1. **B.**
2. **C.**
3. **A.**
4. **A.**
5. **D.**

6. **B.**
7. **B.**
8. **C.**
9. **D.**
10. **D.**

11. **A.**
12. **E.**
13. **C.**
14. **C.**
15. **C.**

16. **C.**
17. **B.**
18. **D.**
19. **B.**
20. **A.**

21. **D.**
22. **D.**
23. **B.**
24. **E.**
25. **A.**

26. **A.**
27. **C.**
28. **A.**
29. **C.**
30. **B.**

1. **B.** Euclid

2. **C.** *De Divine Proportione*

3. **A.** Phidias

4. **A.** Kepler

5. **D.** Binet

6. **B.**
Answer choice A is incorrect because ϕ does not terminate. Answer choice B is the correct answer. Answer choice C is incorrect because it is the limit of the ratio rather than the ratio of two arbitrary successive Fibonacci numbers. Answer choice D is just flat out wrong.

7. **B.**
ϕ can be the root of a polynomial with rational coefficients if $\frac{1+\sqrt{5}}{2}$ is produced as a root (ex. $x^2 - x - 1 = 0$), so the golden ratio is not transcendental. It is, however, irrational, complex, and real.

8. **C.**
The easiest way is to do the calculation the long way. Binet's formula would also work:
$$F_n = \frac{\left(1+\sqrt{5}\right)^n - \left(1-\sqrt{5}\right)^n}{2^n\sqrt{5}}.$$

9. **D.** $A = \pi\phi^2 = \pi(1+\phi) = \pi\left(2+\frac{1}{\phi}\right) = \pi\left(3-\frac{1}{\phi^2}\right)$

10. **D.**
Golden rectangles have side lengths in proportion of ϕ. Thus, the longer side has length
$32\phi = 32\left(\frac{1+\sqrt{5}}{2}\right) \approx 32\left(\frac{1+9/4}{2}\right) = 52$ meters.

11. **A.**
Looking at where the pattern repeats, we arrive at $y = \frac{1}{1+y}$, which has roots at $y = \frac{-1\pm\sqrt{5}}{2}$.
Taking the positive solution, $\frac{\sqrt{5}-1}{2} = \phi - 1 = \frac{1}{\phi}$.

12. **E.** All vertices lie on the plane $x + y = z$. Therefore, the tetrahedron encloses no volume.

13. **C.** By plugging in answer choices and applying $f^2 = f + 1$:

$$f^{\left(f^2 - \frac{1}{f}\right)} - \frac{1}{f} = 2 \rightarrow f^{(f+1-(f-1))} = 2 + \frac{1}{f} \rightarrow f^2 = 2 + \frac{1}{f} \rightarrow f + 1 = 2 + \frac{1}{f} \rightarrow f^2 = f + 1$$

\ $x = 2$

14. **C.**

First assume that the smallest squares have side length 1, representing the first 2 Fibonacci numbers. The largest square is the 9th Fibonacci number, which is 34. Thus, in this example, the side length we are looking for is $(1234)(34) = 41,956$.

15. **C.**

$$SA = (4)\left(\frac{1}{2}\right)(2)\left(\sqrt{1+f}\right) + 4 = 4f + 4$$

$$V = \frac{1}{3}Bh = \left(\frac{1}{3}\right)(4)\left(\sqrt{f}\right) = \frac{4}{3}\sqrt{f}$$

$$\frac{SA}{V} = (4f + 4)\left(\frac{3}{4\sqrt{f}}\right) = 3\left(f^{1/2} + f^{-1/2}\right)$$

16. **C.**

i. $.5 + .5 \times 5^{.5} = \frac{1+\sqrt{5}}{2} = \phi$

ii. $\frac{5+5^{.5}}{5-.5} = 1.608 \neq f$

iii. $a = \left(\frac{5+5^{.5}}{5-5^{.5}}\right)^{.5} \rightarrow a^2 = \frac{5+\sqrt{5}}{5-\sqrt{5}} = \frac{3+\sqrt{5}}{2} = \phi^2 \rightarrow a = \phi$

