Mu Alpha Theta National Convention: Denver, 2001
Sequences & Series Topic Test – Alpha Division

1. What is the common ratio of the geometric sequence 1, 7, 49, …?

   (A) 7 (B) 6 (C) 42 (D) 25 (E) NOTA

2. The common difference of the arithmetic sequence 4, 12, 20, … is

   (A) 12 (B) $\frac{5}{3}$ (C) 8 (D) 3 (E) NOTA

3. Find the sum of the first 50 even natural numbers.

   (A) 2500 (B) 2550 (C) 650 (D) 1275 (E) NOTA

4. What is the sum of the first 48 odd positive integers?

   (A) 576 (B) 4656 (C) 2304 (D) 2352 (E) NOTA

5. Evaluate: $1^2 - 2^2 + 3^2 - 4^2 + 5^2 - 6^2 + 7^2 - 8^2$

   (A) 4 (B) -36 (C) 204 (D) 36 (E) NOTA

6. **Catalan Numbers** are defined explicitly by $C_n = \frac{1}{n+1}\binom{2n}{n}$ (with $n > 0$) and proves to be useful in solving certain counting problems. Find the product of the first three Catalan Numbers.

   (A) 4 (B) 6 (C) 8 (D) 10 (E) NOTA

7. Evaluate: $\sqrt{20+\sqrt{20+\sqrt{20+\cdots}}}$

   (A) 3 (B) 7 (C) -4 (D) 5 (E) NOTA

8. A sequence is defined explicitly by $a_n = 6n + 5(-1)^n$. What is the value of $a_3 + a_7$?

   (A) 40 (B) 50 (C) 60 (D) 70 (E) NOTA

9. A storeowner is setting up an impressive display of spinach cans to help with sales. The cans are to be placed on a stack with 10 levels, each level having one more can than the one above it. Half of the cans in stock will be used in his display, with 2 cans on the top level. How many cans does he have in stock?

(A) 55 (B) 65 (C) 130 (D) 110 (E) NOTA

10. Evaluate $\sum_{n=1}^{\sqrt{k}} n^2$, where k is a perfect square.

(A) $\frac{k(k+1)(2k+1)}{6}$ (B) $\frac{k(k+1)}{2}$

(C) $\frac{k^2(k^2+1)(2k^2+1)}{6}$ (D) $\frac{\sqrt{k}\left(\sqrt{k}+1\right)\left(2\sqrt{k}+1\right)}{6}$ (E) NOTA

11. Two sequences are defined as $a_n = 3^{n+1}$ and $b_n = n^4$. For what values of n is $a_n > b_n$?

(A) $n \geq 1$ (B) $n \geq 5$ (C) $n \geq 4$ (D) $n > 3$ (E) NOTA

12. If $V_k = \sum_{n=1}^{5} n^k$, evaluate $\sum_{k=1}^{3} V_k$.

(A) 145 (B) 295 (C) 300 (D) 430 (E) NOTA

13. To help ease the cost of his prescription drug payments, Wayne decides to join a Drug Club, which helps its members save money on their medication. Members must pay $10 for their first month and that fee goes up $2 each month. If Wayne stayed in the club for 13 months, how much money did he pay on dues alone?

(A) $309 (B) $276 (C) $299 (D) $286 (E) NOTA

14. Let T_n be the nth triangular number (assume 1 is the first) and S_n the nth square number (assume 1 is the first). Which of the following is equal to S_n?

(A) $\frac{T_n}{2}+n$ (B) $T_n - n^2$ (C) $2T_n - n$ (D) $\left(T_n\right)^2 - 1$ (E) NOTA

15. How many numbers must be inserted between 13 and 100 to make an arithmetic sequence with common difference 3?

(A) 28 (B) 29 (C) 30 (D) 31 (E) NOTA

16. Once upon a time, there were seven forests each housing seven owls. Each owl killed seven mice. If left alive, each mouse would have eaten seven ears of corn. When not eaten, each ear of corn produced seven pounds of grain. How many pounds of grain were saved due to the existence of the forests?

