Mu Alpha Theta National Convention: Denver, 2001
Advanced Calculus Topic Test – Mu Division

1. Evaluate: $\int_0^{\pi/2} \cos^2 t \; dt$

(A) $\frac{1}{4}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{2}$ (D) $\frac{1}{2}$ (E) NOTA

2. Evaluate: $\lim_{(r,\theta)\to(-1,3)} \left(\frac{r^2 + 2\theta - r}{\theta^2 - 7r^5} \right)$

(A) $\frac{1}{2}$ (B) 4 (C) 3 (D) $-\frac{2}{425}$ (E) NOTA

3. Determine $\frac{\partial M}{\partial v}$ if $M(u,v) = 2vu^2 - 3\cos(uv)$.

(A) $4uv - 3v\sin(uv)$ (B) $2u^2 + 3u\sin(uv)$

(C) $2u^2 + 3\sin(uv)$ (D) $4uv + 3v\sin(uv)$ (E) NOTA

4. Find $\frac{dy}{dx}$ given that $x^2 + \sin y = xy^3$.

(A) $\frac{2x - y^3}{\cos y + 3xy^2}$ (B) $\frac{2x}{3y^2 + \cos y}$

(C) $\frac{2x - 3y^2}{3y^2 - \cos y}$ (D) $\frac{2x - y^3}{3xy^2 - \cos y}$ (E) NOTA

5. Given that $A = \frac{ab}{2}$, express $(A_a)(A_b)$ in terms of A.

(A) A (B) $\frac{A}{4}$ (C) $\frac{A}{2}$ (D) $2A$ (E) NOTA

6. Let f be a continuous function of n variables. How many orders of integration are there for f?

(A) 1 (B) $(n-1)!$ (C) $n!$ (D) $\frac{n!}{2}$ (E) NOTA

7. What is the directional derivative of the function $z = x^2 + y^3 - 2xy$ at the point (1, 2, 5) as y increases and x remains constant?

(A) 8 (B) –2 (C) 1 (D) 10 (E) NOTA

8. Let $f(x, y) = x^3 \sin(y^2) - 2y\cosh(2x)$. Evaluate $\dfrac{f_{xy}(220, 284)}{f_{yx}(220, 284)}$.

(A) 1 (B) $\dfrac{1}{2}$ (C) $\dfrac{2}{e}$ (D) $\dfrac{3}{4}$ (E) NOTA

9. A triangle in the xy-plane whose vertices are (0, 0), (1, 0), and (0, 1) has density given by $\rho(x, y) = x + y$. Where is the center of mass of this triangle?

(A) $\left(\dfrac{1}{2}, \dfrac{1}{2}\right)$ (B) $\left(\dfrac{1}{3}, \dfrac{1}{3}\right)$ (C) $\left(\dfrac{3}{8}, \dfrac{3}{8}\right)$ (D) $\left(\dfrac{2}{3}, \dfrac{2}{3}\right)$ (E) NOTA

10. Which double integral represents the area of the region in the first quadrant bounded by the graphs of $y = 4 - x^2$, $y = 3x$, and the y-axis?

(A) $\int_0^1 \int_{3x}^1 dx\,dy + \int_0^1 \int_1^{4-x^2} dx\,dy$ (B) $\int_0^1 \int_{3x}^{4-x^2} dy\,dx$

(C) $\int_0^4 \int_{y/3}^{\sqrt{4-y}} dx\,dy$ (D) $\int_0^3 \int_{y/3}^{\sqrt{4-y}} dy\,dx$ (E) NOTA

11. Given that $w = x^3 + 2y^2$, where $x = 2\sin t$ and $y = 5\cos 2t$, find $\dfrac{dw}{dt}$ at $(t, x, y) = \left(\dfrac{\pi}{6}, 1, \dfrac{5}{2}\right)$.

