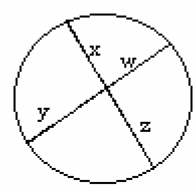
Mu Alpha Theta National Convention: Denver, 2001
Potpourri Topic Test Solutions – Euclidean Division

1. **C** The number of diagonals of a convex polygon is $\frac{n(n-3)}{2}$, for n = number of sides. Therefore, we get $\frac{10(7)}{2}$, or 35.

2. **A** Cross-multiply and we get $(x+2)(2-2x) = (x-1)(3x-1)$. Multiplying this out: $-2x^2 - 2x + 4 = 3x^2 - 4x + 1$. Simplifying: $5x^2 - 2x - 3 = 0$. This factors into: $(5x-3)(x-1) = 0$. Since we cannot use x = 1 (this makes the equation undefined due to the denominator for the left side), our answer is x = $-\frac{3}{5}$.

3. **D** Using the given two points, we use the equation of the line between those points. The slope of the line is $\frac{-3-6}{3-0} = \frac{-9}{3} = -3$. The point -slope equations gives us: $y - 6 = -3(x - 0)$, or $y = -3x + 6$. A line crossing the x-axis looks like (c, 0) for some c. Plugging this into our line equation we get: $0 = -3(c) + 6$. Then c = 2. Therefore, the line crosses at (2,0).

4. **E** By the geometry theorem regarding two chords intersecting inside a circle, xz=wy. Then 6*3=9*x → x = 2.



5. **B** Possible combinations of a sum of 7:
1+6 5+2
6+1 3+4
2+5 4+3 Out of 36 total combinations of the two dice, we have 6 we want. Therefore, the probability is $\frac{6}{36} = \frac{1}{6}$.

6. **D** Kim has her age = 6
Barry = 4 + Kim = 10
Lisa = $\frac{1}{2}$ Barry = 5.

7. **E** Translate the words into an expression. Recall that "is" → "=".
17(number) –4 = 10 (number)+10 .

8. **D** Solve for the number from #7. 17x-4 = 10x + 10
7x = 14 → x = 2.

9. **A** 6 shirts x 3 pairs of pants x 5 pairs of socks = total number of outfits = 90.

Mu Alpha Theta National Convention: Denver, 2001
Potpourri Topic Test Solutions – Euclidean Division

10. **B** Slope of a line: $\frac{\Delta y}{\Delta x} = \frac{0-2}{2-1} = \frac{-2}{1}$.

11. **C** Natural factors of 4288: 1, 2, 4, 8, 16, 32, 64, 67, 134, 268, 536, 1072, 2144, 4288.

12. **D** Distance formula: $\sqrt{(\Delta x)^2 + (\Delta y)^2} = \sqrt{(-4-2)^2 + (6-3)^2} = \sqrt{45} = 3\sqrt{5}$.

13. **C** For an n-sided regular polygon, each interior angle measures $\frac{(n-2)180}{n}$. Then

$$\frac{(13)180}{15} = 156.$$

14. **C** Equilangular → each angle = 60° (180° in a triangle). This also implies the triangle is equilateral (drop an altitude). Therefore, if the perimeter is 6 cm, each side is 2 cm long. The area is then $\frac{(2cm)^2\sqrt{3}}{4} = \sqrt{3}cm^2$.

15. **A** Let's call *x* how much she had left after buying the present. Then $\frac{1}{7}$ of *x* bought a cone and left her $12, which is another way of saying $\frac{6}{7}$ of *x* is $12. Solving for *x* means she had $14 after buying the present. Adding the $6 (cost of the present) means Nancy had $20. Stepping back further and using similar logic, we see that $\frac{2}{3}$ of pre-lunch funds is $20. Then she had $30 before heading off to lunch. So we say $\frac{5}{6}$ of what she had before socks is $30. She had $36 before buying the socks. Now we come to her first purchase: $\frac{4}{5}$ of the money she got from her grandmother is $36. This means she began with $45.

16. **C** $V_{cone} = \frac{1}{3}\pi r^2 h$. Height= 2radius → $V_{cone} = \frac{1}{3}\pi r^2 2r = \frac{2}{3}\pi r^3$. Radius=3.

Then $V_{cone} = \frac{2}{3}\pi(3)^3 = 18\pi$.

17. **D** Units digit pattern: $7^0 = 1$; $7^1 = 7$; $7^2 = 9$; $7^3 = 3$.

The pattern repeats every cycle of four. How many cycles in this problem? $4\overline{)61.0}$ = $15R1$.
The remainder of 1 means we use the second number in our cycle (if it were even we would use the first), which is 7.

18. A $N = \frac{1}{2}N + 9 - \sqrt[4]{81^{(+)}}$

$\frac{1}{2}N = 6 \rightarrow N = 12$.

19. **D** $1101_2 = 2^3 2^2 2^1 2^0$

1 1 0 1. Multiplying each digit by its positional weight and add: 1(8)+1(4)+1(1)=13.

20. **C** Four aces & 12 hearts (ace of hearts already counted)= 16 chances.
$\frac{16}{52} = \frac{4}{13}$.

21. **C** 416 -Take all the digits and one of the repeats
\- 41 -Subtract the nonrepeated part
375

$\frac{375}{900} = \frac{5}{12}$ Put answer into a fraction with denominator's zereos = #of nonrepeated digits.

22. **C** Jane got 320 points on the first four tests. The overall average is then:
$\frac{320 + x + y}{6} = 90$. This implies $x+y=220$. Then the average of the last two tests must be 110. It looks hopeless for Jane to get an 'A' this semester!