(A) 33614 (B) 2401 (C) 117649 (D) 16807 (E) NOTA

17. Which of the following is a possible ordered triplet (a, b, c) if $\sum_{i=0}^{500} 7^{2i} = \frac{7^a - b}{c}$?

(A) (501, 1, 6) (B) (1001, -1, 6)
(C) (500, -1, 48) (D) (1002, 1, 48) (E) NOTA

18. Find the sum of the first 14 terms of the arithmetic series $(x+1)+(2y)+(3x+y)+12+\ldots$.

(A) 245 (B) 378 (C) 315 (D) 355 (E) NOTA

19. A rubber ball dropped from a height of 20 meters rebounded on each bounce $\frac{5}{8}$ of the height from which it fell. How far (in meters) did it travel before coming to rest?

(A) 40 (B) $\frac{160}{3}$ (C) $\frac{220}{3}$ (D) $\frac{260}{3}$ (E) NOTA

20. Solve for x: $\sum_{i=x}^{15} (i-8)^2 = 140$.

(A) 7 (B) 8 (C) 9 (D) 10 (E) NOTA

21. Find the matrix equivalent to $\begin{bmatrix} 1 & \frac{1}{2} \\ \frac{1}{3} & \frac{1}{4} \end{bmatrix} + \begin{bmatrix} \frac{1}{2} & \frac{1}{6} \\ \frac{1}{12} & \frac{1}{20} \end{bmatrix} + \begin{bmatrix} \frac{1}{4} & \frac{1}{18} \\ \frac{1}{48} & \frac{1}{100} \end{bmatrix} + \cdots$

(A) $\begin{bmatrix} 2 & \frac{3}{4} \\ \frac{4}{9} & \frac{5}{4} \end{bmatrix}$ (B) $\begin{bmatrix} 2 & \frac{3}{2} \\ \frac{4}{9} & \frac{5}{16} \end{bmatrix}$ (C) $\begin{bmatrix} 2 & \frac{3}{4} \\ \frac{4}{3} & \frac{5}{16} \end{bmatrix}$ (D) $\begin{bmatrix} 2 & \frac{3}{4} \\ \frac{4}{9} & \frac{5}{16} \end{bmatrix}$ (E) NOTA

22. The fifth term of an arithmetic sequence is 4 and the *x*th term is 2504, where $x > 5$. Given that the common difference of this sequence is an integer, how many possible values are there for *x*?

(A) 15 (B) 8 (C) 30 (D) 16 (E) NOTA

23. A cube is inscribed in a sphere of radius 9. Another sphere is inscribed inside the cube and a second cube is inscribed in this sphere. If this pattern continues, what is the surface area of the 11th sphere?

(A) $\frac{4\pi}{81}$ (B) $\frac{4\pi}{729}$ (C) $\frac{4\pi}{243}$ (D) $\frac{4\pi}{2187}$ (E) NOTA

24. Find the sum of all positive 7-digit palindromes. Express your answer in scientific notation.

(A) 4.95×10^{10} (B) 4.54×10^{10} (C) 4.13×10^{10} (D) 4.01×10^{10} (E) NOTA

25. Evaluate the infinite series $\sum_{n=2}^{\infty}\left[\log_{2n+1}(2n+2)-\log_{2n+3}(2n+4)\right]$.

(A) $\frac{\ln 6}{\ln 5}$ (B) $\log_3 4$
(C) $\log_5 6-\log_7 8$ (D) Diverges (E) NOTA

26. Evaluate $\sum_{n=0}^{243}\left((-1)^n+(-1)^{\frac{n(n+1)}{2}}\right)$.

(A) 1 (B) 0 (C) 2 (D) -2 (E) NOTA

27. Starting with a circle of radius 1, a new circle is formed whose radius in units is numerically equal to the area of the previous circle in square units. What is the area of the 12th circle?

(A) π^{1023} (B) π^{4095} (C) π^{8191} (D) π^{2047} (E) NOTA

28. Let $i=\sqrt{-1}$. For any positive integer *n* greater than 1, which of the following is equal to $\sum_{k=0}^{n-1} e^{\frac{2\pi k}{n}i}$?

(A) $\sqrt{2}$ (B) 1 (C) 0 (D) -1 (E) NOTA

29. If n is odd and a_n is an arithmetic sequence with positive terms, find the maximum value of

$$\frac{a_{\frac{n+1}{2}}}{a_1+a_n}+\frac{a_1+a_n}{a_2+a_{n-1}}+\frac{a_2+a_{n-1}}{a_{\frac{n+1}{2}}}.$$

(A) 2 (B) $\frac{5}{2}$ (C) 3 (D) $\frac{7}{2}$ (E) NOTA

30. Evaluate $\sum_{N=1}^{729}\lfloor \log_3 N\rfloor$, where $\lfloor x\rfloor$ represents the greatest integer less than or equal to x.

