0. 0
1. $\frac{56\pi}{3}$
2. -12
3. 12210
4. 32/3
5. 10.875 or 87/8
6. $e^2 + 4e$
7. 54
8. 6
9. -2/5
10. 79
11. 30
12. -17
13. 3100
14. 49

0. **Answer: 0**
$A = 0$ since $x^2 \sin x^5$ is an odd function.
Parts B and C are now irrelevant, since the problem asks for ABC.
$B = \sqrt{\pi}$ (Base function for normal distribution)
$C = \frac{\pi^2}{6}$ (Well known series)

1. **Answer:** $\mathbf{\frac{56\pi}{3}}$
The region lies in the first quadrant with endpoints at $(0,0)$ and $(4,2)$.
$A = 2\pi \int_0^2 y(2y - y^2)\,dy = 2\pi \int_0^2 2y^2 - y^3\,dy = 2\pi\left(\frac{2}{3}y^3 - \frac{1}{4}y^4\right)\Big|_0^2 = \frac{8\pi}{3}$
$B = \pi \int_0^2 (2y)^2 - (y^2)^2\,dy = \pi \int_0^2 4y^2 - y^4\,dy = \pi\left(\frac{4}{3}y^3 - \frac{1}{5}y^5\right)\Big|_0^2 = \frac{64\pi}{15}$
$C = 2\pi \int_0^2 (y+1)(2y - y^2)\,dy = 2\pi \int_0^2 -y^3 + y^2 + 2y\,dy$
So $C = 2\pi\left(-\frac{1}{4}y^4 + \frac{1}{3}y^3 + y^2\right)\Big|_0^2 = \frac{16\pi}{3}$
$D = \pi \int_0^2 (4 - y^2)^2 - (4 - 2y)^2\,dy = \pi \int_0^2 y^4 - 12y^2 + 16y\,dy$
So $D = \pi\left(\frac{1}{5}y^5 - 4y^3 + 8y^2\right)\Big|_0^2 = \frac{32\pi}{5}$
Finally, $A + B + C + D = \frac{56\pi}{3}$

2. **Answer:** $\mathbf{-12}$
For A, use L'Hospital's rule twice.
$A = \lim\limits_{x\to 1} \frac{15x^4 - 27x^2 + 26x - 14}{20x^3 - 9x^2 - 26x + 15} = \lim\limits_{x\to 1} \frac{60x^3 - 54x + 26}{60x^2 - 18x - 26} = \frac{32}{16} = 2$
Alternatively, synthetically divide out 1 twice for numerator and denominator to arrive at the same answer.
$B = \lim\limits_{x\to 2} \frac{\sqrt{x+2} - 2}{\sqrt{3x+3} - 3} \cdot \frac{\sqrt{x+2} + 2}{\sqrt{x+2} + 2} \cdot \frac{\sqrt{3x+3} + 3}{\sqrt{3x+3} + 3}$, then simplify to get
$B = \lim\limits_{x\to 2} \frac{(x-2)}{3(x-2)} \cdot \frac{\sqrt{3x+3} + 3}{\sqrt{x+2} + 2} = \frac{6}{12} = \frac{1}{2}$
Part C is definition of derivative, so $C = \cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$.
$D = \lim\limits_{x\to\infty} \sqrt{\left(x + \frac{5}{2}\right)^2} - \sqrt{\left(x - \frac{3}{2}\right)^2} = 4$
$E = \lim\limits_{x\to\infty} \left(1 + \frac{1}{\frac{x}{-3}}\right)^{\frac{x}{-3}\cdot -6} = e^{-6}$
So $ABCD \ln E = -12$

3. **Answer: 12210**

Clearly, if (a, b) is on the graph, there is a single line of tangency, with slope of $2a$. If (a, b) is above the graph, there is no line of tangency. Finally, if (a, b) is below the graph, there are two lines of tangency.

Let's examine the last case more carefully by looking at tangent lines to $f(x)$ drawn through (a, b). We'll call a point of tangency (x, x^2). There are two, but they do not need to be handled differently. The slope of the line can be computed in two ways – through derivative at x, and through rise over run over those 2 points.

Thus, $2x = \frac{x^2-b}{x-a}$, or $x^2 - 2ax + b = 0$.

The sum of the x-coordinates for the points of tangency is $2a$, and since $f'(x) = 2x$, $S(a, b) = 4a$ for the case $b < a^2$.

