Mu Alpha Theta National Convention: Denver, 2001
Individual Test – Theta Division

1. In a plane, equilateral triangle *CAD* shares a side with, but does not overlap, isosceles triangle *ABC* where $AB = AC$. If $\angle ABC = 50°$, what is the measure of $\angle ADB$?

   (A) 10° (B) 20° (C) 40° (D) 60° (E) NOTA

2. El Nido Blue, a deep serene hue used often by struggling artists to paint ocean scenes, is made by mixing 3 parts of pure blue paint with 2 parts of seawater. How much seawater is in a 20 ml vial of El Nido Blue?

   (A) 8 ml (B) $6\frac{2}{3}$ ml (C) 10 ml (D) 12 ml (E) NOTA

3. Simplify $\sqrt{x^{3/4}}(x^6)^{1/3}$ for $x > 0$.

   (A) $x^{\frac{3}{2}}$ (B) $x^{\frac{5}{4}}$ (C) $x^{\frac{7}{8}}$ (D) $x^{\frac{9}{16}}$ (E) NOTA

4. Given that $9 \le x \le 12$ and $a \le \dfrac{x}{3} - 1 \le b$ describe the same set, evaluate: $b - a$.

   (A) 1 (B) 2 (C) 3 (D) 4 (E) NOTA

5. Given that $i - 2i^2 + 3i^3 - 4i^4 = a + bi$, where $i = \sqrt{-1}$ and *a* and *b* are real numbers, what is the value of $a^2 + b^2$?

   (A) 40 (B) 4 (C) 8 (D) 82 (E) NOTA

6. How many consecutive zeroes appear at the end of $2^{19} \times 3 \times 5^{21}$?

   (A) 18 (B) 19 (C) 20 (D) 21 (E) NOTA

7. At the University of Washington's Computer Resource Center, it costs three times as much to print a page with color ink than it does to print a page with black ink. One of Richard's reports includes a page of colored charts and two pages of black-and-white text, which, in total, cost 50 cents to print. How much would Richard have to pay to print an additional 50 copies of his color chart?

   (A) \$13.00 (B) \$14.00 (C) \$15.00 (D) \$16.00 (E) NOTA

8. Consider the arithmetic sequence -2, 1, 4, … What is the 49^{th} term?

   (A) 190 (B) 136 (C) 94 (D) 142 (E) NOTA

9. Convert the statement $\log_x y = z$ into exponential form.

(A) $y^z = x$ (B) $x^y = z$ (C) $x^z = y$ (D) $y^x = z$ (E) NOTA

10. Find the equation of the circle with center at (-3, 1) with a radius of $\sqrt{5}$.

(A) $(x-3)^2 + (y+1)^2 = 20$ (B) $(x+3)^2 + (y-1)^2 = 5$
(C) $(x-3)^2 + (y+1)^2 = \sqrt{5}$ (D) $(x+3)^2 + (y-1)^2 = 10$ (E) NOTA

11. Find the greatest prime number less than 149.

(A) 141 (B) 143 (C) 145 (D) 147 (E) NOTA

12. Triangle T_2 is formed by connecting the midpoints of an equilateral triangle T_1. Circles C_1 and C_2 are then circumscribed about T_1 and T_2, respectively. Find the ratio of the area of C_1 to the area of C_2.

(A) 3:1 (B) 4:1 (C) 9:1 (D) 2:1 (E) NOTA

13. Clymer leased a sports car for 2 weeks. The terms of the lease were a nonrefundable $50 down payment and he had to pay $30 per day for the length of the lease. In addition, he paid 10 cents per mile for all miles driven in excess of 1000. What was the total cost of the lease, given that Clymer drove the car 1600 miles?

(A) $320 (B) $420 (C) $530 (D) $600 (E) NOTA

14. Evaluate the expression $-3x^5 + 3x^4 - 9x^3 + x^2 + 2x + 5$ when $x = 2$.

(A) -347 (B) 221 (C) 37 (D) -107 (E) NOTA

15. Find three consecutive integers so that a third of the smallest plus half the middle added to twice the largest equals the sum of the three original integers.

(A) 5, 6, 7 (B) 12, 13, 14 (C) 14, 15, 16 (D) 9, 10, 11 (E) NOTA

16. What is the units digit of $2^{47} \times 3^{85} \times 7^{117} \times 11^{268}$?

(A) 8 (B) 4 (C) 2 (D) 0 (E) NOTA

17. Evaluate: $\log_7 \frac{1}{343} + \log_4 1024 - \log_5 \frac{1}{25}$

(A) 2 (B) 13 (C) 6 (D) 4 (E) NOTA

18. There are 20 married couples in a group. Find the number of ways of selecting a woman and a man who's not her husband.