(A) $-47\sqrt{3}$ (B) 13 (C) $-4\sqrt{3}$ (D) –450 (E) NOTA

12. For $D(x, y) = x^2 y$, what is the value of $D_x(2,0) + D_y(0,1)$?

(A) 2 (B) 1 (C) 0 (D) 3 (E) NOTA

13. What is the volume enclosed by the graphs of $x^2 + y^2 + z = 18$ and $x^2 + y^2 - z = 0$?

(A) $\dfrac{81\pi}{2}$ (B) 72π (C) 36π (D) 81π (E) NOTA

14. Find the gradient of $T(x, y) = 6x^7 + \cos^2 y - 3xy$.

(A) $\dfrac{3x^8 + 2y + \sin 2y - 3x^2 y^2}{4}$ (B) $\left(42x^6 - 3y\right)\mathbf{i} - \left(\sin 2y + 3x\right)\mathbf{j}$

(C) $42x^6 - 3y - \sin 2y - 3x$ (D) $\left(-\sin 2y - 3x\right)\mathbf{i} + \left(42x^6 - 3y\right)\mathbf{j}$ (E) NOTA

15. What is the equation of the tangent plane to the surface $2x^3 - y^2 + 9\sqrt{z} = 25$ at (1, 2, 9)?

(A) $12x - 8y + 3z = 23$ (B) $12x - 8y + z = -6$

(C) $6x - 4y + z = 7$ (D) $6x - 8y + 9z = 15$ (E) NOTA

16. The surfaces $x^2 + y^2 + z^3 = 6$ and $z = x - y$ intersect at many points, including P = (2, 1, 1). Which of the following is a set of parametric equations for the line tangent to both surfaces at P?

(A) $x = 2$, $y = 6t + 1$, $z = 1 - 6t$ (B) $x = 4t + 2$, $y = 2t + 1$, $z = 3t + 1$ (C) $x = 2 - t$, $y = 1 - 7t$, $z = 6t + 1$ (D) $x = t + 2$, $y = 1 - t$, $z = 1 - t$ (E) NOTA

17. Find the surface area of the portion of the plane $2x - y + z = 8$ that is above the diamond in the *xy*-plane given by $|x| + |y| \le 6$.

(A) $36\sqrt{2}$ (B) 144 (C) $72\sqrt{6}$ (D) $144\sqrt{5}$ (E) NOTA

18. What is the directional derivative of the function $z = \operatorname{Arctan}(x) + \operatorname{Arctan}(y)$ at the point $\left(1, 1, \dfrac{\pi}{2}\right)$ as x and y increase such that $24x - 7y = 17$?

(A) $\arctan\dfrac{24}{7}$ (B) $\dfrac{25\pi}{2}$ (C) $\dfrac{31}{50}$ (D) $\dfrac{29\sqrt{2}}{25}$ (E) NOTA

19. Given that $u = xy + yz + xz$, $x = st$, $y = e^{st}$, and $z = t^2$, find $\dfrac{\partial u}{\partial s}$ at $(s, t) = (0, 1)$.

(A) 0 (B) 1 (C) 2 (D) 3 (E) NOTA

20. Reverse the order of integration of $\int_{1}^{10}\int_{-\sqrt{y-1}}^{0}\sqrt{xy}\;dx\,dy$.

(A) $\int_{-\sqrt{y-1}}^{0}\int_{0}^{10}\sqrt{xy}\;dy\,dx$ (B) $\int_{-3}^{0}\int_{x^2+1}^{10}\sqrt{xy}\;dy\,dx$

(C) $\int_{-\sqrt{y-1}}^{0}\int_{1}^{10}\sqrt{xy}\;dy\,dx$ (D) $\int_{-3}^{0}\int_{0}^{9-x^2}\sqrt{xy}\;dy\,dx$ (E) NOTA

21. Evaluate: $\lim_{(m,n)\to(0,0)}\dfrac{m^3+n^3}{m^2+n^2}$

(A) Undefined (B) 0 (C) 1 (D) 2 (E) NOTA

22. Which of the following is a potential function for $\mathbf{V}(x,y)=\left(2xy+y^2\right)\mathbf{i}+\left(x^2+2xy\right)\mathbf{j}$?