Now we can examine the double sum:

$$\sum_{b=0}^{100}\sum_{a=0}^{10} S(a,b) = \sum_{a=0}^{10}\sum_{b=0}^{100} S(a,b) = \sum_{a=0}^{10}\left(\sum_{b>a^2} 0 + \sum_{b=a^2} 2a + \sum_{b<a^2} 4a\right)$$

This is equivalent to

$$\sum_{a=0}^{10} 2a + \sum_{a=0}^{10}\sum_{b=0}^{a^2-1} 4a$$

Since $4a$ is independent of b, the inner sum on the right is simply $a^2(4a) = 4a^3$. So the final sum can be computed as $\sum_{a=0}^{10} 2a + \sum_{a=0}^{10} 4a^3 = 2(55) + 4(55)^2 = 12210$

4. **Answer: 32/3**

Part A: The equation can be rewritten as

$$4(x^2 - 4x + 4) + 9(y^2 - 6y + 9) = -61 + 16 + 81$$

which is $\frac{(x-2)^2}{9} + \frac{(y-3)^2}{4} = 1$. The area of an ellipse is πab, so $A = 6\pi$.

Part B: The graph is a 3-petaled rose. The first petal goes from $\theta = 0$ to $\theta = \frac{\pi}{3}$. So

$B = \frac{3}{2}\int_0^{\frac{\pi}{3}}(3\sin 3\theta)^2\,d\theta = \frac{27}{2}\int_0^{\frac{\pi}{3}}\frac{1-2\cos 6\theta}{2}\,d\theta = \frac{27}{4}\left(\int_0^{\frac{\pi}{3}} d\theta + 2\int_0^{\frac{\pi}{3}}\cos 6\theta\,d\theta\right) = \frac{9\pi}{4}$

Part C: Applying double angle, $y = 6\sin t\cos t$. $\sin^2 t + \cos^2 t = 1$, so $\cos t = \pm\sqrt{1-\sin^2 t}$. Thus, $y = \pm 6x\sqrt{1-x^2}$. As t ranges from 0 to 2π, the graph traces out the a figure resembling a bowtie that is symmetric to both x and y axes. So to find the area enclosed in the figure, we can simply compute the area in the first quadrant, and multiply it by 4.

The area in Q1 is $\int_0^1 6x\sqrt{1-x^2}\,dx = -2(1-x^2)^{\frac{3}{2}}\Big|_0^1 = 2$

Therefore, $C = 8$, and $\frac{A}{B} + C = \frac{32}{3}$.

5. **Answer: 10.875 or 87/8**
$f_1'(x) = \cos x - \sin x\,, f_1''(x) = -\sin x - \cos x$.
So $f_1(0) = 1, f_1'(0) = 1, f_1''(0) = -1$.
$f(0.2) \approx 1.2$, and it is an overestimate since f_1 is concave down on $(0, 0.2)$.
$f_2'(x) = \frac{-x}{\sqrt{25-x^2}}, f_2''(x) < 0$ since it is upper half of a semicircle.
So $f_2(3) = 4, f_2'(3) = -\frac{3}{4}, f_2''(0) < 0$.
$f(3.1) \approx 3.925$, and it is an overestimate since f_2 is concave down on $(3, 3.1)$.
$f_3'(x) = \frac{-8x}{(x^2+3)^2}, f_3''(x) = \frac{24\left(x^2-1\right)}{(x^2+3)^3}$.
So $f_3(-1) = 1, f_3'(-1) = \frac{1}{2}, f_3''(-1) = 0$, but $f_3''(x) > 0$ for $x < -1$.
$f(-1.1) \approx 0.95$, and it is an underestimate since f_3 is concave up on $(-1.1, -1)$.
$f_4'(x) = 3x^2 - 12x + 12, f_4''(x) = 6x - 12$.
So $f_4(1) = 7, f_4'(1) = 3, f_4''(1) = -6$.
$f(0.9) \approx 6.7$, and it is an overestimate since f_4 is concave down on $(0.9, 1)$.
So $E(A) + E(B) + E(C) + E(D) = 1.2 + 3.925 - 0.95 + 6.7 = 10.875$.

6. **Answer: $\boldsymbol{e^2 + 4e}$**
Since $\frac{2n}{n!}$ is 0 when $n = 0$,
$$A = \sum_{n=1}^{\infty} \frac{2n}{n!} = \sum_{n=1}^{\infty} \frac{2}{(n-1)!} = \sum_{n=0}^{\infty} \frac{2}{n!} = 2e$$
$$\text{Since } \sum_{n=0}^{\infty} \frac{x^n}{n!} = e^x, B = e^2$$
Since $\frac{n^2}{n!}$ Is 0 when $n = 0$,
$$C = \sum_{n=1}^{\infty} \frac{n^2}{n!} = \sum_{n=1}^{\infty} \frac{n}{(n-1)!} = \sum_{n=0}^{\infty} \frac{(n+1)}{n!} = \sum_{n=0}^{\infty} \frac{n}{n!} + \sum_{n=0}^{\infty} \frac{1}{n!} = 2e$$
So $A + B + C = e^2 + 4e$.