17. **B.**

Complete the chart:

F_{-8}	F_{-7}	F_{-6}	F_{-5}	F_{-4}	F_{-3}	F_{-2}	F_{-1}	F_0	F_1	F_2	F_3	F_4	F_5	F_6	F_7	F_8
-21	13	-8	5	-3	2	-1	1	0	1	1	2	3	5	8	13	21

18. **D.**

Observe a pattern:

$f(1) = 1 \rightarrow 1$

$f(2) = 1+1 = 2 \rightarrow 2$

$f(3) = 1+1+1 = 1+2 = 2+1 \rightarrow 3$

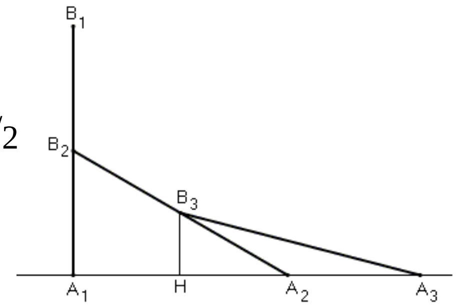
$f(4) = 1+1+1+1 = 2+2 = 1+1+2 = 1+2+1 = 2+1+1 \rightarrow 5$

$\vdots$

$f(n) = f(n-1) + f(n-2)$

This is classic Fibonacci recursion. Therefore, in this problem, you are looking to find the 15th Fibonacci number, which is 610.

19. **B.**



Drop a perpendicular B_3H from B_3 to A_1A_2. Because B_3H is a midline in $A_1A_2B_2$, its length is $1/2$. By Pythagorean theorem, $A_3H = \sqrt{15}/2$ and $A_1H = A_1A_2/2 = \sqrt{3}/2$. Thus, $A_1A_3 = \frac{\sqrt{3}+\sqrt{15}}{2} = \frac{\sqrt{3}\left(1+\sqrt{5}\right)}{2} = f\sqrt{3}$.

20. **A.**

As the index of the Fibonacci sequence approaches infinity, the ratio of each successive Fibonacci number approaches f. Thus, $\lim_{n\to\infty}\frac{F_{n+a}}{F_n} = f^a$. By definition (you could use an easy inductive pattern as well according to $f^a = f^{a-1} + f^{a-2}$), $f^a = F_a f + F_{a-1}$.

21. **D.** $f(x) = \frac{x}{1-x-x^2} \to f\left(\frac{1}{10}\right) = \frac{10}{89} \to F_n = 89 \to n = 11$

22. **D.**

i. $\sqrt{\frac{7+3\sqrt{5}}{2}} - 1 = \phi \to \sqrt{\frac{7+3\sqrt{5}}{2}} = \phi^2 \to \sqrt[4]{\frac{7+3\sqrt{5}}{2}} = \phi$ TRUE

ii. 21 and 47 are the 8th Fibonacci and Lucas numbers, not the 7th ➔ FALSE

iii. $\sqrt[13]{\frac{521+233\sqrt{5}}{2}} = \sqrt[5]{\frac{11+5\sqrt{5}}{2}} = \phi$ TRUE

23. **B.**

For some r, $a > 0$, the three sides could be written as a, ar, and ar^2, where a is irrelevant for this problem. Thus, for the triangle to exist, r must satisfy the following three inequalities:

$$r^2 < 1 + r$$
$$r < 1 + r^2$$
$$1 < r + r^2$$

The first one is equivalent to $r^2 - 1 - r < 0$, which has roots at f and $-\frac{1}{f}$. We find that r should satisfy $0 < r < f$. The second inequality is always true.

The third inequality is equivalent to $r^2 + r - 1 > 0$, which is satisfied when $r > 0$ by $r > \frac{1}{f}$. Thus, the range for r that satisfies the three conditions is $\frac{1}{f} < r < f$. The length of this range is $f - \frac{1}{f} = 1$.

