1. **E** – There is no rule saying that E and F cannot be installed together, and there is no third rule that would preclude both (e.g. if R is installed, both E and S cannot be installed). Similarly, M and S can be installed together – we need two statistical computing packages anyway, given that two browsers are installed. M and R can be co-installed; we just need to use browsers F and G rather than E and F. Finally, A and B can be co-installed; we just need M as one of the statistical computing packages, and either R or S.

2. **B** – When R is installed, we need exactly one browser and one antivirus package; there are 3*3 = 9 combinations of these. However, any combination involving E will not work, and A and M can only be installed together, so that A cannot be installed with R; this leaves us with four possible combinations (BF, BG, CF, CG).

3. **A** – Unless x and y are positive, we can't necessarily say $\sqrt{\frac{x}{y}} = \frac{\sqrt{x}}{\sqrt{y}}$! This is where the error occurs.

4. **E** – We might think initially that choice (D) is unprofitable, since the expected payoff if exactly one more roll occurs is $\frac{5}{6} \cdot \left(48 + \frac{2+3+4+5+6}{5}\right) < 48$. However, note that if the die is rolled once and the game does *not* end, we are guaranteed at least one more risk-free roll; in fact, we can roll risk-free until we hit 58. Hence the relevant number is $\frac{5}{6} \cdot (58) > 48$ (note that the expected value is of course larger than $\frac{5}{6} \cdot 58$, but we don't need to compute the exact value; all we need to do is show that it's better to roll than to quit)

5. **D** – Note that the term in parentheses in the "inductive" step should really have one more term, i.e., it should be $\left(\frac{1}{1 \cdot 2} + \cdots + \frac{1}{n \cdot (n+1)}\right)$. Doing the computations properly will NOT yield $\frac{3}{2} - \frac{1}{n+1}$. This problem comes from Donald Knuth's *Fundamental Algorithms*, and is meant to illustrate the dangers of improperly indexing.

6. **C** – I won't go through the formal proof here (Google it – there are plenty of good results), but roughly, induction states that we start with the smallest element and work our way up; if we can do this, it means that the set of numbers in question (frequently the natural numbers) is well ordered. Similarly, if a set of numbers is well-ordered, we have a well-defined basis case for induction and can then work our way up due to well ordering.

7. **C** – Inequality (I) is true, and can be proved by taking $x = (a_1, a_2, \ldots a_n)$ and $y = (1, 1, \ldots)$ (an n-vector). Inequality (II) is false; the reverse can be proved by the AM-GM inequality, which in turn can be proved using the Cauchy-Schwarz inequality. Inequality (III) is true, and can be shown by first squaring both sides; then rewriting $\|\vec{x} + \vec{y}\|^2 = \vec{x} \cdot \vec{x} + 2\vec{x} \cdot y + \vec{y} \cdot y = \|\vec{x}\|^2 + \|\vec{y}\|^2 + 2\vec{x} \cdot \vec{y}$ which, by Cauchy-Schwarz, is less than or equal to $\|\vec{x}\|^2 + \|\vec{y}\|^2 + 2\|\vec{x}\|\|\vec{y}\| = (\|\vec{x}\| + \|\vec{y}\|)^2$.

8. **D** –

$$\frac{\frac{3}{4}(0.6)^2}{\frac{3}{4}(0.6)^2 + \frac{1}{4}(0.2)^2} = \frac{3^3}{3^3 + 1} = \frac{27}{28}$$

9. **C** – If you attend 2*N* press conferences and hear enhancement *N* times, then your updated conditional probability of the CEO being a shirker is

$$\frac{\frac{3}{4}(0.6)^N(0.4)^N}{\frac{3}{4}(0.6)^N(0.4)^N + \frac{1}{4}(0.2)^N(0.8)^N}$$

We can simplify the expression above to

$$\frac{3(0.6)^N}{3(0.6)^N + 0.4^N} = \frac{3^{N+1}}{3^{N+1} + 2^N}$$

The smallest N for which this exceeds 0.95 is N = 5.

10. **B** – The approach here is to take the total number of permutations and subtract all arrangements in which at least one element appears in its original location. We also have to account for double-counting; hence, given that there are seven letters in WASHING, we have

$$n! - \binom{n}{1}(n-1)! + \binom{n}{2}(n-2)! + \cdots$$

all the way down to 1; (alternating addition and subtraction) we can rewrite $\binom{n}{k}(n-k)!$ as $\frac{n!}{k!}$. Thus our sum becomes

$$7!\left(1 - \frac{1}{1!} + \frac{1}{2!} - \cdots \frac{1}{7!}\right)$$

Evaluating this expression gives 1854.

11. **B** – Consider the following. On the first day, anyone seeing six red hats will know that he himself is wearing the seventh; all seven of those wearing a red hat will only see six other red hats, and know that they too are wearing red hats. If instead they see seven hats, they don't know whether they are wearing a red hat (and there are eight total red hats), or if they are wearing a blue hat and there are seven total. However, if nobody has left after the first day, it means that there must be eight hats; if there were seven, the seven red hat wearers would have realized it on day 1 and left. Thus, there must be 8 red hats, and all those wearing one realize it. Continuing in this way, we see that one more red hat leads to one more day; thus it will take 22 – 7 + 1 = 16 days.

12. **C** – The key here is that *bowls* are drawn randomly, *then* marbles. Hence we can maximize by putting a blue marble in one bowl and nothing else, and 49 blue marbles and 50 red marbles in the second bowl; and we can minimize by doing the opposite (a red marble and nothing else in one bowl, and 49 red marbles and 50 blue marbles in the second bowl). The payoff under the maximal strategy is $\frac{1}{2}(2) + \frac{1}{2}\left(2 \cdot \frac{49}{99} - 1 \cdot \frac{50}{99}\right) = \frac{123}{99}$ while the payoff under the minimal strategy is $\frac{1}{2}(-1) + \frac{1}{2}\left(2 \cdot \frac{50}{99} - 1 \cdot \frac{49}{99}\right) = -\frac{24}{99}$. Hence we have $\frac{123}{-24} = -\frac{41}{8}$.

13. **B** – First set up a 3 by 3 grid to match individuals to company, city, and returns; then use the clues, and the logical steps that follow, to deduce the following relationships:

Alice	\$3	San Francisco	Capslock Capital
Bob	\$1	St. Louis	Magenta Rock
Cat	\$2	New York	Silverman Rucks
Prometheus	\$4	Miami	Average Returns

Hence we have L = 5 (New York), M = 1, N = 4. Hence M+N+L = 10.

14. **B** – From the grid above, we see P = 4, Q = 3. Hence Q/P = ¾.

15. **D** – This is just a numerical representation of tic-tac-toe! In particular, we have

$$\begin{matrix} 1 & 2 & -3 \\ -4 & 0 & 4 \\ 3 & -2 & -1 \end{matrix}$$

Writing the game in this way, it should be obvious that if both players play optimally the only result that will ever occur is a draw. (This game can also be solved by brute force – it won't take too long, one just needs to show that for any first choice by Ms. Herron, Dr. Morris can play such that a draw always occurs).