23. **A** Large one, four using the midpoints of the outside lines, twelve singles.

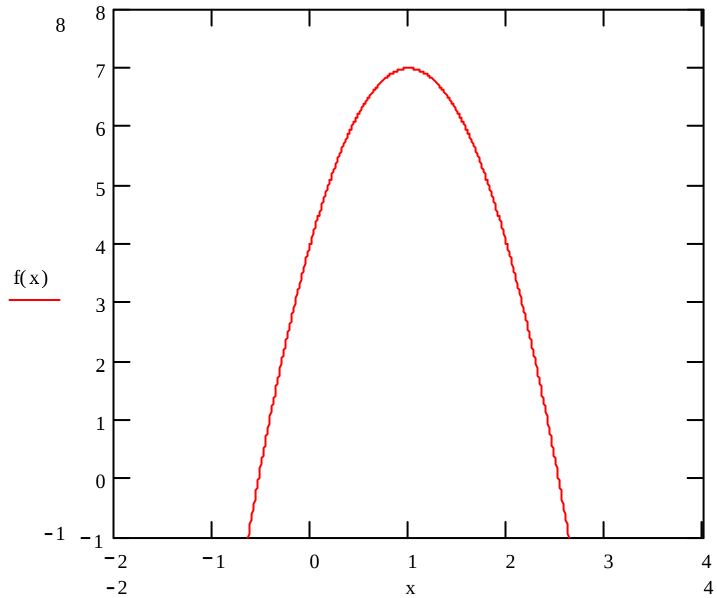
24. **B** $23_4 = 11_{10} = 102_3$.

25. **A** $\dfrac{\frac{15}{20} - \frac{12}{20}}{\frac{1}{6} - \frac{4}{6}} = \dfrac{\frac{3}{20}}{-\frac{3}{6}} = \frac{3}{20}(-\frac{6}{3}) = -\frac{6}{20} = -\frac{3}{10}$.

26. **B** We can eliminate all the x's by adding the two equations: $5y = 25 \rightarrow y = 5$.
Substituting back into one of the equations, we find $x = -1$.

Mu Alpha Theta National Convention: Denver, 2001
Potpourri Topic Test Solutions – Euclidean Division

27. **C** $f(x) = 4 - 3x^2 + 6x$.



28. **D** Slope: $\frac{8-5}{5-3} = \frac{3}{2}$. Point-slope equation: $y - 5 = \frac{3}{2}(x-3)$. Simplifying it: $-3x + 2y = 1$.

29. **B** Plugging *r* in: $6 - 8q + 6 = -4$ → $q = 2$.

30. **B** *f*(2) = 1. Then g(f(2)) = 5.

31. **E** $\sqrt{98} = 7\sqrt{2}$, $\sqrt{27} = 3\sqrt{3}$, $\sqrt{75} = 5\sqrt{3}$. Adding up like irrationals: $7\sqrt{2} + 8\sqrt{3}$.

32. **B** Let $x = \sqrt{6\sqrt{6\sqrt{6...}}}$, the answer. Then $x = \sqrt{6x}$, for in this infinite product, it doesn't matter where x actually starts (or stops!). Then $x^2 = 6x$ → $x = 0$, or $x = 6$. Since the former cannot be true by intuition, $x = 6$.

33. **A** Mode: most common = 5. Mean = average = 9. Median = In ordered set, middle number = 8.

34. **D** Arithmetic sum = $\frac{n}{2}(a_1 + a_n)$. Here, n = 50 terms, $a_1 = -100$, $a_n = -2$.

Sum = -2550.

35. **A** Lucy = Jake +2. (1)
L+5=4(J-5) (2) By (1), J = L-2. Substitute into (2) → L+5 = 4(L-2-5)

Then Lucy = 11.

36. **E** Samantha's rate is $\frac{45\,planes}{75\min} = \frac{3\,planes}{5\min}$. Marlene's rate is $\frac{60\,planes}{90\min} = \frac{2\,planes}{3\min}$. Then their rate together is $\frac{3}{5} + \frac{2}{3} = \frac{19\,planes}{15\min}$. So to get 225 planes, it will take them $\frac{15\min}{19\,planes} * 225\,planes = \frac{3375}{19}\min$ → $\frac{225}{76}\,hours$.

37. **C** Typing in "rectangular coordinate system" or "Descartes" into any web search engine will lead to many sites confirming this, along with many history of math books. One such website – http://plato.phy.ohiou.edu/~dutta/notes/node5.html

38. **B** Typing in "inventor of Calculus" or "Newton/Leibniz" into any web search engine will lead to many sites confirming this, along with many history of math books. One such website – www.encyclopedia.com/articles/02143.html

39. **D** The third term, a_3, = $\frac{5}{32}$ which also = $a_1 r^{3-1}$. So $\frac{5}{32} = a_1(\frac{5}{8})^2$. This means the first term is $\frac{2}{5}$. And using the same formula, $a_5 = \frac{2}{5}(\frac{5}{8})^4 = \frac{125}{2048}$.

40. **C** 20% of 32,160 = 6432, which belong to season ticket holders. Also, 6432 remain empty. The remaining number of seats is 19,296. 25% of the remaining number = 4824, buy tickets for each game. Half of the fans who buy tickets for each game= 2412. We want this number plus the number of season tickets ticket holders: 2412+6432= 8844.