Mu Alpha Theta National Convention: Denver 2001
Euclidean Individual Test – Solutions

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1. **(B)**. "Of" means multiply so $(.10)(.16)(3200) = 51.2$.
2. **(A)**. Simplify each side of the equation to get $-3x-7 = 7x+5$. Solve to get $x = -6/5$.
3. **(C)**. We have $x/z = (x/y)(y/z) = (4/7)(14/3) = 8/3$. Thus, $(x+z)/z = x/z+1 = 8/3+1 = 11/3$.
4. **(C)**. Notice that $\angle AEF$ and $\angle EFC$ are alternate interior angles hence, they're equal.
5. **(C)**. The desired number is the least common multiple of 42, 175, and 392: 29400.
6. **(B)**. Simplify in a bottom-up fashion to obtain 43/30.
7. **(C)**. If the area of the circle is 49π then the radius is 7, making the diameter 14.
8. **(D)**. Simplify the expression to obtain $3(xy)^2/4$. Thus, when $(x, y) = (3, 4)$, its value is $3(144)/4 = 108$.
9. **(C)**. Let c be the cost of the meal. Solving the equation $.12c = 432$, we obtain $c = 3600$.
10. **(B)**. By inspection, $10/3 = 3 + 1/3$, so the answer is 3.
11. **(D)**. Using the area of a trapezoid formula, $(8+3)(10)/2 = 55$.
12. **(B)**. $5292 = 2^2 \times 3^3 \times 7^2$
13. **(C)**. From the given information, $a+b = 16$ and $c+d = 88$. Thus, the average of them all is $(a+b+c+d)/4 = (16+88)/4 = 26$.
14. **(D)**. $3x - 2 \leq -8x + 7 \rightarrow x \leq 9/11$.
15. **(D)**. Let $\lfloor x \rfloor$ be the greatest integer that does not exceed x. Then of the first 56997 counting numbers, $\lfloor 56997/2 \rfloor = 28498$ of them are divisible by 2, $\lfloor 56997/3 \rfloor = 18999$ of them are divisible by 3, and $\lfloor 56997/6 \rfloor = 9499$ are divisible by both 3 and 2, or 6. The number divisible by 3 or 2 is $28498 + 18999 - 9499 = 37998$, having subtracted the number divisible by 6 since they were counted twice.
16. **(A)**. The mode is the element that appears the most, or 5.
17. **(D)**. Setting up the proportions, we find that the bathrooms are $(6\frac{3}{8})(25)/(1\frac{1}{2}) = 106.25$ feet apart.
18. **(B)**. The length is $252/12 = 21$ so the perimeter is $2(12+21) = 66$.

19. **(A)**. When x is positive, y is also positive so the line never passes through the fourth quadrant.

20. **(D)**. By the diagonals formula, $(23)(23-3)/2 = 230$.

21. **(B)**. Multiply top and bottom of the fraction by ab to get $(ab - a^2)/(b^2 - ab) = (a(b-a))/(b(b-a)) = a/b$.

22. **(D)**. According to the definition, the 54th Fibonacci number must equal $y - x$, the 53rd $x - (y-x) = 2x - y$. Proceeding this way, we find that the 51st Fibonacci number is equal to $5x - 3y$.

23. **(C)**. The slope of the line is $(-1-6)/(8-3) = -7/5$. Picking $(8, -1)$ as an arbitrary point on the line and using point-slope form, the equation of the line is $y + 1 = (-7/5)(x-8)$ or in slope-intercept form, $y = -7x/5 + 51/5$.

24. **(A)**. The diagonals of a rhombus divides it into four congruent right triangles where the hypotenuse is the edge length of the rhombus. Since the diagonals bisect each other, the legs of one of the right triangles are 8 and 15 so the hypotenuse is $\sqrt{8^2 + 15^2} = 17$. The perimeter of the rhombus is $4(17) = 68$.

25. **(C)**. Factor the equation to obtain $(5x+6)(2x-7) = 0$; thus, the solutions for x are $-6/5$ and $7/2$.

26. **(A)**. Relative to point A, triangles ABD and ABC have the same height. Therefore, the ratio of their areas is just the ratio of their bases, or $BD/BC = BD/(BD+DC) = 3/8$.

27. **(A)**. Plug in $n = 1, 2, \ldots, 5$ into $a_n = 2n^2 - 3$ to obtain $\{-1, 5, 15, 29, 47\}$.

28. **(D)**. Let x be the smaller number and $52 - x$ the larger. Then we have $(52-x)/x = 9 + 2/x$. Solve to obtain $x = 5$.

29. **(E)**. $(3 \otimes 4) + (6 \otimes 8) = \dfrac{(3)(4)}{\sqrt{3^2+4^2}} + \dfrac{(6)(8)}{\sqrt{6^2+8^2}} = \dfrac{12}{5} + \dfrac{24}{5} = \dfrac{36}{5}$

30. **(D)**. $\dfrac{6a^2 - 7a + 2}{10a - 15a^2} = \dfrac{(2a-1)(3a-2)}{-5a(3a-2)} = \dfrac{1-2a}{5a}$

31. **(D)**. By the total surface area formula, $2\pi(3)^2 + 2\pi(3)(4) = 42\pi$.