7. **Answer: 54**
Note that Simpson's Rule evaluates integrals of cubic functions exactly. So $A - D = 0$.
For brevity, we will refer to $36x - x^3$ as $f(x)$.
$$B = 2\big(f(1) + f(3) + f(5)\big) = 2(35 + 81 + 55) = 342$$
$$C = f(0) + 2f(2) + 2f(4) + f(6) = 0 + 2(64) + 2(80) + 0 = 288$$
So $A + B - C - D = 54$

8. **Answer: 6**

Let W be the amount of water that has seeped out at a given time. The initial amount of water in the container is 160mL. The W mL of water that has seeped out is replaced with W mL of 20% solution of alcohol in water, which contains $0.8W$ mL of water. So the amount of water in the container is $160 - W + 0.8W = 160 - 0.2W$. When the concentration of alcohol inside the container is at 90%, there is 20mL of water left in the container. In other words, $160 - 0.2W = 20$, or $W = 700$.

Since the volume of solution in the container is constant, then the concentration of water in the container is proportional to the amount of water in the container. Thus the rate water seeps out is proportional to the amount of water in the container. In other words, $\frac{dW}{dt} = k(160 - 0.2W)$. Separate the variables, $\frac{1}{160-0.2W}dW = kdt$, or $\frac{5}{800-W}dW = kdt$.

Integrating both sides to get $-5\ln(800 - W) = kt + C$. Since k and C are both arbitrary constants, $\ln(800 - W) = kt + C$, or $800 - W = Ce^{kt}$.

When $t = 0$, $W = 0$. So $C = 800$. When $t = 2$, $W = 400$, so $k = -\frac{1}{2}\ln 2$.

Therefore, $W(t) = 800 - 800(0.5)^{0.5t}$.

Substitute 700 in for W, $100 = 800(0.5)^{0.5t}$, or $t = 6$.

9. **Answer: -2/5**

$f(x) = \frac{1}{2}(x + 8)(x - 2)$, so $A = 2$.

The larger zero of $f(x)$ in terms of b and c is $x = -b + \sqrt{b^2 - 2c}$

$$\frac{dx}{dt} = -\frac{db}{dt} + \frac{1}{2}(b^2 - 2c)^{-\frac{1}{2}}\left(2b\frac{db}{dt} - 2\frac{dc}{dt}\right)$$

At $b = 3, c = -8$,

$$\frac{dx}{dt} = -\frac{db}{dt} + \frac{1}{10}\left(6\frac{db}{dt} - 2\frac{dc}{dt}\right) = -\frac{2}{5}\frac{db}{dt} - \frac{1}{5}\frac{dc}{dt}$$

Substitute in the information, $B = -\frac{4}{5}, C = -\frac{2}{5}, D = -\frac{6}{5}$.

Thus $A + B + C + D = -\frac{2}{5}$

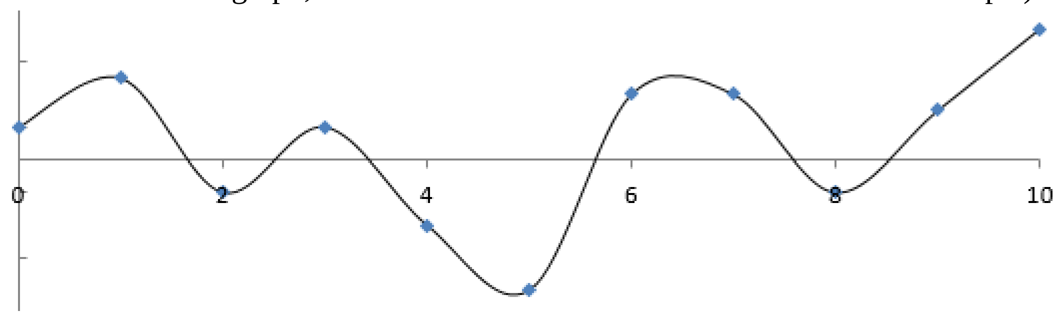
10. **Answer: 79**

Part A: There are clearly zeros on $(0, 2)$ and $(4, 6)$ by IVT, and there is a zero at 8. Since $f'(8) < 0$, there is c slightly less than 8 where $f(c) > 0$. Then by IVT, there is a zero on $(6, c)$, loosening up on the boundaries, that zero is on $(6, 8)$. Similarly, there is another zero on $(8, 10)$. Therefore, $A = 5$.

We will look at parts B, C, D together by inspecting the graph of $f'(x)$. In addition to the derivatives given in the function, additional values are guaranteed within each interval by MVT that are the average rates of change. That results in

x	0	c_1	2	c_2	4	c_3	6	c_4	8	c_5	10
$f'(x)$	1	2.5	-1	1	-2	-4	2	2	-1	1.5	4

Sketching these points on a graph to produce the graph below. (c_k is arbitrarily defined as $2k - 1$ on this graph, but the exact location does not affect the overall shape.)



