Alpha Applications Solutions
FAMAT State Convention 2002

1) B. Use a system of equations $a(-1^2) + b(-1) + c = -6$, $a(2^2) + b(2) + c = 9$, $a(3^2) + b(3) + c = 22$. $a = 2$, $b = 3$, $c = -5$. $2 + 3 + -5 = 0$. B.
2) B. The lines form a triangular region with base from (-5,0) to (5,0) and peak at (0,5). When rotated it forms a cone with height 5 and radius 5. Volume = $(^1/_3)(\pi)(5^2)(5) = \frac{125\pi}{3}$, B.
3) D. Work = Force x Distance = $(17)\sqrt{4^2+3^2}$ = 85.00, D.
4) C. $V_{sphere} = (^4/_3)(\pi)(r^3)$. Initial radius = 6, Final radius = 12. $SA_{sphere} = 4\pi r^2$. $4\pi(12^2) - 4\pi(6^2) = 432\pi$, C.
5) C. Use the distance formula for (3,8), (3,4) to find the radius = 4, center at a,b. $a = 3$, $b = 4$, $r = 4$. 11, C.
6) A. The (x-2) factor shifts the domain right 2, and "+1" shifts the range up one. Thus D:[0,7],R:[0,8], A.
7) C. $\cos(\theta) = \frac{P \bullet Q}{\|P\|\|Q\|} = \frac{-7}{\sqrt{13}\sqrt{26}} = \frac{-7\sqrt{2}}{26}$. $\theta = 112.380^o$, C.
8) C. sin(x) approaches x as x approaches zero. Thus, the quotient of 1, C.
9) A. The possible roots are ± (1,2,3,5,6,10,15,30). Synthetic div. shows the roots to be -3,1,2,5. - 3 + 1 = -2,A.
10) A. After the first two marbles are pulled, the marbles remain are 4 R, 2 G, 4 Bu, 3 Bk. Thus, the prob. of the third red or green = $^4/_{13} + ^2/_{13} = ^6/_{13}$, A.
11) C. The initial height, h(0) = 2. Thus $2 = 11t - \frac{2}{3}t^2 + 2$. Solving yields, $t = 0$, $^{33}/_2 = 16.5$, C.
12) A. The horizontal asym., $y = a$, can be determined by end behavior. The equal degrees for the numerator and denominator yield $a = 1$. The vertical asym. are the roots of the denominator, 2, -1, -3. -1, A.
13) B. Definition of a limacon, B.
14) C. The linear function can be generated with the slope of (3,18), (-2,17). $m = ^1/_5$. $y = ^1/_5 x + b$. Use (3,18) to find b = 17.4. $y = ^1/_5 x + 17.4$. $y = 17.4$ @ $x = 0$, C.
15) B. The description yields two vectors from [0,0] to [-21,7] and [-18,-12]. Thus, $\cos(\theta) = \frac{P \bullet Q}{\|P\|\|Q\|} = \frac{294}{(7\sqrt{10})(6\sqrt{13})} = \frac{7\sqrt{130}}{130}$.
Thus $\theta = 52.12^o \approx 52^o$, B.
16) A. Since the order does not matter, this reduces to ${}_{35}\mathbf{C}_4 = \frac{35!}{(4!)(31!)}$ = 52360, A.
17) D. Orthogonal vectors have a dot product of zero, thus (3)(-2) + (2)(1) + (-1)(-m) = 0. m = 4, D.
18) B. The description yields three points, (0,0,0), (800,-700,0), (0,0,5280). The angle sought is formed by the vectors from (0,0,5280) to (0,0,0) and (0,0,5280) to (800,-700,0). The vectors in unit form are $0\mathbf{i} + 0\mathbf{j} - 5280\mathbf{k}$ and $800\mathbf{i} - 700\mathbf{j} - 5280\mathbf{k}$. Thus the angle formed shows $\cos(\theta) = \frac{P \bullet Q}{\|P\|\|Q\|} = \frac{27878400}{5280(20)\sqrt{72521}}$. Thus $\theta \approx 11.38^o$, B.
19) C. The description "carrying capacity" reduces the problem to the limit of P(t) as t approaches infinity. The denominator limits to 1 – 0 , 1. Thus 227, C.
20) A. Perpendiculars have a zero dot product. (-6.5)(-8) + (3)(-12) + (8)(-2) = 0, Thus perpendicular, A.
21) A. Use law of cosines to find $7^2 = 3^2 + 5^2 - 2(5)(3)\cos(\alpha)$. $\alpha = 120^o$. $\tan(120^o) = -\sqrt{3}$, A.
22) C. 92 minutes = $^{23}/_{15}$ rotations of the hand. 4 inch hand yields a circumference of 8π. $(8\pi)(^{23}/_{15}) \approx 38.54$, C.
23) B. The description yields points in Quad. I, II. The distance is the hypotenuse of a right triangle and legs of 2, 3. Law of cosines yields the length of 3.6, B.
24) D. $f(x) = 2x^2 + 2x - 1$. $f(x+h) = 2x^2 + 4hx + 2x + 2h^2 + 2h - 1$. $f(x+h)-f(x) = 4hx + 2h^2 + 2h$. Divide by "$h$" yields $4x + 2h + 2$. Limit as h goes to zero yields $4x + 2$, D.
25) A. Definition of Parabola, A.
26) C. 3sin( ) + 1 yields a sine wave of amplitude 3, shifted up one. Thus the min. value = -2, C.
27) A. Prob. = P(B)|R + P(B)|W + P(B)|B = $\left(\frac{2}{5}\right)\left(\frac{1}{3}\right)+\left(\frac{2}{5}\right)\left(\frac{1}{3}\right)+\left(\frac{1}{5}\right)\left(\frac{1}{3}\right)=\frac{1}{3}$, A.
28) A. Use Pascal's triangle. 117^{th} term is ${}_n\mathbf{C}_{116}$, so ${}_{150}\mathbf{C}_{116}$, A.
29) A. Center at (1, -1). a = 3, b = 2. Thus $c^2 = 9 + 4$. $c = \sqrt{13}$. Thus Foci are $(1,-1\pm\sqrt{13})$, A.
30) D. Sub. x = y, so $x = \frac{9}{5}x^2 - \frac{10}{9}$. Solve to find $x = \{^{10}/_9, ^{-5}/_9\}$, D.