# SOLUTIONS TO ADVANCED CALCULUS TEST

1. D $\frac{dy}{dx} = \sqrt{x^3 - 1}$;

$$L = \int_1^2 \sqrt{x^3}dx = \frac{2}{5}x^{\frac{5}{2}}\Big]_1^2 = \frac{2}{5}\left(4\sqrt{2} - 1\right)$$

2. A $\lim_{b\to\infty} -2e^{-\sqrt{x}}\Big]_1^b = \frac{2}{e}$

3. B Use integration by parts twice to obtain:

$$\int \sin(\ln x)dx = x\sin(\ln x) - (x\cos(\ln x) + \int \sin(\ln x)dx)$$

$$\int \sin(\ln x)dx = \frac{x\sin(\ln x) - x\cos(\ln x)}{2}\Big]_1^2 =$$

$$\sin(\ln 2) - \cos(\ln 2) + \frac{1}{2}$$

4. B

$$\frac{5x+3}{x(x-3)(x+1)} = \frac{A}{x} + \frac{B}{x-3} + \frac{C}{x+1}$$

$$5x + 3 = A(x-3)(x+1) + Bx(x+1) + Cx(x-3)$$

$$A = -1; B = -1/2; C = 3/2$$

$$-\int \frac{1}{2}dx - \frac{1}{2}\int \frac{1}{x+1}dx + \frac{3}{2}\int \frac{1}{x-3}dx =$$

$$-\ln|x| - \frac{1}{2}\ln|x+1| + \frac{3}{2}\ln|x-3| + C =$$

$$\ln\left(\frac{|x-3|}{|x|\sqrt{x+1}}\right)^{\frac{3}{2}} + C$$

Evaluating from 1 to 2 gives $-\frac{1}{2}\ln 48$

5. C Use L'Hopital's rule to get

$$\lim_{x\to 0} \frac{1}{-3\sin^2 x + \cos x \cdot 3\cos x} = \frac{1}{3}$$

6.C

$$A = \lim_{b\to 0^+} \int_b^1 \left(\frac{1}{x} - \frac{1}{x\left(x^2+1\right)}\right)dx = \lim_{b\to 0^+} \int_b^1 \frac{x}{x^2+1}dx =$$

$$\lim_{b\to 0^+}\left(\frac{1}{2}\ln\left(x^2+1\right)\Big]_b^1\right) = \lim_{b\to 0^+}\left(\frac{1}{2}\ln 2 - \frac{1}{2}\ln(b^2+1)\right) = \frac{\ln 2}{2}$$

7. A

$$\frac{1}{1-t^2} = 1 + t^2 + t^4 + t^6 + \ldots$$

$$\frac{2}{1-t^2} = 2 + 2t^2 + 2t^4 + 2t^6 + \ldots$$

$$\int_0^x \frac{2}{1-t^2}dt = 2t + \frac{2}{3}t^3 + \frac{2}{5}t^5 + \frac{2}{7}t^7 + \ldots\Big]_0^x =$$

$$2x + \frac{2}{3}x^3 + \frac{2}{5}x^5 + \ldots$$

8. B

$$a_0 = 1, a_1 = \frac{2}{1}\cdot 1, a_2 = \frac{2}{2}\cdot\frac{2}{1}\cdot 1, a_3 = \frac{2}{3}\cdot\frac{2}{2}\cdot\frac{2}{1}\cdot 1, \ldots, a_n = \frac{2^n}{n!}$$

$$\sum_{n=0}^{\infty} a_n x^n = \sum_{n=0}^{\infty}\frac{(2x)^2}{n!} = e^{2x} = f(x)$$

$$f'(1) = 2e^2$$

9. A

$f(x) = e^{\sin x}$ $\quad f(0) = 1$

$f'(x) = \cos x \cdot e^{\sin x}$ $\quad f'(0) = 1$

$f''(x) = \cos^2 x \cdot e^{\sin x}$ $\quad f''(0) = 1$

$P_2(x) = 1 + x + \frac{x^2}{2}$ $\quad P(0.5) = 1.625$

10. A The series is geometric with $a = 1$ and $r = \frac{\sqrt{x}}{2} - 1$.

$S = \frac{1}{1-r} = \frac{2}{4-\sqrt{x}}$ on $(0,16)$ since $|r| < 1$.