Mu Alpha Theta National Convention: Denver, 2001
Analytic Geometry Topic Test – Open Division

1. The point (–3, 2) lies in which quadrant?

   (A) I (B) II (C) III (D) IV (E) NOTA

2. Where is the midpoint of (–5, 6) and (1, 2)?

   (A) (–4, 8) (B) (–2, 4) (C) (–3, 4) (D) (–3, 2) (E) NOTA

3. What is the length of the line segment connecting (3, 4) and (–8, 7)?

   (A) $\sqrt{157}$ (B) $\sqrt{146}$ (C) $\sqrt{34}$ (D) $\sqrt{130}$ (E) NOTA

4. What is the slope of the line that passes through (11, 8) and (9, 13)?

   (A) $-\frac{5}{2}$ (B) $\frac{2}{5}$ (C) $\frac{5}{2}$ (D) $-\frac{2}{5}$ (E) NOTA

5. The equation for the axis of symmetry of $x = 2y^2 - 12y + 1$ is

   (A) $y = -3$ (B) $x = 3$ (C) $y = 3$ (D) $x = -3$ (E) NOTA

6. What are the asymptotes of the hyperbola $\frac{(y+1)^2}{16} - \frac{(x-1)^2}{4} = 1$ in slope-intercept form?

   (A) $y = -2x + 3$, $y = 2x - 1$ (B) $y = 2x - 3$, $y = -2x + 1$ (C) $y = -2x - 3$, $y = 2x + 1$ (D) $y = 2x + 3$, $y = -2x - 1$ (E) NOTA

7. Find the volume of a sphere whose equation is $x^2 + y^2 + z^2 - 10x + 6y + 8z + 1 = 0$.

   (A) $\frac{1372\pi}{3}$ (B) $\frac{10000\pi\sqrt{2}}{3}$ (C) $\frac{500\pi}{3}$ (D) 196π (E) NOTA

8. Where is the center of the circle given by $x^2 + y^2 - 6x + 14y = 400$?

   (A) (6, –14) (B) (3, –7) (C) (–6, 14) (D) (–3, 7) (E) NOTA

9. The graph of the polar curve $r = \frac{34}{3 - 4\sin\theta}$ is a(n)

   (A) ellipse (B) line (C) parabola (D) hyperbola (E) NOTA

10. Find the equation of a parabola with vertex at (1, –4) and directrix at $x = -3$.

(A) $x - 1 = 8y(y + 4)$ (B) $x = 16y^2 + 128y + 257$

(C) $16x = y^2 + 8y + 32$ (D) $y = 16x^2 - 32x + 12$ (E) NOTA

11. A line of positive slope and the x-axis meet at a 57° angle. To the nearest integer, what is the slope of this line?

(A) 2 (B) 0 (C) 1 (D) 88 (E) NOTA

12. Find the value of hk if $3x + ky + 2 = 0$ and $5x - y + h = 0$ determine the same line.

(A) $-\frac{2}{3}$ (B) $-\frac{5}{3}$ (C) –2 (D) $-\frac{7}{3}$ (E) NOTA

13. Find the area of the region common to the circles $(x-3)^2 + (y-12)^2 = 144$ and $(x+4)^2 + (y+12)^2 = 169$.

(A) 0 (B) 25π (C) 13π (D) 12π (E) NOTA

14. The focus of $y = -\frac{x^2}{2} - x + \frac{1}{2}$ is (a, b). Find the value of $a + b$.

(A) $-\frac{3}{2}$ (B) $-\frac{1}{2}$ (C) $\frac{1}{2}$ (D) $\frac{3}{2}$ (E) NOTA

15. What is the equation of the square inscribed in the circle $x^2 + y^2 = 81$ whose vertices are on the coordinate axes?

(A) $|x| + |y| = 9$ (B) $|x| + |y| = 9\sqrt{2}$

(C) $|x| + |y| = 81$ (D) $|x| + |y| = 3\sqrt{2}$ (E) NOTA

16. A circle centered at the origin is tangent to the line $\frac{x}{7} + \frac{y}{24} = 1$. What is the length of the radius of the circle?

(A) $\frac{168}{31}$ (B) $\frac{168}{25}$ (C) $\frac{25}{2}$ (D) $\frac{625}{31}$ (E) NOTA

17. The graph of $C(x) = \begin{cases} -x^2 + 8x - 12 & x \le 3 \\ -x^2 + 4x & x \ge 3 \end{cases}$ is a(n)

(A) hyperbola (B) catenary (C) astroid (D) parabola (E) NOTA

18. The foci of the conic $9x^2 + 4y^2 + 36x - 24y + 36 = 0$ lies along the line given by

(A) $x = -2$ (B) $y = \sqrt{5}$ (C) $x = 2$ (D) $y = -\sqrt{5}$ (E) NOTA

19. Suppose Lester starts at the origin and walks to one focus of the ellipse $\frac{x^2}{1681} + \frac{y^2}{81} = 1$, then to any point on the ellipse, then to the other focus and finally, back to the origin. How far has Lester walked, assuming he walks in straight lines?

(A) 121 (B) 98
(C) 162 (D) cannot be determined (E) NOTA

20. Which of the following points is closest to the *yz*-plane?

(A) (20, 20, 2) (B) (13, 1, –15) (C) (–12, 6, –7) (D) (19, –5, 8) (E) NOTA

21. ΔABC has vertices $A(2, 4, 5)$, $B(8, -2, 7)$, and $C(10, 0, -1)$. What is the length of the median to side AC?

