1. C: $2015 = 5^1 \cdot 13^1 \cdot 31^1$ → # positive divisors= $(1+1)\cdot(1+1)\cdot(1+1) = 8$
2. C: $314 = 2 \cdot 157$. Hence, its negative and positive integral divisors are $\pm1, \pm2, \pm157, \pm314$. The product of these can be shown as $(2\cdot157)(-2\cdot-157)(314)(314) = 314^4$
3. B: Divide until remainders repeat (forming a cycle of repeating digits) to find that $\frac{12}{13} = 0.\overline{923076}$
4. D: This is an application of the Pigeon Hole principle. One can minimize the number of oranges picked by Sara K. by letting the number picked by each student be as close to the average as possible → $\frac{31{,}415}{60} = 523 + \frac{35}{60}$. Thus, some of the students must have picked at least 524 oranges, but if Sara K. picked more than any other, she must have picked at least 525.
5. A: $271 = 1(4^4) + 3(4^1) + 3(4^0) = 10033_4$ → Sum of base 4 digits = 7
6. C: Every positive integer has 1 as a factor, so 1 must be the common factor between 30! and Q. Since Q isn't prime, it must have at least two prime factors, neither of which is a factor of 30!. The smallest prime that isn't a factor of 30! is 31, so Q is a multiple of 31. The smallest multiple of 31 that isn't prime and doesn't share any prime factors with 30! is $31 \cdot 31$ but that isn't big enough: $31 \cdot 31 = 961$. The next smallest multiple of two primes is 31 times the next prime, or $31 \cdot 37 = 1147$.
7. B: The number does not have a power or multiple of 11 as its factor. Hence, a should include 11^2. Since 3^3 is also a factor of the number, a must have at least one additional factor of 3 since $6^2 = 3^2 \cdot 2^2$. Thus, a should be at least $11^2 \cdot 3 = 363$
8. A: $1A9_{11} = 1(11^2) + 10(11^1) + 9(11^0) = 240$
9. D: First, notice that anything raised to the 0 power equals 1, so $x = -2$ is a solution. Next, observe that we can only have 1 as a result with an integer base in two cases.

   (1) If $(x^2 - x - 1) = 1$; since 1 raised to any power is 1
   (2) If $(x^2 - x - 1) = -1$ and x itself is even, since -1 raised to any even power equals 1

   For case (1) $(x^2 - x - 1) = 1$, simplifies to $(x-2)(x+1) = 0 \rightarrow x = 2, -1$
   For case (2), $(x^2 - x - 1) = -1$, simplifies to $x(x-1) = 0 \rightarrow x = 0,1$. However, $x = 1$ is NOT a solution, as this will not make the equation have an even power. Hence, there are four solutions, specifically $x = -2, -1, 0, 2$
10. B: The three sexy prime triplets are $(7,13,19)$; $(17,23,29)$; $(31,37,43)$
11. E: Play devil's advocate by picking numbers to get "yes" and "no" answers using both statements. For statement (I), x could be 10 and y could be 1, giving "yes". But it could also be 1 and 1, leading to "no". For statement (II), again x could be 10 and y could be 1, giving "yes". But one could also have the same with negative numbers, -10 and -1, giving "no". The statements together may look to be sufficient (again, 10 and 1 give "yes"). But 2 and 1 also work, which would make $x - 1$ equal to y, not greater. So because one can still get "no" and "yes", the correct answer is E.
12. C: For n odd and $k = 0,1,2,3 \ldots$, $n = (2k+1)^2 = 4k^2 + 4k + 1 \equiv 1 \ mod\ 4$. For n even and $k = 1,2,3 \ldots$, $n = (2k)^2 = 4k^2$, which is divisible by 4.

