Mu Alpha Theta National Convention: Denver, 2001

Gemini Test Solutions – Theta Division

1. Pick a number for the 3-of-a-kind: 13 Cr 1
   Pick their suits: 4 Cr 3
   Pick two other numbers: 12 Cr 2
   Pick 4^{th} suit: 4 Cr 1
   Pick 5^{th} suit: 4 Cr 1

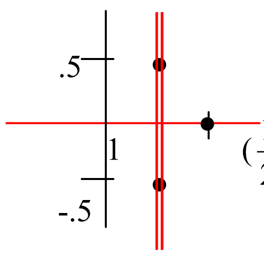
Now multiply them together : $\binom{13}{1}\binom{4}{3}\binom{12}{2}\binom{4}{1}\binom{4}{1}$= 54,912.

2. Out of twenty-five pieces, you want seven: 25 Cr 7
   Out of the five coconut cremes, you want three: 5 Cr 3
   You just want four other pieces of candy, non-coconut: 20 Cr 4

Then the probability is $\dfrac{\binom{5}{3}\binom{20}{4}}{\binom{25}{7}} = \dfrac{51}{506}$.

3. $\dfrac{4-2i}{-3+5i}\dfrac{(-3-5i)}{(-3-5i)}$(complex conjugate)= $\dfrac{-12-20i+6i-10}{34} = \dfrac{-11-7i}{17}$.

4.



The three points are equidistant ($\frac{1}{2}$) from one point: ($\frac{1}{2}$,0). Three points make a circle (see definition of a circle) and the general equation of the circle is $(x-j)^2+(y-k)^2=r^2$.

Plug our center into this equation: $(x-\frac{1}{2})^2+y^2=\frac{1}{4}$.

5. Using the bottom row give one less determinant to figure out:

$8\begin{vmatrix}3 & 4\\ 6 & 7\end{vmatrix}-9\begin{vmatrix}2 & 4\\ 5 & 7\end{vmatrix}$= -24 + 54 = 30.

6. $13(x^2-6x)-5(y^2+4y)=33$

$13(x^2-6x+9)-5(y^2+4y+4)=33+13(9)-5(4)$ -- Complete the squares

$13(x-3)^2-5(y+2)^2=130$ $\dfrac{(x-3)^2}{10}-\dfrac{(y+2)^2}{26}=1$. Hyperbola. Then the center is (3,-2). The foci distance (c) to the center is given by $c^2=a^2+b^2$, where $a^2=10, b^2=26$. Then c= 6. Two times that distance is the distance between the foci : 12. Another option: The foci are located at $(3\pm 6,-2)$. Then they are (9,-2) and (-3,-2). Using the distance formula, we get a distance of 12.

7. Volume of cylinder: πh. Volume of cube: h^3. →Multiplied together= πh^4.

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8. Distance formula works just like in Cartesian coordinates:
$\sqrt{(-2-5)^2+(-1-4)^2}=\sqrt{74}$.

9. 1024 = 5. Since the base is the same, set the exponents equal. 2s-1=5 → s=3.

10. We want the probability of exactly four heads given that at least two appeared. P(at least two heads) = 1-P(one or no heads) = 1-$\frac{6}{32}=\frac{13}{16}$. Then the total probability is the probability of exactly four heads($\frac{5}{32}$)/$\frac{13}{16}=\frac{5}{26}$. In set terms P(A/B) = $\frac{P(A\cap B)}{P(B)}$, where A is exactly four heads, and B is at least two heads.

11. To convert to polar coordinates (r, θ): x=$r\cos\theta$, y =$r\sin\theta$, and $r^2=x^2+y^2$. Then r= 5. This means cos $\theta=\frac{x}{r}=\frac{-3}{5}$ → θ=2.21 Rads.

12. Multiply the fractions through the appropriate matrix: $\begin{bmatrix}\frac{1}{2} & 1\\ -1 & 1\end{bmatrix}*\begin{bmatrix}0 & \frac{5}{4}\\ 1 & -\frac{1}{2}\end{bmatrix}$. Now multipy the matrices appropriately: $\begin{bmatrix}1 & \frac{1}{8}\\ 1 & -\frac{7}{4}\end{bmatrix}$.

13. Simplifies to $\frac{a^8b^{-6}}{a^4b^{-10}}$ → a^4b^4.

14. f(-1) = -5. g(-5) = -68. $\frac{1}{2}(-68)=-34$.

15. Following the layout, Beatrice spends money like this, where *x* is what she began with: *x*-5-$\frac{x}{6}$-15-4 = $\frac{3}{10}x$. Solving for x: $\frac{5x}{6}-24=\frac{3}{10}x$ → x = \$45.

