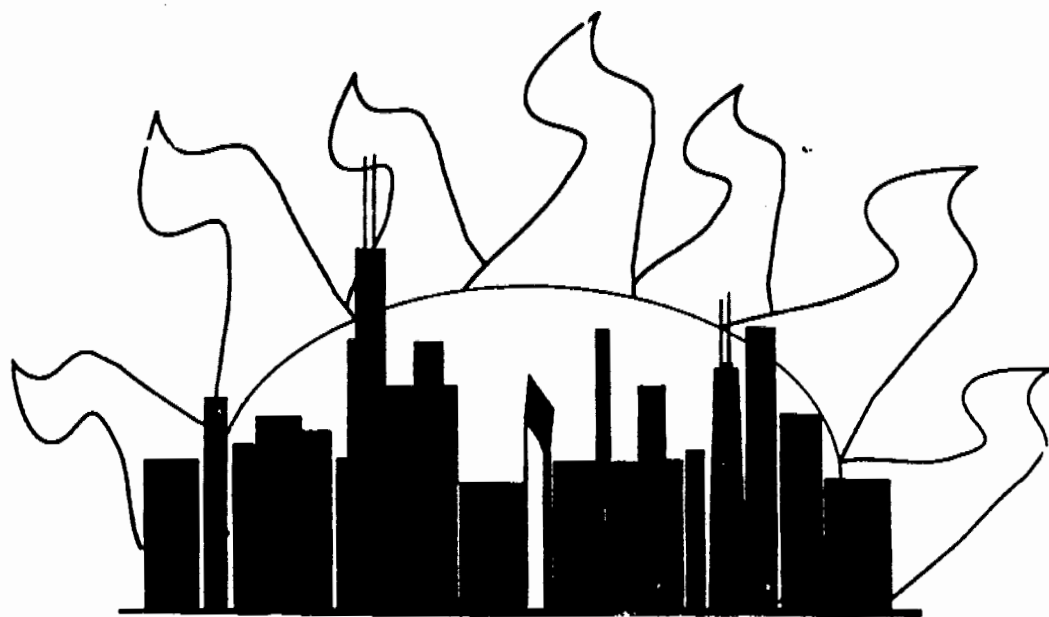


Theta Division

Topic Test 1

**Polynomial
&
Rational Functions**



**Mu Alpha Theta National Convention
Chicago 1998**

General Instructions:

1. Unless otherwise stated all answers should be written as decimals.
2. If you are asked to give your answer as a fraction, please give your answer in $\frac{a}{b}$ form where a and b are relatively prime.

Questions

1. What is the degree of the expression $9x^2y + 13x^3y^2 + 8x^4y^4$?
2. Write $\frac{3(y^3)^{10}}{y^3y^4}$ with a single positive exponent on the variable.
3. Let $A = \frac{x^2 - 9y^2}{x - 3y}$ and $B = \sqrt{x^2 + 2xy + y^2}$. Find the value of $A + B$ if $x = 2.74$ and $y = -3.12$.
4. What is the remainder in the division $\frac{x^4 + x^2 + 1}{x + 1}$?
5. Find the real value of x which satisfies $5x + 3\sqrt{x} - 2 = 0$. Write your answer as a simplified fraction in lowest terms.
6. From a square whose side has a length x inches, create a new square whose side is 3 inches longer. Write a polynomial expression for the difference between the two areas of the two squares as a function of x .
7. The polynomial $x^3 - 23x^2 - 2881x - 37417$ can be factored as $(x - a)(x - b)(x - c)$. Find $|a| + |b| + |c|$.
8. Simplify $\frac{x - \frac{x}{x+1}}{x + \frac{x}{x+1}}$. Assume that the fraction is real-valued.
9. Let $f(x) = 3x - 2$ and $g(x) = 2x^2 + 1$. Evaluate $(g + f)(-3)$.

10. Find the sum of all real values of x such that $\frac{\sqrt{x}-1}{\sqrt{x}+1} = \frac{x-1}{3x}$. Give your answer rounded to four significant digits.
11. Simplify the following complex fraction, assuming $xy \neq 0$ and $x+y \neq 0$: $\frac{\frac{y-x}{x}}{\frac{1}{x} + \frac{1}{y}}$.
12. If $a, 3a, 5a, b, b+3, b+5$ are all roots of a fourth degree polynomial equation where $0 < a < b$, compute the sum of all possible values of a .
13. For how many ordered triples of positive integers (x, y, z) where $x \neq y \neq z$ is a positive integral value attained by $\frac{x}{(x-y)(x-z)} + \frac{y}{(y-x)(y-z)} + \frac{z}{(z-x)(z-y)}$.
14. Find the value of x which satisfies $\sqrt[3]{x\sqrt{x}} = 4$.
15. Solve for n if $3 = \sqrt{1+2\sqrt{1+3\sqrt{1+4\sqrt{n}}}}$.
16. Find the positive number x for which $\sqrt{x} = \sqrt[3]{y}$ and $\sqrt{y} = 8$.
17. Find the sum of all rational numbers x which satisfy the equation $\frac{x+\sqrt{x}}{x-\sqrt{x}} = \frac{81x(x-1)}{4}$.
Give your answer as a simplified fraction in lowest terms.
18. Solve for x : $\sqrt{x+\sqrt{x+\sqrt{x+\dots}}} = 2$.
19. Determine, in simplest form, the numerical value of $\sqrt[3]{7-\sqrt{50}} + \sqrt[3]{7+\sqrt{50}}$.
20. A quadratic polynomial, p , satisfies $p(x) = \left[\frac{p(x+1) - p(x-1)}{2} \right]^2$, for all complex x .
Find the value of $[p(0) - p(-1)] + [p(0) - p(1)]$. Give your answer as a simplified fraction in lowest terms.