( A^2 + B^2 + C^2 + D^2 \).
$f$ and $g$ are invertible functions that are locally differentiable at 1, 2, 3, and 4. The table below shows the evaluation of those functions and their derivatives. Let

\[
\begin{align*}
h_1(x) &= f(2x) + g(3x) \\
h_2(x) &= f\left(f(f(x))\right) \\
h_3(x) &= xf(x^2) \\
h_4(x) &= f(x)g^{-1}(x) \\
h_5(x) &= f(x)(g(x))^{-1} \\
h_6(x) &= f(f^{-1}(x))
\end{align*}
\]

<table>
<thead>
<tr>
<th>$x$</th>
<th>$f$</th>
<th>$f'$</th>
<th>$g$</th>
<th>$g'$</th>
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<tr>
<td>0</td>
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<td>4</td>
<td>-3</td>
</tr>
<tr>
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<td>4</td>
<td>2</td>
<td>3</td>
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<tr>
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<td>3</td>
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Compute $h'_1(1) + h'_2(3) + h'_3(2) + h'_4(4) + h'_5(3) + h'_6(4)$.

---

$f$ and $g$ are invertible functions that are locally differentiable at 1, 2, 3, and 4. The table below shows the evaluation of those functions and their derivatives. Let

\[
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\]

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<td>4</td>
<td>3</td>
<td>3</td>
<td>2</td>
<td>-6</td>
</tr>
</tbody>
</table>

Compute $h'_1(1) + h'_2(3) + h'_3(2) + h'_4(4) + h'_5(3) + h'_6(4)$. 
\[
A = \int_{-2}^{1} x\sqrt{x + 3} \, dx \\
B = \int_{0}^{3} x\sqrt{9 - x^2} \, dx \\
C = \int_{0}^{2} x e^{x/2} \, dx \\
D = \int_{0}^{\pi} \sin 2x \, e^{\sin x} \, dx \\
\text{Compute } 5A - B + C - 2D.
\]
Let $f(x) = x^2 e^x$. Let $f^{(k)}(x)$ denote the $k^{th}$ derivative of $f(x)$, then the sum
\[ \sum_{k=1}^{20} f^{(k)}(x) \]
can be expressed as $Ax^2 e^x + Bxe^x + Ce^x$, where $A, B, C$ are real. Compute $A + B + C$. 
Let \( \ell_1 \) be the line described by \( x = \frac{y-3}{2} = \frac{z-3}{2} \), and \( \ell_2 \) be the line \( \frac{x}{2} = \frac{y+1}{3} = \frac{z+4}{6} \).

Particle A moves along \( \ell_1 \) at a constant speed of 6 units per second. Particle B moves along \( \ell_2 \) at a constant speed of 7 units per second. At \( t = 0 \), particle A is at point \((0, 3, 3)\), and particle B is at the point \((0, -1, -4)\). Both particles are moving in the direction such that their \( x \)-coordinates are increasing. At time \( t = T \) seconds, the two particles make their closest approach to each other, where they are \( D \) units apart. Compute \( T^2 + D^2 \).