(A) 190 (B) 90 (C) 100 (D) 380 (E) NOTA

19. What is the domain of the function $f(x) = e^{\ln(x^2+x-2)}$?

(A) $x \in (-\infty, -2) \cup (1, \infty)$ (B) $x < -1$ or $x > 2$
(C) $x \in (-\infty, \infty)$ (D) $-2 < x < 1$ (E) NOTA

20. How many integer squares are there between 81 and 2401, exclusive?

(A) 40 (B) 38 (C) 41 (D) 39 (E) NOTA

21. Evaluate: $\begin{bmatrix} 4 & 2 \\ -8 & 9 \end{bmatrix}\begin{bmatrix} 9 & -3 \\ 2 & 5 \end{bmatrix}$

(A) $\begin{bmatrix} 36 & -6 \\ -16 & 45 \end{bmatrix}$ (B) $\begin{bmatrix} 40 & -2 \\ -54 & 69 \end{bmatrix}$ (C) $\begin{bmatrix} 60 & -9 \\ -40 & 49 \end{bmatrix}$ (D) $\begin{bmatrix} 13 & -1 \\ -6 & 14 \end{bmatrix}$ (E) NOTA

22. How many ways are there to arrange three nickels and four quarters in a line so that quarters are on both ends?

(A) 5 (B) 35 (C) 60 (D) 10 (E) NOTA

23. An odd and an even number are picked at random from the first ten positive integers. Find the probability their sum is 11.

(A) $\frac{1}{9}$ (B) $\frac{2}{5}$ (C) $\frac{1}{5}$ (D) $\frac{2}{9}$ (E) NOTA

24. If $x + y = 7$ and $\frac{1}{x} + \frac{1}{y} = \frac{3}{8}$, what is the value of $x^2 + xy + y^2$?

(A) $\frac{91}{3}$ (B) $\frac{1}{9}$ (C) 45 (D) $\frac{371}{8}$ (E) NOTA

25. Find the solution set: $(3x+3)^{-1} > 3(20-5x)^{-1}$

(A) $x < \frac{11}{14}$ (B) $x < -1$ or $\frac{11}{14} < x < 4$
(C) $x > 4$ or $-1 < x < \frac{11}{14}$ (D) $-1 < x < 4$ (E) NOTA

26. Given that $\binom{n}{r} = \frac{n!}{(n-r)!r!}$, evaluate: $\log_4\left(\binom{1024}{0} + \binom{1024}{1} + \ldots + \binom{1024}{1023} + \binom{1024}{1024}\right)$

(A) 512 (B) 1024 (C) 256 (D) 10 (E) NOTA

27. David has a bag containing 6 fair coins and 4 double-headed coins. He takes a coin at random from the bag and tosses it in the air. Given the outcome of the toss was heads, what's the probability David picked a double-headed coin?

(A) $\frac{4}{7}$ (B) $\frac{6}{7}$ (C) $\frac{2}{5}$ (D) $\frac{4}{5}$ (E) NOTA

28. The following is a "proof" that 2 = 1. Which line does not follow logically from the one before it?

A. $x = y$
B. $x^2 = xy$
C. $x^2 - y^2 = xy - y^2$
D. $(x-y)(x+y) = y(x-y)$
E. $x + y = y$
F. Let $x = y = 1$. Then 2 = 1, and the proof is complete!

(A) B (B) C (C) E (D) F (E) NOTA

29. The conic with equation $4x^2 - y^2 - 8x + 2y = 1$ is rotated 90° clockwise about the origin. Find the equation of the new conic.

(A) $(y+1)^2 - \frac{(x-1)^2}{4} = 1$ (B) $\frac{(y-1)^2}{4} - (x-1)^2 = 1$
(C) $(y-1)^2 - \frac{(x+1)^2}{4} = 1$ (D) $\frac{(y+1)^2}{4} - (x+1)^2 = 1$ (E) NOTA

30. A random real number x is chosen between 6 and 10, inclusive. What is the probability that x is a solution to $\dfrac{4x^2+23x+15}{2x^2+11x+5}=\dfrac{4x^2-5x-6}{2x^2-3x-2}$?

(A) 0 (B) $\dfrac{2}{5}$ (C) $\dfrac{1}{4}$ (D) 1 (E) NOTA

31. A set of seven integer test scores from 0 to 100 inclusive has a mode of 28, a median of 40, and a mean of 51. What is the minimum possible value of the greatest element of the set?

(A) 69 (B) 71 (C) 75 (D) 77 (E) NOTA

32. Let f be a linear function of a real number x. Which of the following is always true [Note: $\Re$ denotes the set of real numbers.]:

(A) The inverse of f exists. (B) The domain of f is $\Re$.
(C) The range of f is $\Re$. (D) All of A-C (E) NOTA

33. Find the greatest integer k such that 2^k is less than 1,000,000.

(A) 18 (B) 19 (C) 20 (D) 21 (E) NOTA

34. The corners of a square with side 5 are cut off, producing a regular octagon. Find the octagon's side length.

(A) $5\sqrt{2}-5$ (B) $\dfrac{5\sqrt{2}+5}{3}$ (C) $\dfrac{10\sqrt{2}-5}{7}$ (D) $\dfrac{5}{3}$ (E) NOTA

35. For the ordered pair (a, b) such that $\begin{matrix} 64a^2-81b^2=2268 \\ a^2-b^2=45 \\ 8a+9b=126 \end{matrix}$, find a^2+b^3.