32. **(C)**. $\dfrac{3}{k} - k^2 = 3x^2 - \dfrac{1}{x^4} = \dfrac{3x^6 - 1}{x^4}$

33. **(B)**. A zero at the end of a number is created by multiplying by 10, or 5×2. Since there are four 5×2 pairs in the given number, it has four terminal zeroes.

34. **(A)**. $(1-\sqrt{3})^3 = (1-\sqrt{3})^2(1-\sqrt{3}) = (4 - 2\sqrt{3})(1-\sqrt{3}) = 10 - 6\sqrt{3}$

35. **(C)**. $a^2n + b^2m = ct^2 + mcn \rightarrow a^2n + b^2m = c(t^2 + mn) \rightarrow c = \dfrac{a^2n + b^2m}{t^2 + mn}$

36. **(D)**. Set $f(g(x)) = 6g(x) - 7 = x$. Solve for $g(x)$ to obtain $x/6 + 7/6$.

37. **(B)**. The side length of the hexagon is $54\sqrt{3}/6 = 9\sqrt{3}$. We can now break up the hexagon into six equilateral triangles with side length equal to that of the hexagon. Thus, the area is $6((9\sqrt{3})^2\sqrt{3}/4) = 729\sqrt{3}/4$.

38. **(C)**. To minimize z, we must make x and y as large as possible. Because x is at most 73 and $y \leq 152$, the smallest attainable value of z is $284 - 73 - 152 = 59$.

39. **(A)**. $g(2) = 13 \rightarrow f(3 + g(2)) = f(16) = 2(16)^2 - 6(16) = 416$

40. **(A)**. Converting to slope-intercept form, we find that the lines have slopes of $-2a/3b$ and -3. If they intersect at right angles, then their slopes have to be perpendicular, or mathematically speaking, $(-2a/3b)(-3) = -1$. Solving, we get $a/b = -1/2$ so $a^2/b^2 = 1/4$.

41. **(B)**. Since Mike's rate is $1/60$ and JJ's is $1/40$, then their rate working together is $1/60 + 1/40 = 1/24$ lawns per minute. So it will take them 24 minutes to mow one lawn.

42. **(E)**. By the exterior-angle formula, $(360)/(2x + 6) = x$. Rewrite this equation as $180 = x(x+3) \rightarrow x^2 + 3x - 180$. Factoring produces $(x - 12)(x + 15) = 0$. Thus $x = 12$ and the number of sides is $2(12) + 6 = 30$.

43. **(E)**. $a\left(\dfrac{de - bf}{ad - bc}\right) + b\left(\dfrac{af - ce}{ad - bc}\right) = \dfrac{ade - abf + abf - bce}{ad - bc} = \dfrac{e(ad - bc)}{ad - bc} = e$

44. **(B)**. $\displaystyle\sum_{n=1}^{12^2} n = \frac{12^2(12^2 + 1)}{2} = 10440$

45. **(B)**. Drop a perpendicular to the longest side of the triangle. Because the triangle is isosceles, this altitude bisects the side. By the Pythagorean Theorem, the length of the altitude is $\sqrt{10^2 - (15/2)^2} = 5\sqrt{7}/2$. Using the classic formula, the area of the triangle is $(1/2)(15)(5\sqrt{7}/2) = 75\sqrt{7}/4$.

46. **(B)**. Break up the problem into two cases. If a blue fish is transferred from A then the probability a fish taken from B is blue is $(2/6)(3/4) = 1/4$; otherwise, it's $(4/6)(2/4) = 1/3$. The total probability is $1/4 + 1/3 = 7/12$.

47. **(B)**. The length of the base does not change, as well as the height because l is parallel to BC. Thus, the area of the triangle stays the same.

48. **(D)**. $S = \dfrac{a_1}{1 - r} = \dfrac{12}{1 - 5/6} = 72$

49. **(D).** $AB = 10$, $\triangle ABC$ being a famous Pythagorean triple. By the Angle Bisector Theorem, $AC/AD = CB/DB \rightarrow 6/AD = 8/(10 - AD)$. Solve to get $AD = 30/7$.

50. **(E).** Use the definition of harmonic mean and the given information to obtain the system

$$\frac{2}{1/x + 1/y} = \frac{1}{2} \rightarrow \frac{1}{x} + \frac{1}{y} = 4$$
$$\frac{2}{1/(3x) + 1/(5y)} = 2 \rightarrow \frac{5}{x} + \frac{3}{y} = 15$$

Let $u = 1/x$ and $v = 1/y$ and the system becomes

$$u + v = 4$$
$$5u + 3v = 15$$

Solving this, we get the ordered pair $(u, v) = (3/2, 5/2)$. Thus, $xy = 1/(uv) = (2/3)(2/5) = 4/15$.