24. **E.**

Since BO is an angle bisector, O divides AC in the ratio of the sides $\frac{AB}{BC}$:

$$\frac{AO}{CO} = \frac{AB}{BC} = \frac{5}{3}$$

From here, $AO = \frac{5}{2}$ and $CO = \frac{3}{2}$. Thus the circle's radius r is $\frac{3}{2}$. By the power of the point theorem $BP \cdot NQ = BC^2$:

$$\left(BO - \frac{3}{2}\right)\left(BO + \frac{3}{2}\right) = 3^2 \rightarrow BO = \frac{3\sqrt{5}}{2}$$

From which we find $BP = \frac{3}{2}\left(\sqrt{5} - 1\right)$

Finally, $\frac{PQ}{BP} = \frac{2r}{\left(\frac{3}{2}\left(\sqrt{5} - 1\right)\right)} = \frac{2}{\sqrt{5} - 1} = \frac{\sqrt{5} + 1}{2} = f$

25. **A.**

This expression for $f(n)$ is showing that the n[th] Fibonacci number is the sum of the n[th] diagonal in Pascal's triangle. Thus, $f(10) = F_{10} = 55$.

If you did not see that right away, you can arrive at the same conclusion algebraically:

$$f(0) = \sum_{k=0}^{0} \binom{-1-k}{k} = 0$$

$$f(1) = \sum_{k=0}^{0} \binom{-k}{k} = \frac{0!}{0!0!} = 1$$

$$f(2) = \sum_{k=0}^{0} \binom{1-k}{k} = \frac{1!}{1!0!} = 1$$

$$f(3) = \sum_{k=0}^{1} \binom{2-k}{k} = \binom{2}{0} + \binom{1}{1} = 2$$

$\vdots$

$$f(n) = F_n$$

26. **A.**

Each interior angle of regular pentagon is $\frac{3p}{5}$

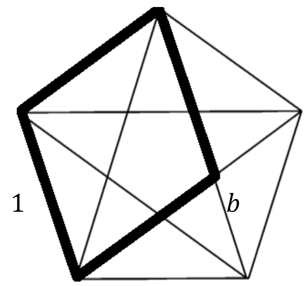
By Law of Cosines:

$a^2 = 1^2 + 1^2 - 2(1)(1)\cos(3p/5)$, where a is side of pentagram

$$a = \sqrt{2 - 2\cos(3p/5)} = \sqrt{\frac{1}{2}\left(3 + \sqrt{5}\right)} = \sqrt{f + 1} = \sqrt{f^2} = f$$

Therefore, the planned amount of flowers to be planted is $5a = 5f$

Notice how a rhombus of side length 1 is formed due to parallel sides:



Therefore, $1+b=f \rightarrow b=f-1$

Planted flower perimeter = $10b=10(f-1)=\frac{10}{f}$, according to $f^2=1+f$

27. **C.**
Construct the following chart:

t	0	1	2	3	4	5	6
A_t	8	13	21	34	55	89	144
C_t	5	8	13	21	34	55	89

Notice how A_t follows the pattern of the Fibonacci sequence.

28. **A.**
Number of adult bunies at time t in terms A of and C:

$A_t = A_{t-1} + C_{t-1} + N$

Number of child bunnies at time t in terms of A:

$C_t = A_{t-1}$

Consolidating…

$A_t = A_{t-1} + A_{t-2} + N$

29. **C.**
The number of adult and child bunnies at each timestep are each reduced by $1-a$. Therefore, the snakes must target adult bunnies only, before they get the chance to procreate. If the species were *P. eatsalottus*, then the child bunnies, A_{t-2}, would be affected by a factor of a^2.

30. **B.**
$a=55$

$b=34$

$$\begin{vmatrix} b & a \\ a & a+b \end{vmatrix} = \begin{vmatrix} 34 & 55 \\ 55 & 89 \end{vmatrix} = 34\cdot 89 - 55^2 = 1$$