Euclidean Applications Answer Key FAMAT State Convention 2002

1(C) Triangle is a 16-30-34 relation. Median to hypotenuse is half of hypotenuse=17.
2(B) Solve 3x+7=-7x-1, x= -0.8; and solve 3x+7=7x+1, x=1.5: Sum= 0.7
3(B) Add 11+13+...+47+49=(20)(11+49)/2=600
4(A) Given height of triangle and rectangle=h, area triangle=.5(2a)(h) and area of rectangle=(3a)(h)
5(B) Area of each face with hole=8. Inside of each hole=4. Total area=6(4+8)
6(E 2/9) Note that triangles DGF and BHE are isosceles right triangles and share common side with square GHFE and diagonal= $\sqrt{2}$. Therefore DG=GH=BH= $\frac{\sqrt{2}}{3}$. Area of GHEF=2/9.
7(C) Change both expressions to powers of 2. Then $3x^2+9x+30=2x^2-2x$. (x=-6,-5) Sum=-11 and product=30.
8(C) Area shaded=$4x^2$-xy. Divide by x to find length
9(C) $\frac{3}{7}=\frac{x}{180-x}$, x=54, comp=36. $\frac{54}{36}=\frac{3}{2}$
10(C)There are a total of 18 angles. Only the pentagon does not have multiples of 30.
11(A) Two possibilities: 70-70-40 and 40-40-100.
12(D) Use Hero's Formula to find area=$8\sqrt{14}$. This is equal to $\frac{1}{2}(8)(h)$.
13(C)Because of parallel chords GF=HA. Thus, AD=100 and angle P=$\frac{1}{2}((110+50)-100)$.
14(D) Circumscribed circles radius=5 (half 10 which is diameter). Inscribed radius is 2 (area of triangle/semi perimeter).
15(A) Radius of sphere=6 ($\frac{4}{3}\pi r^3=288\pi$), which means height of cylinder=8 ($\pi r^2 h=288\pi$). Surface area=$36\pi+36\pi+96\pi$.
16(D) With the rope being raised 10 ft above the circumference the diameter must increase by 20. Therefore old circumference=$d\pi$ and new circumference=$(d+20)\pi$.
17(E $12+8\sqrt{2}$) If length of square=x then diagonal=x+2, but diagonal also=$x\sqrt{2}$. Therefore, x=$2+2\sqrt{2}$ and Area=$(2+2\sqrt{2})^2$
18(A)Shaded region is equivalent to 30 or $\frac{1}{12}$ of area of circle.
19(B) Triangles are similar thus if NE=x, $\frac{2}{x}=\frac{4}{10-x}$ and x=10/3.
20(B) Area=$\frac{1}{2}\begin{vmatrix}-4 & 2 & 4 & 3 & -4\\ 2 & 5 & 3 & -4 & -2\end{vmatrix}$=50.5 or you can circumscribe a rectangle about the pentagon and deal with triangular and rectangular areas.
21(B) The large triangle and the unshaded triangles are similar. If the base of the unshaded triangle is x then, $\frac{1}{2}=\frac{2/3}{x}$. Shaded area=$1-(\frac{1}{2})(\frac{4}{3})(\frac{2}{3})$.
22(B) New volume of water displaced= 2(2)(h)=1.5. Height= 0.375 feet=4.5 inches.
23(C) Consider a diagonal of a rectangular prism to represent distance from pad with height of 0.25, length 2 and width 1. Distance=$\sqrt{1+4+.625}$.
24(A) Triangles ABC and EBD are similar. Note that EDB is a 3-4-5 relationship. If x=CE, then $\frac{3}{5}=\frac{4}{x+5}$.
25(B) Roots are 2,-2, and -3 (factor by grouping). Use 2 for side.
26(C) To reflect a line over y=x, substitute x for y and y for x in original equation and solve for y.
27(C) If you triple your side the area is 9 times the original. The increase is 8 compared to 1= 800%
28(A) AE:ED=1:3 means that the height of triangle EFD=3h and the height of triangle BFC=4h (similar triangles). Thus, the height of ABCD=7h. Area of ABCD=56=7h(4x) which simplifies to xh=2. Area of EFD=.5(3h)(3x)=9.
29(B) If you connect A,B, and C you form an equilateral triangle.
30(C) Complete the square for both x and y and $(x+4)^2+(y-5)^2=36$. Thus radius=6. This corresponds to diagonals of square=12. Area=.5(12)(12)