Limits & Derivatives
FAMAT State Convention 2002

1. Given $y = \sqrt{x^3}$, what is $y'''(4)$?

   a) -4 b) $\frac{-3}{64}$ c) $\frac{-1}{48}$ d) $\frac{1}{6}$ e) NOTA

2. $\lim_{h\to 0} \frac{\sec\left(\frac{p}{3}+h\right)-2}{h}$

   a) $\frac{1}{2}$ b) $\sqrt{3}$ c) 2 d) $2\sqrt{3}$ e) NOTA

3. $\lim_{x\to 0} \frac{-\cos x + 1 - \sin x}{x}$

   a) –1 b) 0 c) undefined d) 1 e) NOTA

4. If $g(x) = \csc x - \cot x$, then $g'\left(\frac{p}{6}\right) =$

   a) $4-2\sqrt{3}$ b) $2-2\sqrt{3}$ c) $2-\sqrt{3}$ d) 1 e) NOTA

5. $y' = \frac{4x}{y}$ and $x = -1$ when $y = 4$. What can x be when $y = 6$?

   a) –6 b) $-\sqrt{6}$ c) –2 d) 2 e) NOTA

6. What is the slope of the tangent line to the curve $y = \cos^2(3x)$ at the point $\left(\frac{p}{4}, \frac{1}{2}\right)$?

   a) –3 b) ½ c) 0 d) 2 e) NOTA

7. $\frac{d}{dx}\left(\int_0^{\frac{x}{2}} \sin(t)dt\right) =$

   a) $\frac{1}{4}\sin\left(\frac{x}{2}\right)$ b) $\frac{1}{2}\sin\left(\frac{x}{2}\right)$ c) $\sin\left(\frac{x}{2}\right)$ d) $2\sin\left(\frac{x}{2}\right)$ e) NOTA

8. If $f(x) = 5x^4 - 2x^3 + 3$ and $g(x) = f^{-1}(x)$, then $g'(10) =$

a) –26 b) $\frac{-1}{26}$ c) 3 d) 19,400 e) NOTA

9. $\lim_{x \to 0} \frac{\sin^2(3x)}{x^2} =$

a) 0 b) 1 c) 3 d) 9 e) NOTA

10. Let $f(x)$ be a function with a continuous derivative on the interval (0,5) such that $f'(0) = 3$, $f'(1) = 2$, $f'(2) = -3$, $f'(3) = -4$, $f'(4) = 1$. Which of the following must be true about $f(x)$?

I. $f(x)$ has a critical point between x = 1 and x = 2
II. $f(x)$ has a critical point between x = 0 and x = 1
III. $f(x)$ has a critical point between x = 2 and x = 3

a) I only b) III only c) I & II only d) I & III only e) NOTA

11. On what interval is the function $y = \frac{6x}{x^2 + 9}$ increasing?

a) $(-\infty, 0)$ b) $(-3, 0)$ c) $(0, \infty)$ d) (0, 3) e) NOTA

12. Let $f(x) = \frac{x}{x+1}$. Which of the following are true?

I. $f(x)$ has exactly one local maximum
II. $f(x)$ has a point of inflection at x = 0
III. $f(x)$ has a vertical asymptote at x = –1

a) I only b) III only c) I & III only d) II & III only e) NOTA

13. At what points does the function $x^3 - 3xy + y^2 = 0$ have a horizontal tangent line?

a) (0, 0) only b) (2, 4) only c) (2, 2) only d) (0, 0) & (2, 4) only e) NOTA

14. What is the slope of tangent line to the curve $y = x\tan x$ when $x = \frac{p}{4}$ (to the nearest 100^{th})?

a) 1.39 b) 1.79 c) 2.57 d) 2.79 e) NOTA

15. A spherical balloon is being blown up, with air entering at a constant rate. Let $r(t)$ be the radius of the balloon. Which of the following is positive?