13. A: Any odd integer multiplied by another odd will be odd. The set is not closed under either addition since $1 + 3 = 4$, which is even, or division since $\frac{1}{3} = 0.333$, which is not an odd integer.
14. B: $\prod(36) = 12$. The numbers 1, 5, 7, 11, 13, 17, 19, 23, 25, 29, 31, and 35 are relatively prime with 36.
15. D: A is false if $m = 2$. B is false if $m = 1$. C is false if $m = 3$. For D, use Heron's formula to determine the area of the triangle. Specially, the semi-perimeter $S = \frac{8m^2+4}{2} = 4m^2 + 2 \rightarrow$ $Area = \sqrt{(4m^2+2)(2m^2+1)(2m^2)} = \sqrt{16m^6 + 16m^4 + 4m^2} = 2m(2m^2+1)$, which is an integer.
16. B: Note that b has to be even and that $\$a67.9b$ must be divisible by 4. Hence, $b = 2\ or\ 6$. $\$a67.9b$ must also be divisible by 9, which means that the sum of the digits of $\$a67.9b$, i.e. $a + b + 22$, is divisible by 9. Let $= 2 \rightarrow a = 3 \rightarrow \frac{\$367.92}{72} = \$5.11$. Thus, $a + b = 5$
17. D: The triangular numbers are: 1,3,6,10,15,21,28… The perfect numbers are: 6, 28,496,…
18. A: Use geometric series. $0.222\ldots_7 = \frac{2}{7} + \frac{2}{7^2} + \frac{2}{7^3} + \cdots = \frac{\frac{2}{7}}{1-\frac{1}{7}} = \frac{1}{3} = 0.333 \rightarrow p = 3$
19. E: The left-hand side of the equation is divisible by 3m but the right-side is not. Hence, there are no integer solutions to the equation.
20. B: There are 60 numbers from 10 to 99 which are divisible by 2 or (or both), so the probability is $\frac{60}{90} = \frac{2}{3}$
21. C: Since $n^5 - 5n^3 + 4n = n(n-2)(n-1)(n+1)(n+2)$, the product will be divisible by 3,5, and 8.
22. C: RuPaul can't make ¥1 or ¥3, so those options are out. Notice too that ¥70 is the maximum possible sum (that would use all the coins), so ¥70 - ¥1, or ¥69, and ¥70 - ¥3, or ¥67 are impossible too. (¥67 would require at least 11 ¥5 coins or at least 11 ¥2 coins). RuPaul can make all other combinations with her coins. Hence, at most 66 sums are possible.
23. D: $3{,}999{,}991 = 2000^2 - 3^2 = (2000+3)(2000-3) = 2003 \cdot 1997$. The others are in fact prime.
24. A: Using Descartes Rules of Signs, we see that $f(x)$ has no sign changes, implying $f(x)$ has no positive root. Thus, statement II is false. $f(-x)$ has 5 sign changes, so there could be up to 5 negative roots and no imaginary roots. However, the function actually has imaginary roots—hence, Statement I is true.
25. B: $\sum_{k=0}^{10} \binom{10}{k} = \binom{10}{0} + \binom{10}{1} + \cdots + \binom{10}{10} = 2^{10} = 1{,}024$
26. B: There are a total of $6 \cdot 6 = 36$ total outcomes. If $D_1 = 1$, then it will be relatively prime will all values of D_2. If $D_1 = 2$, then it will be relatively prime with D_2 if $D_2 = 1,3,5$. If $D_1 = 3$, then it will be relatively prime with D_2 if $D_2 = 1,2,4,5$. Continuing in this way, there are 23 total pairs where D_1 and D_2 are relatively prime. Hence, probability that D_1, D_2 are relatively prime $= 23/36$.
27. A: Since $(B+2)(C+3)$ is even, either $(B+2)$ is even, which implies B is even, or $(C+3)$ is even, which implies C is odd. If B is even, then the product $4BC$ is divisible by

at least 8. If C is odd (and assuming B is odd as well), then the only factors of 2 in the product $4BC$ would come from 4. Thus, 4 is the only factor that must be a factor of $4BC$

28. D: Notice the multiples of 11 that appear in the calculation when $n = 1 \rightarrow 22{,}023 - n = 22{,}022$
29. E: Statement (1) does not reveal too much; x could be 1+2, 2+3, etc. For Statement (2), if y is an integer, than $x = y + (y+1) + (y+2) + (y+3) + (y+4) = 5y + 10$. This means that x is 10 greater than a multiple of 5, e.g. 10 or 15. Taking the two statements together, one is looking for any multiple of 5 that is the sum of two consecutive integers. Two possible solutions are 5, which is 3+2 and -1+0+1+2+3, or 15, which is 7+8 and 1+2+3+4+5. Thus, even together, the two statements are NOT sufficient, and the correct answer is E.
30. B: $2014^2 + 2015 - 2015^2 + 2014 = 2014(2014+1) + 2015(1-2015)$
$$= 2014(2015) + 2015(-2014) = 0$$