(A) 487 (B) 91 (C) 297 (D) 113 (E) NOTA

36. Find the maximum volume of a rectangular box with a total surface area of 24 square feet.

(A) $9\sqrt{3}$ ft^3 (B) 12 ft^3 (C) $6\sqrt{2}$ ft^3 (D) 8 ft^3 (E) NOTA

37. What is the probability that when two cards are drawn simultaneously from a standard, 52-card deck, they are an ace and a king?

(A) $\frac{16}{663}$ (B) $\frac{5}{221}$ (C) $\frac{1}{51}$ (D) $\frac{8}{663}$ (E) NOTA

38. A **lattice point** is an ordered pair (x, y) where x and y are integers. How many lattice points are inside the region satisfying the constraints $y < \frac{4}{3}x$, $y < -\frac{2}{3}x + 6$, and $y > 0$?

(A) 11 (B) 12 (C) 13 (D) 14 (E) NOTA

39. A magical hat contains all the integers. What is the smallest number of integers that must be chosen from the hat to ensure that either you have two numbers with a sum that is a multiple of 10 or you have two numbers with a difference that is a multiple of 10 (or both)?

(A) 7 (B) 6 (C) 5 (D) 4 (E) NOTA

40. What is the third term of an infinite geometric sequence with a sum of 27 and a common ratio of $\frac{1}{2}$?

(A) $\frac{9}{16}$ (B) $\frac{27}{8}$ (C) $\frac{9}{4}$ (D) $\frac{3}{4}$ (E) NOTA

41. Find the sum of all real solutions to $(x-2)(x-3)(x-4)(x-5) = 360$.

(A) 8 (B) 4 (C) 7 (D) 5 (E) NOTA

42. Find the length of the median to the shortest side of a triangle with sides 9, 4, and 7.

(A) $2\sqrt{7}$ (B) $\sqrt{61}$ (C) $2\sqrt{14}$ (D) $\sqrt{57}$ (E) NOTA

43. Find the magnitude of $\frac{10-24i}{4+3i}$, where i represents the imaginary unit.

(A) $\frac{38}{9}$ (B) $\frac{26}{5}$ (C) $\frac{14}{9}$ (D) $-\frac{14}{5}$ (E) NOTA

44. Given a recursive sequence $a_{n+1} = a_n(2-2a_n)$ with $a_1 = .40$, find $.50 - a_4$ in scientific notation.

(A) 6.30×10^{-3} (B) 1.28×10^{-6} (C) 8.00×10^{-4} (D) 1.32×10^{-6} (E) NOTA

45. Express 224_7 as a base 3 number.

(A) 215_3 (B) 2021_3 (C) 2111_3 (D) 11022_3 (E) NOTA

46. What is the minimum value of $f(x) = \dfrac{x^2 + \frac{1}{x}}{x^2 - \dfrac{1-\frac{1}{x^2}}{\frac{1}{x}+\frac{1}{x^2}}}$ on the interval $1 \le x \le 2$?

(A) 1 (B) $\dfrac{6}{5}$ (C) $\dfrac{3}{2}$ (D) 2 (E) NOTA

47. In triangle *ABC*, *BD* bisects angle *B*, where *D* lies on *AC*. If *AB* = 4, *AC* = 5, and *BC* = 6, what is the length of *AD*?

(A) $\dfrac{7}{3}$ (B) 2 (C) $\dfrac{5}{3}$ (D) $\dfrac{3}{2}$ (E) NOTA

48. A *diverse number* is a positive three-digit integer where the hundreds digit is prime, the tens digit is odd, and the units digit is even. How many diverse numbers have digits that sum up to 14?

(A) 9 (B) 11 (C) 13 (D) 15 (E) NOTA

49. Determine the area of the region in the Cartesian plane satisfying the equation $|x| + |y| < 20$.

(A) 400 (B) 800 (C) 1200 (D) 1600 (E) NOTA

50. Let $A = \begin{bmatrix} 4 & 1 \\ -7 & -3 \end{bmatrix}$, $B = \begin{bmatrix} 1 & -2 \\ 3 & 4 \end{bmatrix}$, $C = \begin{bmatrix} 5 & -4 \\ 8 & -6 \end{bmatrix}$, and $D = ABC$. Evaluate: $|D|$.

(A) -100 (B) -216 (C) -81 (D) -729 (E) NOTA