I. $r(t)$ II. $r'(t)$ III. $r''(t)$

a) I only b) II only c) III only d) I and II only e) NOTA

16. If $f(x) = \cos(x-1)e^{x^2-1}$, what is $f'(1)$?

a) 2 b) 1 c) –1 d) –2 e) NOTA

17. Where does the function $\ln(x^2 + 2x + 2)$ have a point of inflection?

a) At x = -2 & 0 only b) At x = 1 & -1 only c) At x = 0 only d) At x = -2 only e) NOTA

18. If the function $f(x)$ is both continuous and differentiable = $\begin{cases} ax^3 - 6x, & \text{if } x \le 1 \\ bx^2 + 4, & \text{if } x > 1 \end{cases}$ then $a =$

a) 0 b) 1 c) –14 d) –24 e) NOTA

19. Find a positive value c, for x, that satisfies the conclusion of the Mean Value Theorem for Derivatives of $f(x) = 3x^2 - 5x + 1$ on the interval [2, 5].

a) 1 b) $\frac{13}{6}$ c) $\frac{11}{6}$ d) $\frac{23}{6}$ e) NOTA

20. If $f(x) = \sqrt{1+\sqrt{x}}$, find $f'(x)$.

a) $\frac{-1}{4\sqrt{x}\sqrt{1+\sqrt{x}}}$ b) $\frac{1}{2\sqrt{x}\sqrt{1+\sqrt{x}}}$ c) $\frac{1}{4\sqrt{1+\sqrt{x}}}$ d) $\frac{1}{4\sqrt{x}\sqrt{1+\sqrt{x}}}$ e) NOTA

21. If $7 = xy - e^{xy}$, then $\frac{dy}{dx} =$

a) $x - e^{y}$ b) $y - e^{x}$ c) $\frac{ye^{xy} + y}{x - xe^{xy}}$ d) $\frac{-y}{x}$ e) NOTA

22. If $y = 5x^2 + 4x$ and $x = \ln t$, then $\frac{dy}{dt} =$

a) $\frac{10}{t} + 4$ b) $10t \ln t + 4t$ c) $\frac{10 \ln t + 4}{t}$ d) $\frac{5}{t^2} + \frac{4}{t}$ e) NOTA

23. $\lim_{x \to \infty} 4x \sin\left(\frac{1}{x}\right)$ is

a) 0 b) 2 c) 4 d) $4p$ e) NOTA

24. The graph of $y = (x^2 - 1)^3$ has an absolute minimum at

a) $x = \frac{-1}{\sqrt{5}}$ b) $x = -1$ c) $x = \frac{1}{\sqrt{5}}$ d) x = 1 e) NOTA

25. If $xy + y^2 = 2$, find $\frac{d^2y}{dx^2}$ at (1,1).

a) $\frac{1}{3}$ b) $\frac{4}{27}$ c) $\frac{-1}{3}$ d) $\frac{-4}{27}$ e) NOTA

26. $\lim_{x \to 1} \frac{\ln x}{\tan(p x)} =$

a) 0 b) 1 c) $\frac{-1}{p}$ d) $\frac{1}{p}$ e) NOTA

27. A particle's position is given by $s(t) = 2t^3 - 12t^2 + 18t - 6$. At what time(s) is it changing direction?

a) t = 1 only b) t = 2 only c) t = 3 only d) t = 1 and t = 2 e) NOTA

28. If $y = x^{\ln x}$, then $\frac{dy}{dx} =$

a) $\frac{x^{\ln x - 1}}{\ln x}$ b) $x^{\ln x - 1} \bullet \ln x$ c) $x^{\ln x} \bullet \frac{\ln x}{x}$ d) $x^{\ln x} \bullet \frac{2\ln x}{x}$ e) NOTA

29. Evaluate: $\sum_{n=0}^{\infty} e^{-n}$

a) 1 b) $\frac{1}{e-1}$ c) $\frac{1}{e+1}$ d) $\frac{e}{e-1}$ e) NOTA

30. If $y\frac{dy}{dx} = (4 + y^2)x^2$, and $y(0) = 12$, find y^2.

a) $148e^{\frac{2x^3}{3}} + 4$ b) $148e^{\frac{2x^3}{3}} - 4$ c) $140e^{\frac{2x^3}{3}} + 4$ d) $140e^{\frac{2x^3}{3}} - 4$ e) NOTA