(A) 7 (B) $3\sqrt{5}$ (C) $\sqrt{66}$ (D) $\sqrt{29}$ (E) NOTA

22. What is the name of the shape of the polar graph of $r = 3 - 3\sin\theta$?

(A) limacon (B) lituus (C) 2-leaved rose (D) cardioid (E) NOTA

23. What is the volume of the solid formed when the first-quadrant portion of the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ is revolved about the *x*-axis?

(A) $\frac{22\pi}{3}$ (B) 8π (C) $\frac{128\pi}{9}$ (D) 12π (E) NOTA

24. What is the eccentricity of the graph of $\frac{x^2}{16} - \frac{y^2}{9} = 1$?

(A) $\frac{3}{2}$ (B) $\frac{\sqrt{13}}{2}$ (C) $\frac{5}{4}$ (D) $\frac{\sqrt{7}}{4}$ (E) NOTA

25. How many parabolas pass through the points (3, 5), (1, 4), and (-3, 8)?

(A) 0 (B) 1 (C) 2 (D) infinitely many (E) NOTA

26. A line passes through the point P (–3, 1) and is tangent to the circle $2x^2 + 2y^2 + 3x - y = 5$ at Q. Determine the distance between P and Q.

(A) $2\sqrt{2}$ (B) $\frac{\sqrt{10}}{2}$ (C) $\frac{5\sqrt{2}}{2}$ (D) $\frac{3}{2}$ (E) NOTA

27. The graph of the surface $16x^2 - y^2 + 25z^2 = 144$ is a(n)

(A) elliptic hyperboloid (B) hyperbolic paraboloid
(C) hyperboloid of one sheet (D) hyperboloid of two sheets (E) NOTA

28. Find all values of k so that it's impossible to draw a circle passing through the points (1, 2k), (3k, 4), and (5, 6k).

(A) 2 and $-\frac{2}{3}$ (B) $\frac{1}{3}$ and $\frac{3}{5}$ (C) 1 and $-\frac{4}{3}$ (D) $\frac{2}{3}$ and $\frac{6}{5}$ (E) NOTA

29. What is the largest possible number of intersection points of two graphs in the Cartesian plane which are both non-degenerate conic sections, assuming the two graphs are not identical?

(A) 4 (B) 5 (C) 6 (D) infinitely many (E) NOTA

30. Which of the following are descriptions of hyperbolae?

I. Given points A and B, the locus of points P such that $PA - PB = 4$
II. Given point A and line L, the locus of points which are twice as far from L as they are from A
III. The graph of $xy = 1$ in the Cartesian plane

(A) I only (B) I & II only (C) I & III only (D) I, II, & III (E) NOTA

31. From which of the following points could one draw tangents of equal length to the circles $x^2 + y^2 + 8x + 12y - 28 = 0$ and $x^2 + y^2 - 4x - 12y + 20 = 0$?

(A) (–8, 2) (B) (–13, 3) (C) (0, 4) (D) (5, –7) (E) NOTA

32. What are the coordinates of the focus of the parabola $y = 2x^2 - 12x + 25$?

(A) $\left(3, \frac{57}{8}\right)$ (B) $\left(\frac{25}{8}, 7\right)$ (C) $\left(3, \frac{15}{2}\right)$ (D) $\left(\frac{7}{2}, 7\right)$ (E) NOTA

33. Which of the following is an equation for the area of an ellipse with a minor axis of length *m* and a major axis of length *M*?

(A) πmM (B) $4\pi\left(\frac{mM}{m+M}\right)^2$ (C) $\pi\left(\frac{m+M}{2}\right)^2$ (D) $\frac{\pi}{4}\left(\frac{m+M}{mM}\right)^2$ (E) NOTA

34. For nonzero integer *n*, let $P(n)$ equal the number of petals on the rose curve defined by the polar graph of the equation $r = \cos n\theta$. Find the value of $\sum_{n=1}^{20} P(n)$.

(A) 320 (B) 210 (C) 310 (D) 220 (E) NOTA

35. What is the area of the convex pentagon with vertices (0, 6), (4, 3), (2, –1), (–1, –2), and (–2, 2)?

(A) $\frac{57}{2}$ (B) 22 (C) $\frac{63}{2}$ (D) 35 (E) NOTA

36. *P* is a point on an ellipse the foci of which are four units apart and the major axis of which is eight units long. Let *x* and *y* be the distances from *P* to each of the foci. What is the smallest possible value of $x^2 + y^2$?

(A) 16 (B) 32 (C) 40 (D) 56 (E) NOTA

37. A parabola with an equation of the form $y = ax^2 + bx + c$ passes through the points (2, 9), (3, 16), and (4, 27). What is the sum of *a*, *b*, and *c*?

(A) 3 (B) 4 (C) 5 (D) 6 (E) NOTA

38. If the lines $5cx + 3y = 1$ and $2x - cy = 1$ intersect in the first quadrant, what is the sum of all possible integer values of c?

(A) 0 (B) –3 (C) –2 (D) –1 (E) NOTA

39. A strand of DNA from an alien life form is in the shape of a helix given by the parametric equations $x = 3\cos t \;\; y = 3\sin t \;\; z = 4t$ where $0 \le t \le 5$. How long is this strand?

(A) 15 (B) $10\sqrt{5}$ (C) 25 (D) $5\sqrt{10}$ (E) NOTA

40. A tunnel beneath a bridge is in the shape of a parabolic arch. The highest point is six meters above the road, and the tunnel is six meters wide at its base. What is the height, in meters, of the tallest three-meter-wide truck that can drive through this tunnel?

(A) 4 meters (B) $3\sqrt{2}$ meters (C) $\frac{9}{2}$ meters (D) $\frac{7\sqrt{2}}{2}$ meters (E) NOTA