1. **For this test, unless the problem states otherwise, assume all variables to represent positive integers.** In other words, arguing otherwise is not proper grounds for a dispute. Find the sum of the solutions to $2x^3 - x^2 - 8x + 4 = 0$.
   A. 0 B. $\frac{1}{2}$ C. 2 D. $\frac{5}{2}$ E. NOTA

2. Define $\mathbb{Z}_n$ as the least residue system modulo n. That is $\mathbb{Z}_n = \{0, 1, 2, \ldots, n-1\}$ is the set of possible remainders when dividing by n. Solve $2x^3 - x^2 - 8x + 4 = 0$, where $x \epsilon \mathbb{Z}_7$, and sum the solutions as integers.
   A. 2 B. 4 C. 6 D. 11 E. NOTA

3. Compute $13 \cdot (19)^{-1}$ in $\mathbb{Z}_{29}$.
   A. 9 B. 13 C. 19 D. 26 E. NOTA

4. Find the sum of all integers $20 \leq x \leq 50$ such that $6x + 5 \equiv 23\ (mod\ 10)$.
   A. 3 B. 99 C. 159 D. 213 E. NOTA

5. It is known that $3x + 5y$ is divisible by 37. Based on this, it can be concluded that $ax + 7y$ is also divisible by 37. Find the smallest possible value of a.
   A. 4 B. 18 C. 19 D. 33 E. NOTA

6. When 59117, 87937, and 131167 are divided by x, they leave the same remainder. Find sum of the digits of the largest possible value of x.
   A. 1 B. 2 C. 4 D. 10 E. NOTA

7. For how many ordered pairs (a, b) is $\frac{1}{a} + \frac{1}{b} = \frac{1}{12}$?
   A. 9 B. 15 C. 17 D. 29 E. NOTA

8. Find the sum of all possible values of $x + y$ if $13x + 31y = 901$.
   A. 61 B. 104 C. 129 D. 183 E. NOTA

9. Let A, B, C be digits, $x_{10} = ABC_4$, and $y_{10} = ABC_6$. If $y = 2x$, and the sum of all possible values of x is z, find the sum of the digits of z.
   A. 0 B. 4 C. 11 D. 14 E. NOTA

10. $\mathbb{Z}[i] = \{a + bi | a, b \in \mathbb{Z}\}$ defines the set of *Gaussian integers*. The norm of a Gaussian Integer $z = a + bi$ is defined as $a^2 + b^2$. Compute the norm of the Gaussian integer $(2 + i)(2 + 2i)(7 + i)(3 + i)$.

A. $20\sqrt{5}$ B. $100\sqrt{2}$ C. 2000 D. 20000 E. NOTA

11. A Gaussian integer *z* is a *Gaussian prime* if it is only divisible by its associates (that is, $z, iz, -z, -iz$) and the units (that is, $1, i, -1, -i$). Like the integers, a unit is not prime. How many numbers in the set $\{2, 3, 5, 2 + 3i, 2 + 5i, 3 + 5i\}$ are Gaussian primes?

A. 2 B. 3 C. 4 D. 5 E. NOTA

12. How many of the sets below have the same cardinality as the set of integers?

- The set of real numbers.
- The set of natural numbers.
- The set of rational numbers.
- The set of irrational numbers.
- The set of Gaussian integers.
- The set of 3-dimensional lattice points.
- The set of algebraic numbers.

A. 2 B. 3 C. 4 D. 5 E. NOTA

13. Find the number of ordered pairs (a, b) such that the least common multiple of *a* and *b* is 720.

A. 30 B. 60 C. 135 D. 240 E. NOTA

14. The fraction $\frac{103_b}{136_b}$ can be reduced to $\frac{14_b}{18_b}$ (which is simplest since it is irreducible in base 10). What is the simplest form of the fraction $\frac{149_b}{338_b}$, written as a quotient of integers, each in base *b*?

A. $\frac{16}{37}$ B. $\frac{19}{39}$ C. $\frac{21}{52}$ D. $\frac{149}{338}$ E. NOTA

15. Find the number of positive integers less than or equal to 2016 that are divisible by 2 or 3, but not 5.

A. 1076 B. 1277 C. 1344 D. 1345 E. NOTA

16. Find the remainder when 8^{321} is divided by 800.

A. 8 B. 208 C. 408 D. 608 E. NOTA

17. x has exactly 2 primes among its N positive integral factors. x^2 has $3N$ positive integral factors. How many positive integral factors does x^7 have?
A. $13N$ B. $29N$ C. $71N$ D. $127N$ E. NOTA

18. Let S be the sum of all x less than 1000 that satisfies the condition $x^2 \equiv x \ (mod\ 363)$. Find the sum of the squares of the digits of S.
A. 62 B. 65 C. 67 D. 70 E. NOTA