This graph yields the lowest amount of zeroes (corresponding to local extrema of f) and local extrema (corresponding to points of inflections of f). Therefore,

$B = 3$, which is the number of zeroes transitioning from positive to negative.

$C = 3$, which is the number of zeroes transitioning from negative to positive.

$D = 6$, which is the number of local extrema.

So $A^2 + B^2 + C^2 + D^2 = 79$.

11. **Answer: 30**

$h_1'(x) = 2f'(2x) + 3g'(3x)$, so $h_1'(1) = 2f'(2) + 3g'(3) = 13$

$h_2'(x) = f'\left(f\big(f(x)\big)\right) f'\big(f(x)\big) f'(x)$,

so $h_2'(3) = f'\big(f(2)\big)f'(2)f'(3) = f'(4)f'(2)f'(3) = -6$

$h_3'(x) = f(x^2) + xf'(x^2) \cdot 2x$, so $h_3'(x) = f(4) + 8f'(4) = 27$

$h_4'(x) = f(x)a'(x) + f'(x)a(x)$, where $a(x) = g^{-1}(x)$. $a(4) = 1$ since $g(1) = 4$, and $a'(4) = \frac{1}{g'(1)} = -\frac{1}{3}$, so $h_4'(4) = 3\left(-\frac{1}{3}\right) + 3(1) = 2$

$h_5'(x) = \frac{g(x)f'(x) - f(x)g'(x)}{\big(g(x)\big)^2}$, so $h_5'(3) = \frac{g(3)f'(3) - f(3)g'(3)}{\big(g(3)\big)^2} = -7$

$h_6(x) = x$, so $h_6'(x) = 1$ and $h_6'(4) = 1$.

Finally, $h_1'(1) + h_2'(3) + h_3'(2) + h_4'(4) + h_5'(3) + h_6'(4) = 30$

12. **Answer: -17**

Part A. Make the substitution $u = x + 3$, then

$$A = \int_1^4 (u-3)\sqrt{u}\,du = \int_1^4 \left(u^{\frac{3}{2}} - 3u^{\frac{1}{2}}\right) du = \left(\frac{2}{5}u^{\frac{5}{2}} - 2u^{\frac{3}{2}}\right)\Big|_1^4 = -\frac{8}{5}$$

Part B. Make the substitution $u = 9 - x^2$, then

$$B = -\frac{1}{2}\int_9^0 u^{\frac{1}{2}}\,du = \frac{1}{3}u^{\frac{3}{2}}\Big|_0^9 = 9$$

Part C. Make the substitution $u = \frac{x}{2}$, then

$$C = 4\int_0^1 ue^u\,du = 4$$

Part D. Make the substitution $u = \sin x$, and note that $\sin 2x = 2\sin x\cos x$, then

$$D = 2\int_0^1 ue^u\,du = 2$$

Note that C is also twice the value of D after u substitution, so

$$5A - B + C - 2D = -17$$

13. **Answer: 3100**

Take a few derivatives of f to seek a pattern in the coefficients of the derivative:

$$f^{(1)}(x) = x^2e^x + 2xe^x + 0e^x$$
$$f^{(2)}(x) = x^2e^x + 4xe^x + 2e^x$$
$$f^{(3)}(x) = x^2e^x + 6xe^x + 6e^x$$
$$f^{(4)}(x) = x^2e^x + 8xe^x + 12e^x$$

In general,

$$f^{(k)}(x) = x^2e^x + 2kxe^x + k(k-1)e^x$$

So

$$A + B + C = \sum_{k=1}^{20}\big(1 + 2k + k(k-1)\big) = \sum_{k=1}^{20}(1 + k + k^2)$$
$$A + B + C = 20 + \frac{20(21)}{2} + \frac{20(21)(41)}{6} = 3100$$

14. **Answer: 49**

A moves along the vector $< 1, 2, 2 >$, which has a magnitude of 3, so the velocity of A is $< 2, 4, 4 >$. The motion of A can be described parametrically as $< 2t, 4t + 3, 4t + 3 >$. B moves along the vector $< 2, 3, 6 >$, which has a magnitude of 7, so the velocity of B is $< 2, 3, 6 >$. The motion of B can be described parametrically as $< 2t, 3t - 1, 6t - 4 >$. Therefore, the distance between A and B is $< 0, t + 4, -2t + 7 >$, which has magnitude $\sqrt{(t+4)^2 + (-2t+7)^2} = \sqrt{5t^2 - 20t + 65}$. This quantity is minimized at $t = 2$, with minimum value of $\sqrt{45}$.

So $T^2 + D^2 = 2^2 + 45 = 49$.