(A) $4(x+y)$ (B) $2x+2y$

(C) yx^2+xy^2 (D) $(x+y)^2+2xy$ (E) NOTA

23. What is the divergence of the vector field $\mathbf{G}(x,y,z)=\left(zx^3\right)\mathbf{i}-\left(2xz\right)\mathbf{j}+\left(yz\right)\mathbf{k}$ at (5, 12, 13)?

(A) 987 (B) 1962 (C) 1651 (D) 142 (E) NOTA

24. Use first-order differentials for the function $h(a,b)=\sqrt{a^2+b^2}$ at (a, b) = (3, 4) to approximate $h(3.1,4.1)$.

(A) $\dfrac{1287}{250}$ (B) $\dfrac{257}{50}$ (C) $\dfrac{126}{25}$ (D) $\dfrac{513}{100}$ (E) NOTA

25. What is the maximum value of the function $A=4xy$ under the constraint $\dfrac{x^2}{49}+\dfrac{y^2}{16}=1$?

(A) 56 (B) 42 (C) 28 (D) 14 (E) NOTA

26. Which of the following is an integrating factor of the linear differential equation $\frac{dy}{dx} + y \ln x = \frac{1}{\sqrt{x^2+1}}$?

(A) x (B) $x+\sqrt{x^2+1}$ (C) $\ln x$ (D) $x^x e^{-x}$ (E) NOTA

27. Which of the following is a set of symmetric equations for the line perpendicular to the surface $2x^2 + y^2 - 9z^4 = 8$ at (–2, 3, 1)?

(A) $\frac{x+2}{19} = \frac{y-3}{8} = \frac{1-z}{12}$ (B) $\frac{x+2}{108} = \frac{y-3}{67} = \frac{1-z}{15}$

(C) $\frac{x+2}{80} = \frac{3-y}{135} = \frac{z-1}{27}$ (D) $\frac{x+2}{4} = \frac{3-y}{3} = \frac{z-1}{18}$ (E) NOTA

28. Rewrite the triple integral $\int_0^2 \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_0^1 \sqrt{x^2+y^2}\ dz\,dy\,dx$ using cylindrical coordinates.

(A) $\int_{-\pi/2}^{\pi/2} \int_0^2 \int_0^1 r^2\ dz\,dr\,d\theta$ (B) $\int_0^{\pi} \int_0^2 \int_0^1 r\ dz\,dr\,d\theta$

(C) $\int_0^{\pi/2} \int_0^4 \int_0^1 r^2\ dz\,dr\,d\theta$ (D) $\int_{-\pi/2}^{\pi/2} \int_0^2 \int_0^1 r^3\ dz\,dr\,d\theta$ (E) NOTA

29. Evaluate: $\lim_{n\to\infty}\left[\int_{1/(n+1)^2}^1 \cdots \int_{1/16}^1 \int_{1/9}^1 \int_{1/4}^1 dx_1\,dx_2 \cdots dx_{n+1}\right]$

(A) ∞ (B) $\frac{1}{2}$ (C) 0 (D) 1 (E) NOTA

30. Evaluate the line integral of the vector field $\mathbf{W}(x,y) = \left(e^y + y^2\cos x\right)\mathbf{i} + \left(xe^y + 2y\sin x\right)\mathbf{j}$ along the graph of $y=\sin x$ from (0, 0) to $\left(\frac{3\pi}{2}, -1\right)$.

(A) $\frac{3\pi e}{2}+1$ (B) $\frac{3\pi}{2e}+1$ (C) $\frac{3\pi}{2e}-1$ (D) $\frac{3\pi e}{2}-1$ (E) NOTA

31. Find the Jacobian $\dfrac{\partial(x,y)}{\partial(u,v)}$ for the change of variables $u = 4x + y$ and $v = 4x - y$.

(A) $-\dfrac{1}{16}$ (B) $-\dfrac{1}{8}$ (C) $\dfrac{1}{16}$ (D) $\dfrac{1}{8}$ (E) NOTA