19. Define $f(x) = \begin{cases} (x-1)! & \text{if } x \text{ is prime} \\ (x+1)! & \text{if } x \text{ is not prime} \end{cases}$
and $r(a,b) =$ remainder when a is divided by b.
Compute

$$r\left(\sum_{n=1}^{25} r(f(n), n), 5\right)$$

A. 0 B. 1 C. 2 D. 3 E. 4

20. A topic of interest in number theory is continued fractions. The continued fraction of a positive real number x is $[a_0; a_1, a_2, a_3, \ldots]$ means that

$$x = a_0 + \cfrac{1}{a_1 + \cfrac{1}{a_2 + \cfrac{1}{a_3 + \cdots}}}$$

$a_0, a_1, a_2, a_3, \ldots$ are called the entries of the continued fraction. For example, $\frac{8}{3} = [2; 1, 2]$.
Find the continued fraction of $\frac{9}{34}$ and compute the sum of the squares of the entries.
A. 17 B. 23 C. 27 D. 36 E. NOTA

21. Continued fractions for irrational numbers do not terminate, but some of them are periodic. When a continued fraction is periodic, a bar is placed on the numbers that form a period, much like repeated decimals. Find the value of $[2; \overline{1, 2}]$.
A. $1 - \sqrt{3}$ B. $\sqrt{3}$ C. $4 - \sqrt{3}$ D. $1 + \sqrt{3}$ E. NOTA

22. Approximating an irrational number using rational numbers is another application of continued fraction. These approximations are called *convergents*. The two well known rational approximations for π are $\frac{22}{7}$ and $\frac{355}{113}$. They are the second and fourth convengents of π (the first is 3). Find the third convergent of π, which is much less well known. You will need to know that $\pi = [3; 7, 15, 1, 292, 1, 1, 1, 2, 1, 3, 1, \ldots]$.
A. $\frac{179}{57}$ B. $\frac{223}{71}$ C. $\frac{267}{85}$ D. $\frac{311}{99}$ E. NOTA

23. Convergents can be used to solve Pell's Equation. Given that $\sqrt{2} = [1; \overline{2}]$, and let the n^{th} smallest solutions to $x^2 - 2y^2 = 1$ be (x_n, y_n). Find $x_3 + y_3$. (Of course, you don't have to use convergents to solve this particular problem.)

A. 12 B. 29 C. 70 D. 169 E. NOTA

24. Let $f(x) = 2880x^6 - 18816x^5 + 11852x^4 + 80356x^3 - 26877x^2 - 67590x - 5400$. All roots of f are distinct and rational. A rational number is randomly chosen from the list according to rational root theorem. What is the probability the number chosen is a root?

A. $\frac{1}{672}$ B. $\frac{1}{336}$ C. $\frac{1}{80}$ D. $\frac{1}{40}$ E. NOTA

25. In trapezoid $ABCD$, $AB = 4, CD = 10$, and $m\angle C < m\angle D \leq 90°$. Point E is on CD such that $BE \perp CD$. If $BC, CE,$ and DA are all integer lengths. How many possible such trapezoids are there?

A. 1 B. 2 C. 3 D. 4 E. NOTA

26. Find the cube root of 31,217,193,218,303. I assure you, it's one of the answer choices.

A. 31427 B. 31487 C. 31527 D. 32087 E. 32323

27. The product of any two of the elements of the set $\{30, 54, N\}$ is divisible by the third. Find the number of possible values of N.

A. 3 B. 6 C. 9 D. 12 E. NOTA

28. Define a sequence G_n recursively as $G_1 = 7, G_2 = 13$, and for $n > 2, G_n = 3G_{n-1} + 4G_{n-2}$. The sequence can also be defined explicitly as $G_n = a \cdot b^n + c \cdot d^n$ for $a, b, c, d \in \mathbb{R}$, find the value of $a + b + c + d$.

A. 1 B. 5 C. 10 D. 14 E. NOTA

29. 0.1_{10} can be expressed as $\left(0.A_1A_2 \ldots A_m\overline{A_{m+1}A_{m+2} \ldots A_{m+n}}\right)_2$. That is, a repeating decimal in base 2. In that representation, m is the shortest possible length of the non-repeating part while n is the minimal length of the repeating part. For example, $0.11010101\ldots$ is to be represented as $0.1\overline{10}$, and not as $0.110\overline{1}$ or $0.1\overline{1010}$. Compute

$$m + n + \sum_{k=1}^{n} A_{m+k}2^k$$

A. 8 B. 10 C. 17 D. 29 E. NOTA

30. Find $A + B + C + D$ if

- A minimum of *A* colors is needed to color any map consisting of contiguous regions to guarantee that no two regions sharing a border also share a color.
- It is guaranteed that any positive integer can be written as the sum of at most *B* perfect squares.
- A graph with an Eulerian path has at most *C* vertices of odd degree.
- A graph with an Eulerian circuit has at most *D* vertices of odd degree.

A. 10 B. 11 C. 12 D. 13 E. NOTA