16. Slope of the line with the points: $\frac{-3-1}{-2}=2$. Equation of the line: $y-1=2(x-2)$
→ $y=2x-3$. Parallel line has the same slope (A or D). Perpendicular line has negative reciprocal (A or C). A fits both → check:

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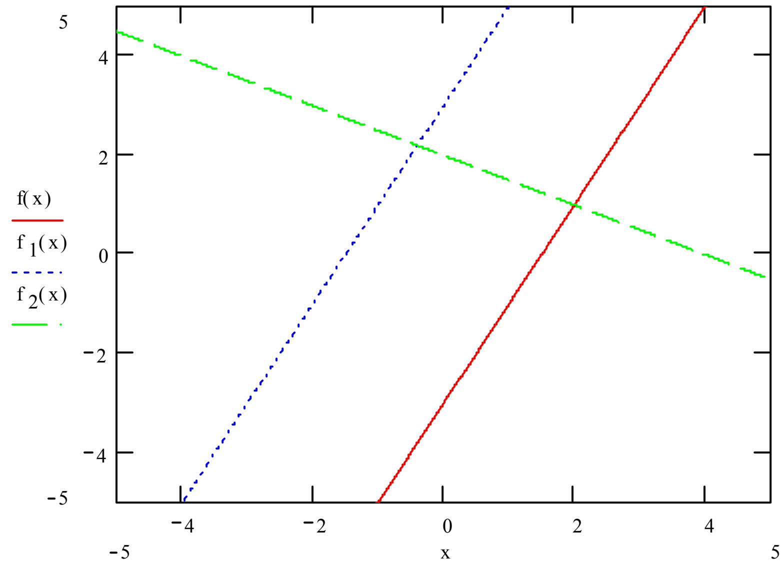

$f(x) := 2x - 3$

$f_1(x) := 2x + 3$

$f_2(x) := \frac{-x}{2} + 2$

17. x-intercept: (0,-3), y-intercept: (-4, 0). Distance formula yields $\sqrt{9+16} = 5$.

18. Not log properties: III and IV.

19. Rearrange the exponent: $e^{\ln A^2 + \ln B^{-2}} = e^{\ln A^2} e^{\ln B^{-2}} = A^2 B^{-2} = \frac{A^2}{B^2}$.

20. First square both sides → $3 + \sqrt{-2 + \sqrt{w}} = 4$. Then $\sqrt{-2+\sqrt{w}} = 1$. Square again: $-2 + \sqrt{w} = 1$. Then $\sqrt{w} = 3$ → $w = 9$. $\frac{2001}{9} = 222R3$.

21. $1230_4 = 108_{10} = 213_7$.

22. Numbers with exactly 9 + integral factors (odd number means they are square numbers): 36+ 100+ 196 = 332.

23. PST and RQT are similar triangles with ratio 2:1. → PT=10. So we have a triangle with sides 8, 10, and an integer > 2. The largest value it could be is 18, if the sides were on a line. But since we can't have a triangle like that, the next largest is 17. Then TQ=$\frac{17}{2}$. →PQ= $\frac{37}{2}$.

24. This looks like the infinite sum: $S = \frac{1}{1-\frac{t}{2}} = \frac{2}{2-t}$. Also,

$$\sum_{x=0}^{\infty} \left(\frac{t}{2}\right)^x \rightarrow \frac{-1}{\left(\frac{1}{2}\cdot t - 1\right)} \rightarrow \frac{-2}{t-2} = \frac{2}{2-t}.$$

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25. If $3+\sqrt{5}$ and $3-i\sqrt{5}$ are roots, then $3-\sqrt{5}$ and $3+i\sqrt{5}$ are too. That means our lowest order polynomial looks like x^4 + … Multiplying the $3\pm\sqrt{5}$ roots together implies x^2-6x+4 is a factor, and similarly the second pair imply $x^2-6x+14$. Multiplying these two together gives us $x^4-12x^3+54x^2-108x+56$. The quadratic term's coefficient is 54.

26. The grapefruit will spend half of its time in the air going up, the other half going down. Factoring and solving for t will give us the total time in the air: t = 6.3094348007… (t cannot be negative, so we ignore the other choice). Divide this by two and plug into the height equation: h = 162.25 ft.

27. This is a tough one, since we don't get "nice" numbers. But here goes…
To get the center of this circumscribed circle, we need the intersection of the perpendicular bisectors. A perpendicular bisector of a side will be equidistant from the two vertices on that side. So, the point where two perpendicular bisectors intersect is the center of the circumcircle. Take two slopes of two of the triangle's sides, and take their negative reciprocals. To further illustrate, the lines connecting (-2,1)&(5,-2) (line 1) and (5,-2)&(3,5) (line 2) will be used. To get the perpendicular bisector, we need the midpoints of these lines. Mdpt1= $(\frac{3}{2},\frac{-1}{2})$, Mdpt2 = $(4,\frac{3}{2})$. Now we need the lines' slopes: Slope1= $\frac{-3}{7}$, Slope2= $\frac{-7}{2}$. So the perpendicular lines are L1= $\frac{7}{3}(x-\frac{3}{2})-\frac{1}{2}$ and L2= $\frac{2}{7}(x-4)+\frac{3}{2}$. To find their *x*-intersection we set the y-values equal: $\frac{7}{3}(x-\frac{3}{2})-\frac{1}{2}=\frac{2}{7}(x-4)+\frac{3}{2} \rightarrow x=\frac{183}{86}$. Plug this into either line equation to get the y-value $y=\frac{83}{86}$. With our center at $(\frac{183}{86},\frac{83}{86})$, we can find the distance to one of the vertices to get the radius of the circle. Using the distance formula with $(\frac{183}{86},\frac{83}{86})$ and (-2,1) (though any vertice should work), our radius is ~4.12805. $\pi r^2=\pi * 17.0408 \approx 53.5$.