(A) 2390 (B) 2482 (C) 3288 (D) 3056 (E) NOTA

31. What is the harmonic mean of the altitudes of a triangle whose area is numerically equal to its perimeter? Assume consistent units (i.e. length is units, area is square units).

(A) 9 (B) 12 (C) 3 (D) 6 (E) NOTA

32. Express $\sum_{j=1}^{999} j(j+1)(j+2)$ in terms of binomial coefficients $\binom{n}{r}$.

(A) $24\binom{1003}{3}$ (B) $6\binom{1001}{3}+2\binom{1001}{2}$

(C) $6\binom{1002}{4}$ (D) $\binom{1000}{3}+2\binom{1000}{2}+\binom{1001}{2}$ (E) NOTA

33. For which value of v does the equality $(1+2+3+\ldots+n)^2=1^v+2^v+3^v+\ldots+n^v$ hold true? (Assume $n > 3$.)

(A) No value (B) 4 (C) 3 (D) 2 (E) NOTA

34. Let a_n and b_n be two arithmetic progressions with $n>0$, the sum of the first n terms of which are $S_a(n)$ and $S_b(n)$, respectively. Given that $\frac{S_a(n)}{S_b(n)}=\frac{5n+9}{2n+8}$, find $\frac{a_{15}}{b_{15}}$.

(A) $\frac{7}{3}$ (B) $\frac{42}{19}$ (C) $\frac{15}{4}$ (D) $\frac{93}{46}$ (E) NOTA

35. A recursive sequence is defined by $a_n = 3a_{n-1} + 4a_{n-2} - 12a_{n-3}$ where $a_0 = 2$, $a_1 = 5$, and $a_2 = 13$. What's the remainder when a_{1492} is divided by 10?

(A) 2 (B) 7 (C) 3 (D) 5 (E) NOTA

36. Ryan and Brian decide to play a game. At the start of each turn, the player rolls a fair 6-sided die. If a prime number is rolled, that player loses (and the other player is declared the winner). If a 6 is rolled, that player wins. If neither of these events occurs, the next player's turn begins. The game continues until someone wins. Ryan decides to go first. What is the probability that a single game goes on indefinitely?

(A) 2/3 (B) 1/4 (C) 1/3 (D) 1/2 (E) NOTA

37. Which of the following is equal to $x^2 + 2x^3 + 3x^4 + 4x^5 + 5x^6 + 6x^7 + 5x^8 + 4x^9 + 3x^{10} + 2x^{11} + x^{12}$ for $|x| < 1$?

(A) $\dfrac{x^2(1-x)}{(1-x^2)}$ (B) $\left(\sum_{i=0}^{6} x^i\right)^2$ (C) $\dfrac{1-x^6}{1-x}$ (D) $\dfrac{x^2(1-x^6)^2}{(1-x)^2}$ (E) NOTA

38. Given that $\sum_{n=1}^{\infty} \dfrac{1}{n^2} = \dfrac{\pi^2}{6}$ and $\sum_{n=1}^{\infty} \dfrac{1}{(2n-1)^2} = \dfrac{\pi^2}{8}$, find the sum $\sum_{n=1}^{\infty} \dfrac{6-24n}{4n^4 - 4n^3 + n^2}$.

(A) $-4\pi^2$ (B) $-3\pi^2$ (C) $-2\pi^2$ (D) $-\pi^2$ (E) NOTA

39. Evaluate $1 + \dfrac{\binom{50}{1}}{2} + \dfrac{\binom{50}{2}}{2^2} + \ldots + \dfrac{\binom{50}{50}}{2^{50}}$.

(A) $\dfrac{3^{50}}{2^{50}}$ (B) $1 + \dfrac{2^{50}}{50}$ (C) $\left(\dfrac{3}{2}\right)^{50}\binom{100}{50}$ (D) $50\left(\dfrac{3}{2}\right)^{50}$ (E) NOTA

40. What is the integer part of the sum $\dfrac{1}{\sqrt{100}} + \dfrac{1}{\sqrt{101}} + \ldots + \dfrac{1}{\sqrt{399}} + \dfrac{1}{\sqrt{400}}$?

(A) 19 (B) 21 (C) 23 (D) 25 (E) NOTA