32. Change the order of integration of $\int_0^2 \int_0^{4-2x} \int_0^{4-y-2x} \sqrt{xyz}\; dz\,dy\,dx$ to $dx\,dy\,dz$.

(A) $\int_0^2 \int_0^{4-y} \int_0^{4-y-2x} \sqrt{xyz}\; dx\,dy\,dz$ (B) $\int_0^{4-y-2x} \int_0^{4-2x} \int_0^2 \sqrt{xyz}\; dx\,dy\,dz$

(C) $\int_0^2 \int_0^{4+2x} \int_0^{4-\frac{y}{2}+\frac{x}{2}} \sqrt{xyz}\; dx\,dy\,dz$ (D) $\int_0^4 \int_0^{4-z} \int_0^{2-\frac{y}{2}-\frac{z}{2}} \sqrt{xyz}\; dx\,dy\,dz$ (E) NOTA

33. Evaluate $\int_C \left(3x^2 y + \cos x\right) dx + \left(x^3 + 4xy^3 + \sin 5y\right) dy$, where C is the path from (0, 0) to (1, 1) along the graph of $y = x^2$, from (1, 1) to (0, 1) along the line $y = 1$, and from (0, 1) to the origin along the y-axis.

(A) $\dfrac{8}{9}$ (B) $\dfrac{6}{7}$ (C) $\dfrac{4}{5}$ (D) $\dfrac{2}{3}$ (E) NOTA

34. What is the flux of the vector field $\mathbf{S}(x,y,z) = (2x+3y)\,\mathbf{i} + (5x+y)\,\mathbf{j} - (2z+1)\,\mathbf{k}$ across the surface of the sphere given by $x^2 + y^2 + z^2 = 81$?

(A) 972π (B) 81π (C) 324π (D) 18π (E) NOTA

35. Solve the differential equation $y'' - 8y' + 15y = 0$, where y is a function of x and a and b are arbitrary constants.

(A) $y = ae^{-3x} + be^{-5x}$ (B) $y = ae^{3x} + be^{5x}$

(C) $y = a\cos 3x + b\sin 5x$ (D) $y = ae^{8x} + bxe^{8x}$ (E) NOTA

36. Let $\mathbf{E}(x,y,z) = \left(2\sin xyz + x^3 e^{y^2}\right)\mathbf{i} + \left(y - 2z^2 + 1\right)\mathbf{j} + \left(9e^{y^2-4}\right)\mathbf{k}$. Find $\operatorname{div}(\mathbf{curl}\,\mathbf{E})$.

(A) 3 (B) 2 (C) 1 (D) 0 (E) NOTA

37. A critical point of $C(x, y) = x^3 - 3xy + y^3$ is (1, 1, –1). This point is

(A) a saddle point (B) a relative maximum

(C) a relative minimum (D) nothing (E) NOTA

38. Evaluate $\iiint_Q z \, dV$, where Q is the solid bounded by the first-octant portion of the cylinder $z^2 + y^2 = 9$, $x = 0$, and the plane $y = 3x$.

(A) $\frac{17}{24}$ (B) $\frac{153}{24}$ (C) $\frac{27}{8}$ (D) $\frac{243}{8}$ (E) NOTA

39. A planar lamina consists of the third-quadrant portion of the circle $x^2 + y^2 - 25 = 0$ where the density at the point (x, y) is numerically equal to the distance between the point and the origin. Find the mass of the lamina.

(A) $\frac{25\pi}{4}$ (B) $\frac{125\pi}{6}$ (C) $\frac{5\pi}{2}$ (D) $\frac{625\pi}{4}$ (E) NOTA

40. For two functions $y_1(t)$ and $y_2(t)$, the **Wronskian** of y_1 and y_2 at t is denoted by $W(y_1, y_2)(t)$ and is equal to the determinant of $\begin{pmatrix} y_1(t) & y_2(t) \\ y_1'(t) & y_2'(t) \end{pmatrix}$. Find $W(t, te^t)(2)$.

(A) e^2 (B) $2e^2$ (C) $3e^2$ (D) $4e^2$ (E) NOTA