28. We are looking for the first hundred numbers divisible by 2, 3 or both. In the first six natural numbers (1-6), we have four that are divisible by two or three. So to get 100 such numbers, we need the first 150 natural numbers. To find the total sum, we need to add the numbers divisible by two to the numbers divisible by three, and then subtract the numbers divisible by 6 (since we counted them twice). Since there were three out of the six numbers divisible by two, then there are seventy-five total numbers divisible by two. Similarly, there are 50 numbers divisible by three, and 25 numbers divisible by six. Then Sum2= $\frac{75}{2}(2+150)$=5700, Sum3= $\frac{50}{2}(3+150)$=3825, and Sum6= $\frac{25}{2}(6+150)$=1950. Our answer is then 5700+3825-1950=7575.

29. Non-negative integers: 0, 1, 2, 3, 4, 5. Negative integers: -5, -4, -3, -2, -1. Total: 11.

30. There are three ways you could get letters:

2& 2 (e.g. DDII); 2& 1& 1 (e.g. DDIE); and 1& 1& 1& 1 (e.g. DIVE).

For the 2& 2 case, there is only one way to pick the letters: you must get two D's and two I's.
For the 2& 1& 1 case, there are 2*3Cr2 ways to pick the letters (two ways to pick which double (D or I) you're going to get, and 3Cr2=3 ways to pick which two of the remaining three –letter types for the two spots left.
For the 1& 1& 1& 1 case, there is only one way to pick the letters: you must take one of each kind.
For the 2& 2 case, there are 4Cr2 = 6 ways to arrange the letters you get. For the 2& 1& 1 case, there are 4!/2! = 12 ways. For the 1&1& 1& 1 case, there are 4! = 24 ways. 6*1 + 12*6 + 24*1 = 102.

31. $\log_6 6^{-3} + \log_2 2^8 - \log_4 4^4 = -3 + 8 - 4 = 1$.

32. –5 < 2*t* and 5*t*<10 → $\frac{-5}{2} < t < 2$.

33. By columns 1 and 3 (left to right), we can see that C+A=E → add to less than 10 (the last column is just A still). This and column 2 → B=0. To get the biggest C, A must be the smallest. A can't be 0 because of column 1. Then if A=1, then C=8.

34. Number of diagonals: $\frac{10*7}{2} = 35$.

35. Continuous compounding: C= Pe^{rt}. Quarterly: Q= $P(1+\frac{r}{4})^{4t}$. With P=$100, we get

C=$100e^{.16} = 117.35$, Q=$100(1+\frac{.08}{4})^8 = 117.16$. The difference is $.19.

36. First get rid of a variable: $\begin{matrix} -6x-4y+2z=-2 \\ 5x+4y-2z=2 \\ -x+6y-z=5 \end{matrix}$ add the first two→ $-1x=0$ → $x=0$. Get rid of z in the second two to solve for y: $\begin{matrix} 5x+4y-2z=2 \\ 2x-12y+2z=-10 \end{matrix}$ → $7x-8y=-8$. With x = 0, y = 1. Plug these into any of the original equations to get z = 1. Sum = 0 + 1 + 1 = 2.

37. There are four roots: h, j, k, p. We want $\frac{1}{h}+\frac{1}{j}+\frac{1}{k}+\frac{1}{p}=\frac{jkp+hkp+hjp+hjk}{hjkp}$. Compare this to (x-*h*)(x-*j*)(x-*k*)(x-*p*). The denominator is *hjkp*=$\frac{d}{a}$ in the original equation. Similarly, we

can get the coefficients of the other terms. The numerator is *hjk*+*hjp*+*hkp*+*jkp*= $\frac{-c}{a}$. Then

$$\frac{\frac{-c}{a}}{\frac{d}{a}} = \frac{-c}{d}.$$

38. Wendy's hamburger: Volume= s^3. McDonald's hamburger: Volume= $\pi r^2 h$, but we set s=2r, so V=$\pi(\frac{s}{2})^2 h$. Depths are equal, so V=$\pi\frac{s^3}{4}$. Then our ratio is: $\frac{\frac{\pi s^3}{4}}{s^3} = \frac{\pi}{4}$.

39. Tony's rate : $\frac{1 crane}{5.6 \min}$. Mark's rate: $\frac{1 crane}{4.2 \min}$. Their rate together is $\frac{1}{5.6} + \frac{1}{4.2} = \frac{5 cranes}{12 \min}$. Then for 250 cranes, they'll need $\frac{12 \min}{5 cranes} * 250 cranes = 600 \min$, or 10 hours.

40. The real wall has dimensions 1.6m x 2.4m ($\frac{1m}{25cm} * 40cm = 1.6m$). Its area is 3.84m^2. A poster covers .6m^2. So we can fit $\frac{3.84}{.6} = 6.4$ posters. But we can't use .4 of a poster, so we can fit 6 posters.